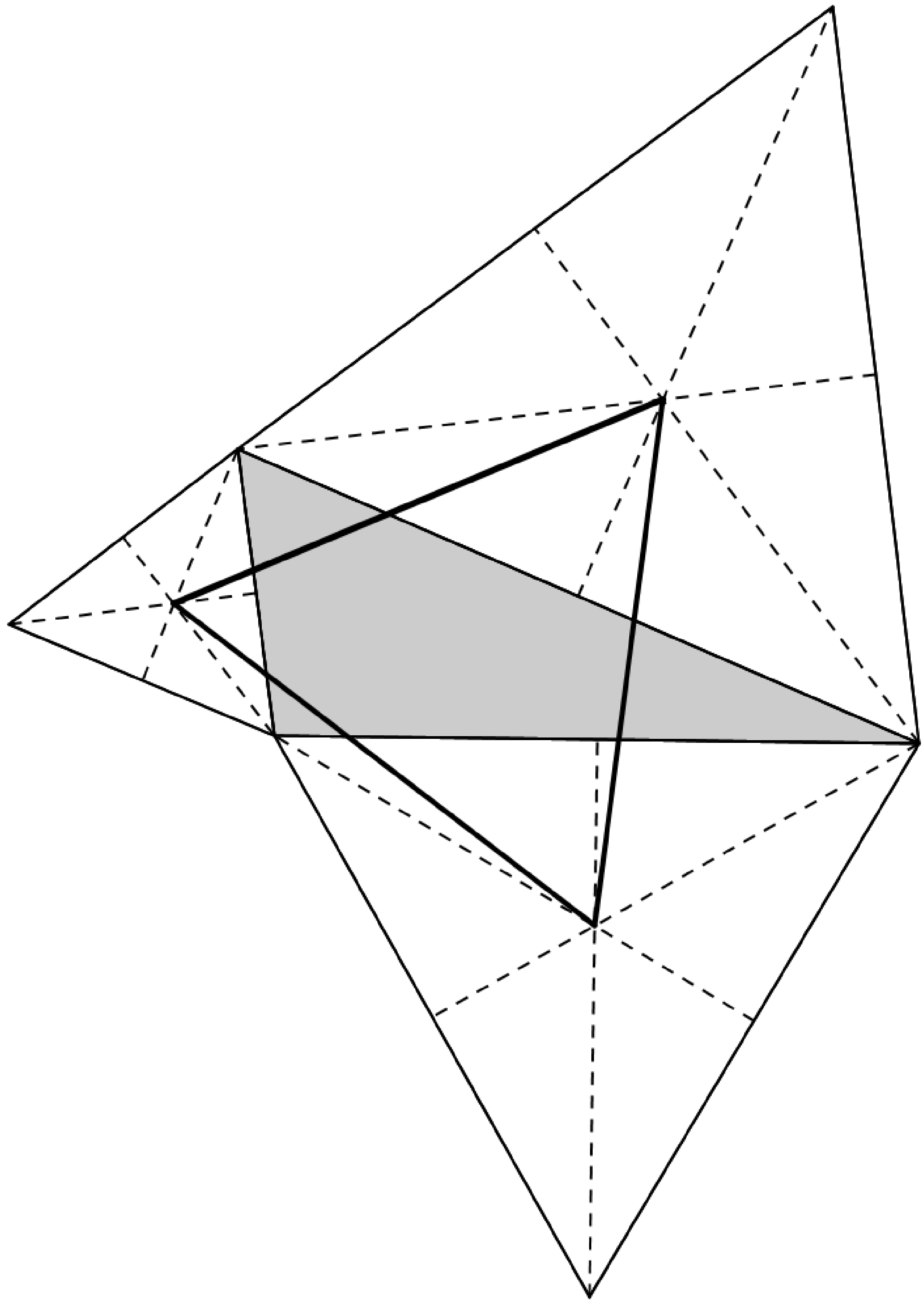
Geometry Part 1



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Advice for students:

1. This class is about problem-solving, which often involves applying material learned to unfamiliar problems. We will cover basic concepts thoroughly, practice with routine problems, and then often attack harder problems where you have to apply new and previously-learned concepts. These can be difficult, and one of my main goals this semester is to help you develop important problem-solving skills. In the face of challenges, try to be persistent, patient, and creative. Be proactive about seeking help when you need it. ***We do not expect everyone to get all homework problems correct; the important thing is to try your best!***

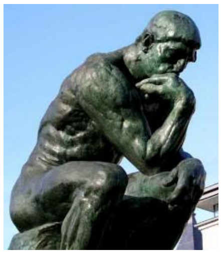
2. In sports, you have practices. In performing arts, you have rehearsals. In academics, you have homework. Homework in this class is designed to help you cement your understanding of the material by practicing straightforward problems and develop your problem-solving skills. Do your best to try all the homework questions. If you have difficulty, come to the next class with specific questions that will help you advance your understanding.

3. The problem sets in this book comes with answers. They are at the end of each individual problem set. Checking your answers is essential. We recommend that you check answers after every few problems to make sure you are on the right track. We all make mistakes; please let us know if you think you have found an error in the answers.

4. You will never need a formal textbook in class or for homework. All problems will be assigned from this book or supplemental handouts.

5. HELP!!! Get help when you need it. Some places (in no particular order):
- -Classmates
- -Parents
- -Your teacher
- -Internet (believe it or not, Google can help you find great explanations and practice problems)
- -Khan Academy videos: goto www.khanacademy.org

6. Think more… memorize less!



7. Problems with boxes around their numbers, like 17. or e. tend to be harder. We encourage those who are considering taking accelerated math courses to try them.

Note: the diagram on the cover illustrates Napoleon's Theorem, from the famed Frenchman Napoleon Bonaparte. Given any triangle (the gray one), construct equilateral triangles on each of its sides. In each of these three equilateral triangles, find the centroid—where the medians to the sides intersect. The centroids of the three triangles will form the vertices of an equilateral triangle, no matter on the shape of the original triangle!

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Geometry Unit 1 Vocabulary List

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Acute Angle

Obtuse Angle

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Coplanar

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Addition and Subtraction Properties of Equality

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Unit 1 Handout #1: Points, Lines, Planes, Segments, Rays

Fundamental shapes of geometry include ***points***, ***lines*** (and line segments), and ***planes*** (and planar regions).

-Points on the same line are ***collinear***; points in the same plane are ***coplanar***.

-Two distinct lines in a plane may be ***parallel*** or intersect in a point.

-Two lines in space may also be ***skew***—meaning they are not in the same plane and do not intersect.

-Two distinct planes may be parallel or may meet in a line.

A segment is ***bisected*** when it is cut into two equal (congruent) pieces and ***trisected*** when it is cut into three congruent pieces. The ***midpoint*** of a segment bisects that segment, by definition.

Example #1. The diagram below shows a three-dimensional object. Triangles ABC and EDF are congruent, with EDF positioned directly above ABC. They are joined by segments $\overline{EA}$, $\overline{DB}$, and $\overline{FC}$. Answer the following questions:

a. List all drawn segments parallel to $\overline{DE}$.

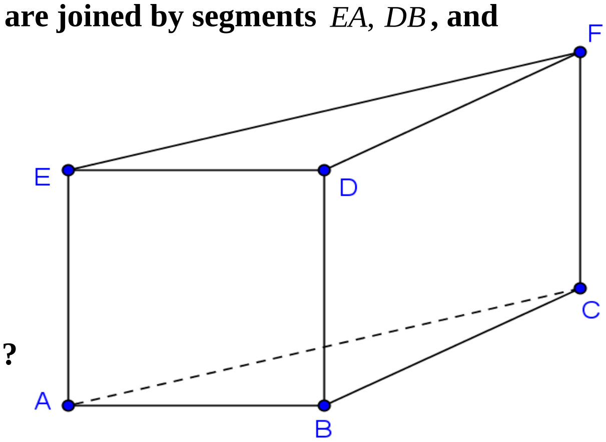
b. List all drawn segments skew to $\overline{DE}$.

c. Name all points not coplanar with B, A, and D.

d. What is the intersection of plane BCF and plane DEA?

e. What is the intersection of plane ABD and segment $\overline{FE}$?

f. What is the intersection of plane ABC and plane DEF?



Solution:

1a. $\overline{AB}$ is parallel to $\overline{DE}$ since it is in the same plane and the lines containing the segments do not intersect.

b. Skew segments are not in the same plane and do not intersect. Segments $\overline{BC}, \overline{FC},$ *and* $\overline{AC}$ are all skew.

c. E is coplanar with B, A, and D because plane BAD represents the rectangular face ABDE. But F and C are not in this plane, so they are not coplanar with points B, A, and D.

d. Plane BCF is the right face of the solid and DEA is the front face. They intersect in segment $\overline{DB}$.

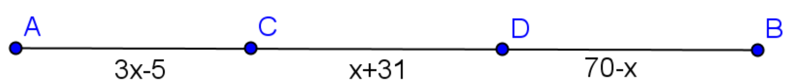
e. The line and plane intersect in point E.

f. Those planes are parallel and do not intersect.

Example #2. Given that C is the midpoint of $\overline{AD}$ answer the following:

a. What is the value of *x*?

b. Is D the midpoint of $\overline{CB}$?



Solution:

a. For C to be the midpoint of $\overline{AD}$, AC must be equal to CD, so $3x-5 = x+31$

this occurs when $2x=36$ and thus $x=18$

b. If x is 18, then CD=49 and DB=52. Since they are not equal, D is not the midpoint of $\overline{CB}$.

1. Each of the following is best defined by which undefined term (point, line, or plane). Note: "line" can also refer line segment and "plane" can also refer to planar region.

a. A grain of sand

b. The edge of a desk

c. The whiteboard

d. A sheet of paper

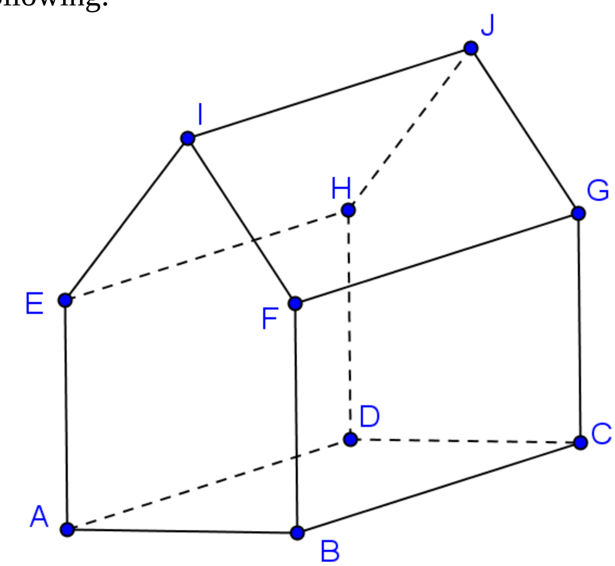
e. The corner of the roof.

f. A pencil's tip

g. The surface of the desk

h. A piece of string

2. The diagram on the right below is a hotel from the board game Monopoly. (so points A, B, F, I, and E are coplanar, as are points C, D, G, H, and J). Answer the following:



a. Are lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ parallel, skew, or neither?

b. Are lines $\overleftrightarrow{AE}$ and $\overleftrightarrow{FG}$ parallel, skew, or neither?

c. Are lines $\overleftrightarrow{CF}$ and $\overleftrightarrow{DE}$ parallel, skew, or neither?

d. Are lines $\overleftrightarrow{IF}$ and $\overleftrightarrow{AE}$ parallel, skew, or neither?

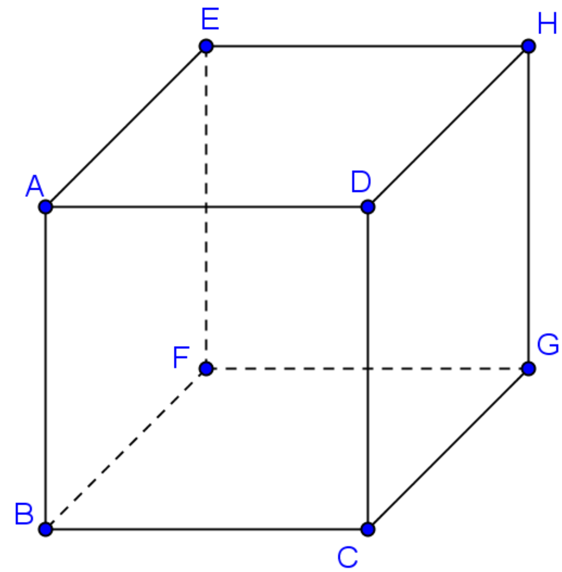
e. Are lines $\overleftrightarrow{EF}$ and $\overleftrightarrow{CD}$ parallel, skew, or neither?

f. What is the intersection of planes FGJ and BCG?

g. Are planes ADHE and FGJI parallel?

h. Name one plane parallel to ADHE.

3. Given the cube on the right, name the following:



a. A plane parallel to CDH

b. A point coplanar with C, F, and E

c. The intersection of planes BFC and ABF

d. The intersection of segment $\overline{CD}$ with plane ADH

e. Two planes that intersect in $\overline{EH}$

f. A point coplanar with segment $\overline{CG}$ and point A

g. All segments in the diagram parallel to segment $\overline{DH}$.

h. All segments in the diagram perpendicular to $\overline{DH}$.

i. All segments in the diagram parallel to segment $\overline{DE}$.

j. All segments (drawn) skew to segment $\overline{BF}$.

4. Sketch and Label

a. Points A, B, and C are collinear and A, B, C, and D are coplanar

b. Line *m* intersects plane P at point *x*

c. Plane *m* contains intersecting lines *x* and *y*

d. Planes P and Q intersect at line $\overleftrightarrow{AB}$.

e. $\overline{CD}$ and $\overline{RS}$ do not intersect but $\overleftrightarrow{CD}$ and $\overline{RS}$ do.

5. Fill in each blank with either "sometimes", "always", or "never".

a. Two lines that intersect in one point are ____ coplanar

b. Two different ("distinct") lines ___ intersect in more than one point

c. Two planes ____ intersect in a point

d. Two points are __ collinear

e. Three points are ___ coplanar

f. Four points are ___ coplanar.

g. A plane containing one point of a line ___ contains the whole line

h. A plane containing two points of a line ___ contains the whole line

i. Two parallel lines are ___ coplanar

j. Two skew lines are __ coplanar

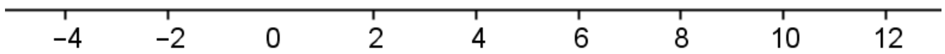
k. Two lines in parallel planes are __ parallel

l. Two lines parallel to the same line are __ parallel

m. Two lines perpendicular to the same line are __ parallel

n. A line not in a plane ____ intersects it exactly one time.

6. On a number line, A is located at -3 and B is located at 11. Point C is between A and B.



a. What is the sum of the lengths of $\overline{AC}$ and $\overline{BC}$?

b. Write an inequality showing possible locations of point C.

c. If C is the midpoint of $\overline{AB}$ then where is it located?

7. Given that points E, F, and G are collinear and that $EF = 10$ and $EG = 12$:

a. Give one possible length of $\overline{FG}$.

b. Is there more than one possible answer to part *a*? Explain.

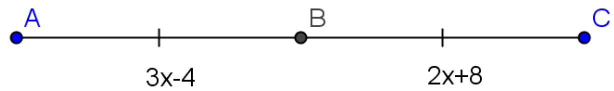
8. A, B, C, and D are in that order on a line. If $\overline{AB} \cong \overline{BD}$ and $\overline{BC} \cong \overline{CD}$ and $BC = 5$ then find the following:

a. *BA*

b. *AD*

c. The ratio of *CD* to *AC*

9. In the diagram below, points B bisects segment $\overline{AC}$. ***This means it divides it into two segments of equal length. The "slashes" on the segments indicate this.*** Find the length of $\overline{AC}$.



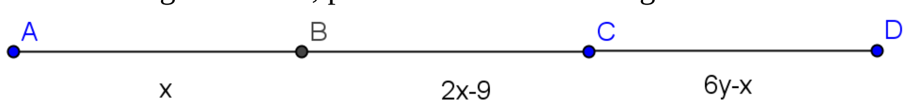
10. B is between A and C on segment $\overline{AC}$. If AB = 2(BC), then find:

a. The ratio of AC to BC.

b. The ratio of AC to AB.

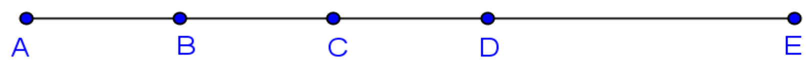
11. A triangle's perimeter is 60 and its sides are in the ratio 2:3:4. What is the length of the longest side? Hint: call its sides 2*x*, 3*x*, and 4*x*. This often works well when dealing with proportions.

12. In the diagram below, points B and C trisect segment $\overline{AD}$. Find the values of *x* and *y*:

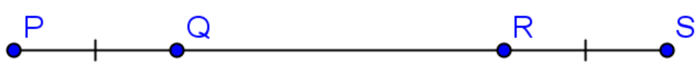


13. Draw a diagram where line segment $\overline{AB}$ bisects segment $\overline{CD}$; $\overline{CD}$ and point E are trisect $\overline{AB}$.

14. In line segment $\overline{AE}$ below, B and C trisect segment $\overline{AD}$ and D bisects $\overline{BE}$. If AB=6, find AE.



15. On the line segment below, PR=4x-1, QS=3x+6, and QR=2x+5. Find RS. Note that $\overline{PQ} \cong \overline{RS}$.



16. Draw a diagram where point C is the bisector of $\overline{AB}$ and also the bisector of $\overline{DE}$, where $\overline{DE}$ is longer than $\overline{AB}$ if:

a. A, B, D, and E are collinear.

b. A, B, D, and E are not collinear.

17. A, B, and C are points on a number line. AB = x cm and BC = y cm.

a. How long is $\overline{AC}$ if B is between A and C?

b. How long is $\overline{AC}$ if C is between A and B?

18. On a number line, find the coordinates of the point that is:

a. 2/3rd of the way from -7 to 17

b. 2/3rd of the way from 4 to m.

c. 2/3rd of the way from m to n.

d. w% of the way from m to n.

Answers

1a. point b. line c. plane d. plane e. point f. point g. plane h. line

2a. parallel b. skew c. parallel d. neither e. parallel f. line FG g. no h. plane BCGF

3a. BAEF b. D c. $\overline{BF}$ d. point D e. ADHE and FGHE f. E g. $\overline{AE}$, $\overline{BF}$, $\overline{CG}$

h. $\overline{CD}$, $\overline{GH}$, $\overline{AD}$, $\overline{HE}$, i. $\overline{CF}$ j. $\overline{CD}$, $\overline{GH}$, $\overline{AD}$, $\overline{HE}$

5a. always b. never c. never d. always e. always f. sometimes g. sometimes h. always i. always j. never k. sometimes l. always m. sometimes (always if in the same plane) n. sometimes

6a. 14 2b. -3<C<11 c. 4 7a. 2 or 22 b. yes: F can be between E and G or E can be between F & G

8a. 10 b. 20 c. 1:3 9. 64 10a. 3:1 b. 3:2 11. 80/3 12. x=9 and y=3 14. 30

15. x=7 so RS=8 17a. x+y b. x-y 18a. 9 b. $4+\frac{2}{3}(m-4)$ c. $m+\frac{2}{3}(n-m)$ d. $m+\frac{w}{100}(n-m)$

Unit 1 Handout #2: Angles; Intersection and Union

An ***angle*** is defined as the union of two line segments or rays with a common endpoint, called the vertex.

Note: ***union*** ($\cup$) is like the mathematical "or". The union of two sets is anything in ***at least one*** set (including those in both sets)

The ***intersection*** ($\cap$) is the mathematical "and". The intersection of two sets is anything in both sets.

There are many types of angles:

-***Obtuse*** angles measure more than 90°; ***acute*** angles measure less than 90°; ***right*** angles measure 90°.

-Two angles are ***complementary*** if they add up to 90° and are ***supplementary*** if they sum to 180°.

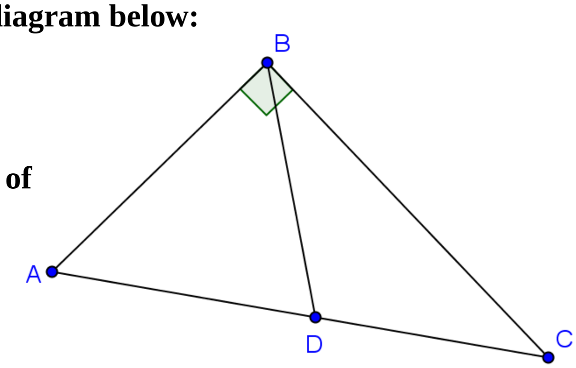
-Two pairs of ***vertical*** angles are created by two intersecting lines or line segments; the angles in each pair are congruent.

Example #1: Answer the following questions based on the diagram below:

a. Angle ACB is the union of what two segments?

b. What is $\overline{AC} \cap \overline{CD}$? $\overline{AC} \cup \overline{CD}$?

c. The ratio of the measure of angle ABD to the measure of angle DBC is 3:2. What is the measure of $\angle DBC$?



Solution

a. $\overline{AC}$ *and* $\overline{CB}$. They have a common endpoint C.

b. The intersection is all points in both sets, which is $\overline{CD}$. The union is all points in at least one set, which is $\overline{AC}$.

c. Call them 3*x* and 2*x*. We know, since $\angle ABC$ is a right angle, that 3*x*+2*x*=90 so *x*=18 and $\angle DBC$ measures 2*x*, or 36°.

Example #2: The supplement of an angle is 20° less than three times its complement. Find the angle.

Solution

-First translate the sentence into a mathematical equation: Supplement = 3*Complement - 20

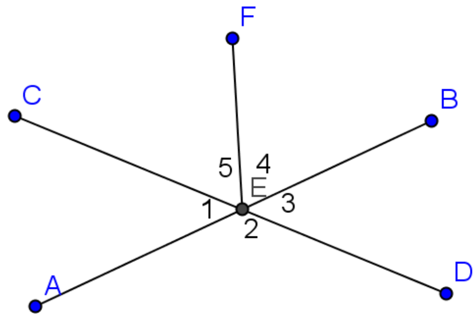
-Now define *x* as the angle. Thus the supplement is (180°-*x*) and the complement is (90°-*x*).

-Substitute these into the equation: $180 - x = 3(90 - x) - 20$

-Solve the equation: $180 - x = 270 - 3x - 20$ so $2x = 70$ and $x = 35^\circ$.

-Check: 145 = 3*(55)-20.

Example #3: Segments $\overline{AB}$ and $\overline{CD}$ intersect at E. $\overline{EF}$ bisects $\angle CEB$. $\angle 3$ measures 40°. Find the measure of all other numbered angles.



Solution

Angles 1 and 3 are vertical angles, so are congruent (they are both supplementary to angle 2) so angle 1 measures 40°. Angle 2 is a supplement of angle 3 so it measures 140°. Angle BEC also measures 140° since it is vertical to angle 2. Since it is bisected, angles 4 and 5 are congruent and each measure 70°.

1. If A is the set of all green things and B is the set of all fruits and vegetables then answer the following:

a. Is a ripe orange in $A \cap B$?

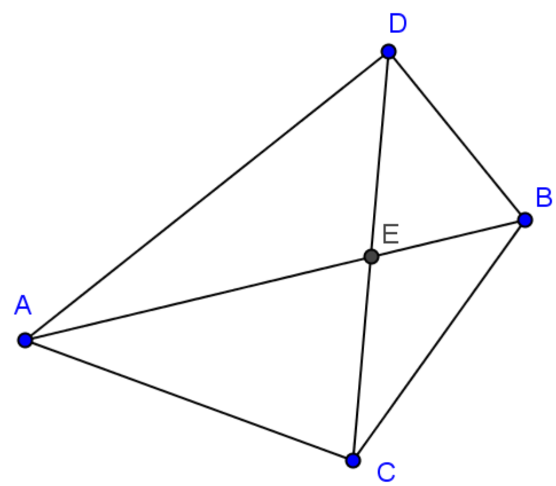
b. Is a ripe orange in $A \cup B$?

c. Is a basketball in $A \cup B$?

d. Name three things in $A \cap B$.

e. Name three things in $A \cup B$ that are not in $A \cap B$.

2. Answer the following questions about the diagram below. It is in one plane:



a. How many different triangles are there?

b. Give all other names for $\angle CBA$.

c. What is $\overline{AC} \cap \overline{CB}$?

d. What is $\overline{AC} \cup \overline{CB}$?

e. What is $\overline{AB} \cap \overline{EB}$?

f. What is $\overline{AB} \cup \overline{EB}$?

3. Identify each statement below as always true, sometimes true, or never true:

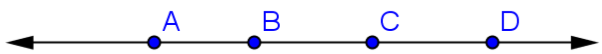
a. Line $\overleftrightarrow{AB}$ is the same as $\overleftrightarrow{BA}$

b. Ray $\overrightarrow{AB}$ is the same as $\overleftarrow{BA}$

c. Ray $\overrightarrow{AB}$ is the same as $\overleftarrow{AB}$

d. Two rays intersecting create exactly one angle.

4. Use the points on the line below to answer the following questions:



a. What is $\overline{AD} \cap \overline{BC}$?

b. What is $\overline{AC} \cup \overline{BD}$?

c. What is $\overrightarrow{BC} \cap \overrightarrow{CD}$?

d. What is $\overrightarrow{BC} \cup \overrightarrow{CD}$

e. Is it true that $\overrightarrow{BD} \cup \overrightarrow{CA} = \overleftrightarrow{BD}$?

5. For each part below, draw a diagram where the statement is true. Or state that it is not possible. The points R, S, C, and D do not need to be collinear!

a. $\overline{RS} \cap \overline{CD} = \overline{RS}$

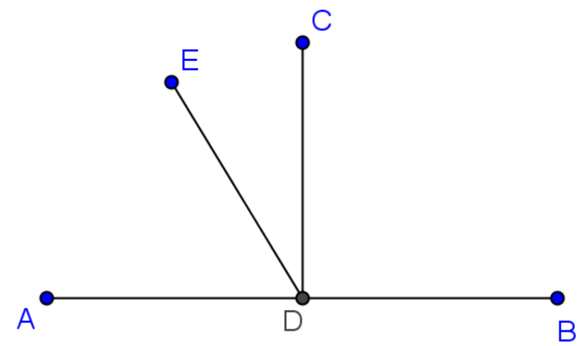
b. $\overline{RS} \cap \overline{CD} = \overline{CR}$

c. $\overline{RS} \cup \overline{CD} = \overline{RC}$

d. $\overline{RS} \cap \overline{CD} = R$

e. $\overline{RS} \cup \overline{CD} = \overline{CD}$

6. Given that $\overline{CD} \perp \overline{AB}$, name the following:



a. A right angle

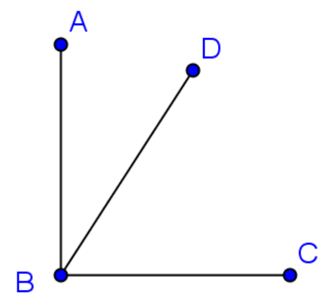
b. An acute angle

c. An obtuse angle

d. Two complementary angles

e. Two supplementary angles that are not congruent

7. In the diagram below, angle ABC is a right angle. The ratio of the measure of $\angle DBC$ to $\angle DBA$ is 5:3. What is the measure of $\angle DBC$?



8. An angle is twelve degrees more than twice its supplement.

a. Let x be the angle. What is the supplement (in terms of x)?

b. Write an equation that describes the given scenario.

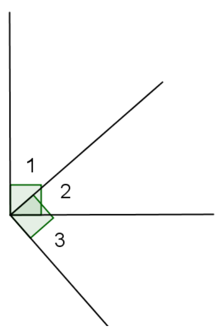
c. Solve it to find the angle.

9. When $\angle ABC$ is bisected, one resulting angle measures 8° more than one third of the measure of $\angle ABC$. What is the measure of $\angle ABC$? Hint: let x represent the measure of $\angle ABC$.

10. Can an angle's supplement be equal to twice its complement? Explain.

11. Angles A and B are complementary. The sum of A's supplement and twice B's supplement equals 402 degrees. Find the measure of angles A and B. Hint: you may want to write two equations involving A and B and solve the system.

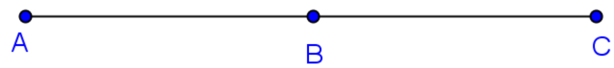
12. Angle 1 is complementary to angle 2; angle 3 is also complementary to angle 2 (see diagram.) Does that mean that angles 1 and 3 must be congruent? Explain briefly.



13. Two supplementary angles differ by 40 degrees. What is the measure of the larger one? Two supplementary angles differ by *n* degrees. What is the measure of the larger one?

14. Two rays with endpoints at point B trisect straight angle ABC. $\overrightarrow{BE}$ is one of them, and $m\angle ABE < m\angle EBC$. Also, $\overrightarrow{BG}$ is a bisector of angle ABE. Do the following:

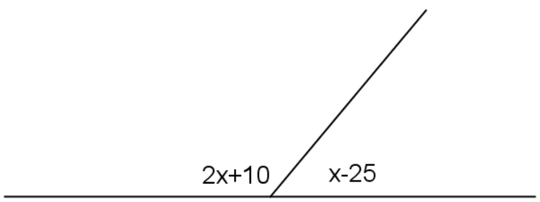
a. Draw a diagram that represents this.



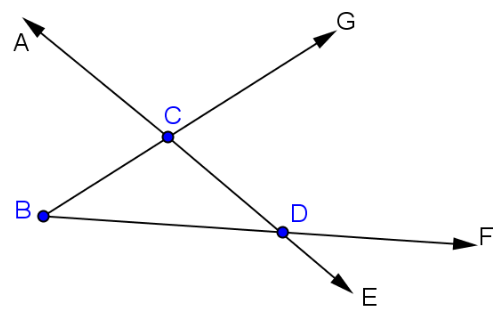
b. Find the measure of $\angle$ EBC.

c. Find the measure of $\angle$ GBC.

15. Find the value of *x* below.



16. Use the diagram below, to answer the following questions:



a. Angle B is the union of what two things?

b. What is $\overrightarrow{DA} \cap \overrightarrow{BG}$?

c. What is $\overrightarrow{DA} \cap \overrightarrow{CE}$? (it looks like "dance"!)

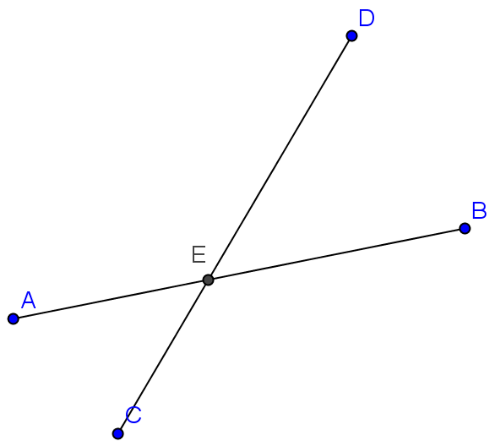
d. What is $\overrightarrow{DA} \cup \overrightarrow{CE}$?

e. If CD=*x* and CB=0.5(CD) and BD=1.25(CD) and the perimeter of BCD is 30, then how long is BD?

f. If the measure of angle EDB is 30 degrees more than twice the measure of angle CDB, then find the measure of EDB.

17. Segments $\overline{AB}$ *and* $\overline{CD}$ intersect at E.

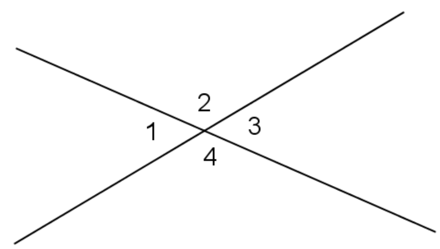
a. Given that ∠DEB measures 65°, find the measures of angles BEC and CEA.



b. Given that ∠DEB measures *x*°, find the measures of angles BEC and CEA.

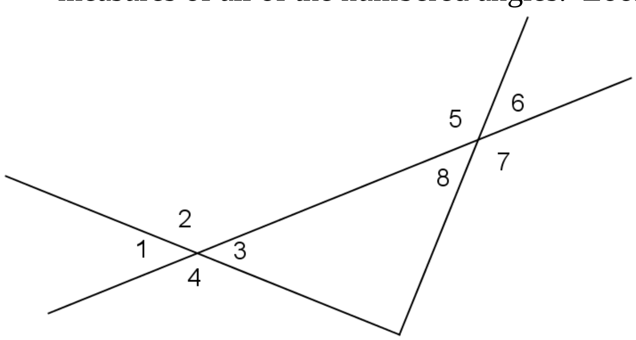
18. The diagram below shows four angles created when two line segments intersect.

a. Name the vertical angles.

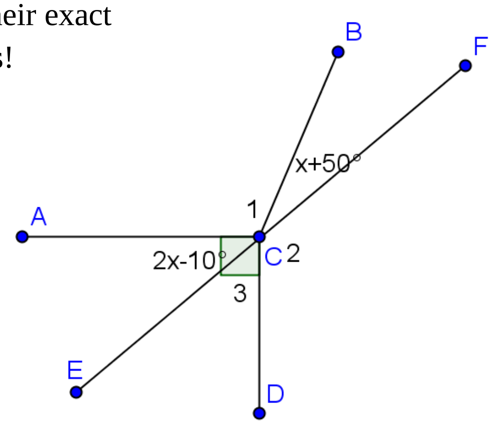


b. Must it be the case that angles 2 and 4 are congruent? Justify your answer carefully.

19. In the diagram below, angles 3 and 8 are complements and the measure of angle 5 is 140°. Find the measures of all of the numbered angles. Look for supplementary angles!



20. In the diagram below, C is on $\overline{EF}$. Find the measures of angles 1, 2, and 3 in the diagram below ***in terms of x***. Note: if you think you can find their exact measure, then either you or I have goofed! Watch your parentheses!



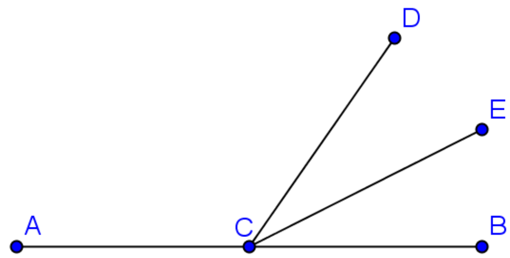
21. Find the following in terms of *x*.

a. The complement of an angle measuring 2*x*-18°.

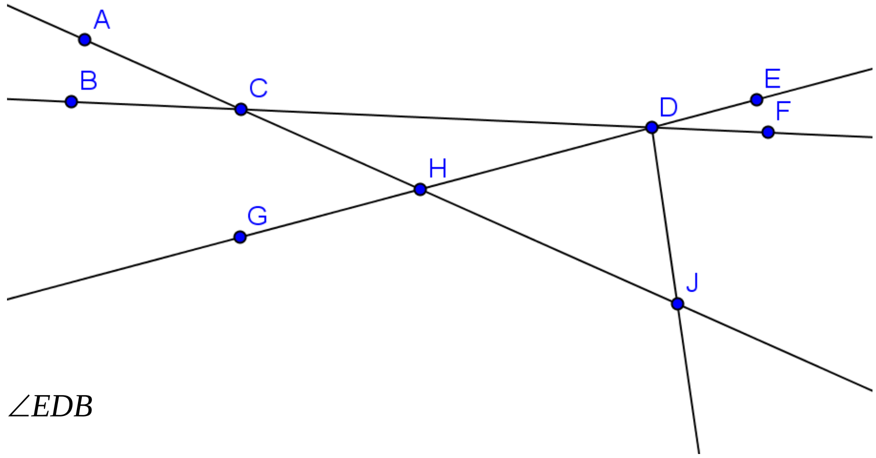
b. The supplement of an angle measuring 120-3*x*.

c. The measure of one angle created when an angle of 5x+40 is bisected.

22. Points A, B, and C are collinear. $\overline{CE}$ bisects $\angle BCD$. Angle DCB measures 3x+40; find the measure of angles DCE, ACD, and ACE in terms of *x*.



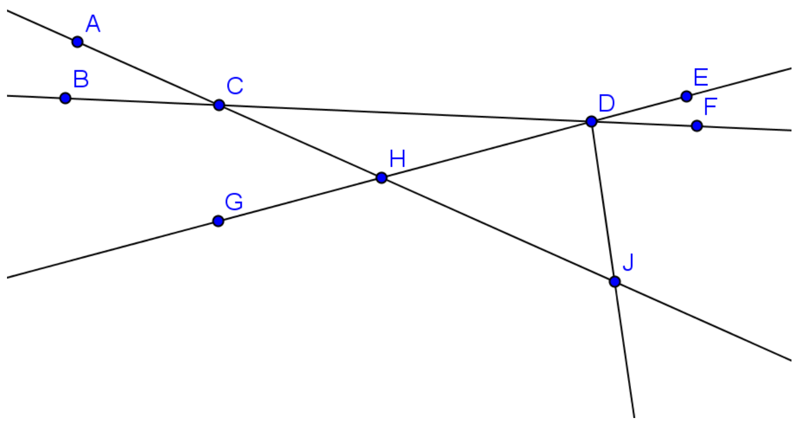
23. The diagram below includes $\overline{GE}, \overline{BF}, and\ \overline{AJ}$. Given $\angle HCD \cong \angle HDC$ and $\overrightarrow{DJ}$ bisects $\angle HDF$.
If $\angle HCD$ measures 40° then find the following:



a. $\angle EDF$ b. $\angle EDB$

c. $\angle FDJ$ d. $\angle ACD$ e. $\angle BDJ$

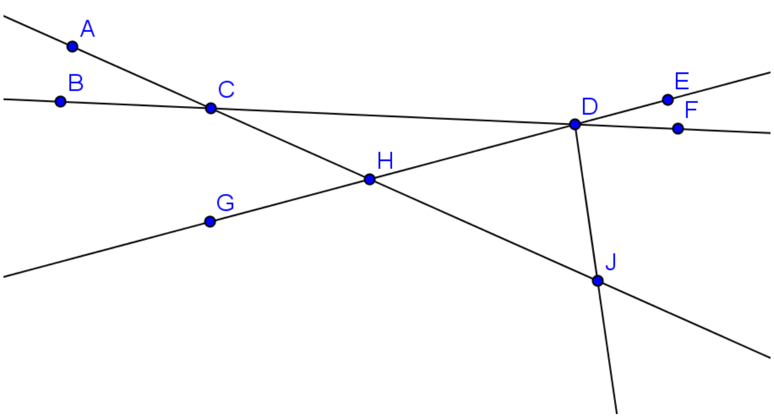
24. Repeat question 23 assuming that $\angle HCD$ measures x° instead of 40°. Your answers should all be in terms of x.



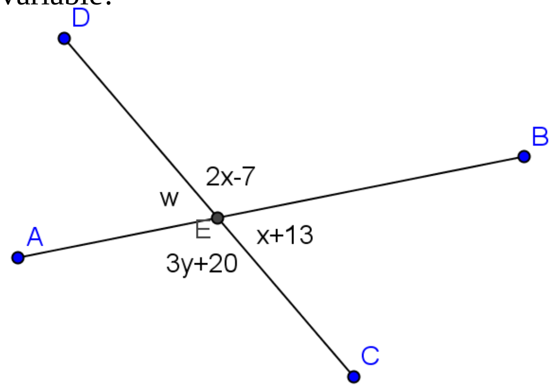
a. $\angle EDF$ b. $\angle EDB$

c. $\angle FDJ$ d. $\angle ACD$ e. $\angle BDJ$

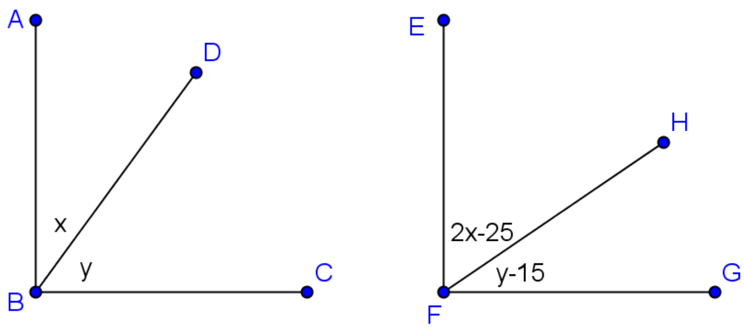
25. Same diagram: now assume that $\angle FDJ$ measures x°. Find the measure of $\angle ACD$ in terms of x.



26. Two line segments intersect at point E. Find the values of *w*, *x*, and *y* below. Before writing too many equations, see if there is an easy way to find the value of one variable!



27. In the diagrams below, $\overline{AB} \perp \overline{BC}$ *and* $\overline{EF} \perp \overline{FG}$. Find the measures of angle EFH. Hint: find *x* and *y* by writing two equations and algebraically solving the system.



28. In the clock below, the hands move continuously (note that the hour hand is between 11 and 12—two-thirds of the way to 12 because it is 40 minutes after the hour).

a. How many degrees does the minute hand advance in 10 minutes?

b. How many degrees does the hour hand advance in 10 minutes?

c. What is the (smallest) angle that the hour hand and minute hand make with each other at 4:00?

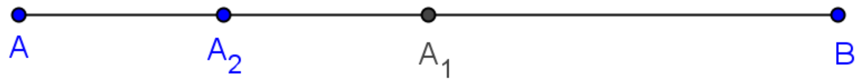
d. What is the (smallest) angle that the hour hand and minute hand make with each other at 11:40?

e. What is the (smallest) angle that the hour hand and minute hand make with each other at 6:30?

f. Give all times between 7 and 8 where the minute hand and hour hand point in the same direction.

g. Give all times between 4 and 5 that the hour hand and minute hand make right angles with each other.

29. On the number line below, $\overline{AB}$ measures 30 units. A_1 is the midpoint of $\overline{AB}$, and A_2 is the midpoint of $\overline{AA_1}$. Additional points are added, so A_3 is the midpoint of $\overline{AA_2}$, A_4 is the midpoint of $\overline{AA_3}$.



a. How long is $\overline{AA_5}$?

b. For any positive integer *n*, how long is $\overline{AA_n}$? (answer in terms of *n*)

c. For any integer *n>4*, how long is $\overline{A_nA_{n-4}}$? (answer in terms of *n*; simplify)

Answers

1a. no b. yes c. no (unless it is green) d. cucumber, zucchini, spinach e. radish, lemon, sprite bottle

2a. 8 b. $\angle CBE$, $\angle EBC$, $\angle ABC$ c. C d. $\angle ACB$ e. $\overline{EB}$ f. $\overline{AB}$

3a. always b. always c. never d. sometimes (if they have the same endpoint)

4a. $\overline{BC}$ b. $\overline{AD}$ c. $\overrightarrow{CD}$ d. $\overrightarrow{BC}$ e. yes

6a. $\angle ADC$ or $\angle BDC$ b. $\angle ADE$ or $\angle EDC$ c. $\angle EDB$ d. $\angle ADE$ & $\angle EDC$ e. $\angle ADE$ & $\angle EDB$

7. 56.25° 8a. 180-x b. x=2(180-x)+12 c. 124° 9. 48° 10. Only if it measures 0°

11. A=42° and B=48° 12. yes; if the original angle is x, then they are both 90°-x so they are the same

13. 110°; 90+0.5n 14b. 120° c. 150° 15. 65°

16a. rays BG & BF (other names are OK) b. C c. $\overline{CD}$ d. $\overline{AE}$ e. x=120/11; BD=150/11 f. 130°

17a. 115° and 65° b. 180°-x; x° 18a. 1&3; 2&4 b. yes; both are supplementary to 1

19. 5=140°, 8=40°, 7=140°, 6=40°, 3=50°, 2=130°, 1=50°, 4=130°

20. 1=140-3x 2= 80+2x 3=100-2x 21a. 108-2x b. 60+3x c. 2.5x+20

22. DCE=1.5x+20; ACD=140-3x; ACE=160-1.5x

23a. 40° b. 140° c. 70° d. 140° e. 110° 24a. x b. 180-x c. (180-x)/2 d. 180-x e. x +(1/2)(180-x)

25. 2x 26. x=58; w=71; y=89/3 27. x+y = 90 and 2x-25+y-15=90 so x=40 and EFH =55°

28a. 60° b. 5° c. 120° d. 110° e. 15° f. 420/11 minutes after 7 g. 60/11 and 420/11 minutes after 4

29a. 15/16 b. $30(0.5)^n$ c. $450(0.5)^n$ which is $30[(0.5)^{n-4} - (0.5)^n]$

Unit 1 Handout #3: The Coordinate Plane; Parallel Lines

Some important formulas when working on the coordinate plane:

-The ***slope*** between points with coordinates (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$ or $\frac{\Delta y}{\Delta x}$, where Δ, the Greek letter "delta", represents the change in the variable.

Parallel lines have the same slope; the slopes of ***perpendicular lines*** are negative reciprocals.

-The ***midpoint*** of a segment with endpoints (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$, or the average of the *x*'s and the average of the *y*'s.

-The length of a segment with endpoints (x_1, y_1) and (x_2, y_2) comes from the ***distance formula***, an application of the Pythagorean Theorem, and is given by $d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$, which is NOT $\Delta x + \Delta y$!

To determine if three points are collinear, you may write the equation of the line through any two points and see if the third point is on that line. You may also compute the slopes between any two pairs of points; if they are equal then the points are collinear.

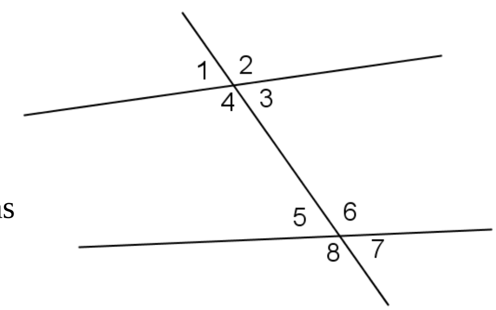
Finding the ***intersection of two lines*** algebraically means finding the point that is on both lines, so it satisfies both equations. Algebraically, this is solving a system of two linear equations in two variables, and can be done either by ***substitution*** or ***elimination*** (sometimes called "linear combination").

Many angles are created when two lines are crossed by a third line, called a ***transversal***.

-***Corresponding angles*** occupy the same relative position. So angles 1 and 5 are corresponding, as are 2 and 6, 3 and 7, and 4 and 8

-***Alternate interior angles*** are between the two lines on opposite sides of the transversal. So angles 3 and 5 are alternate interior, as are angles 4 and 6.

-***Alternate exterior angles*** are outside the two lines on opposite sides of the transversal. So angles 1 and 7 are alternate exterior, as are angles 2 and 8.



If the two lines crossed by the transversal are ***parallel***, then each pair of corresponding angles, alternate interior angles, and exterior angles are congruent. Conversely, if any pair of such angles are congruent, then the lines must be parallel.

Example #1: Given points A(3,7) and B(-1,4) do the following:

a. If C(5,*w*) is collinear with A and B, find *w*.

b. Is D(8,10) collinear with A and B?

c Find the midpoint of $\overline{AB}$.

d. Find the length of $\overline{AB}$.

Solution

a. Find the equation of the line through A and B. The slope is $\frac{\Delta y}{\Delta x} = \frac{3}{4}$. So $y = \frac{3}{4}x + b$. Plug in either point. Using point A: $7 = \frac{3}{4}(3) + b$ so $b = \frac{19}{4}$ and the equation is $y = \frac{3}{4}x + \frac{19}{4}$. For point C, when *x*=5, what is *y*? $y = \frac{3}{4}(5) + \frac{19}{4} = \frac{34}{4} = \frac{17}{2}$ so *w*=17/2 or 8.5.

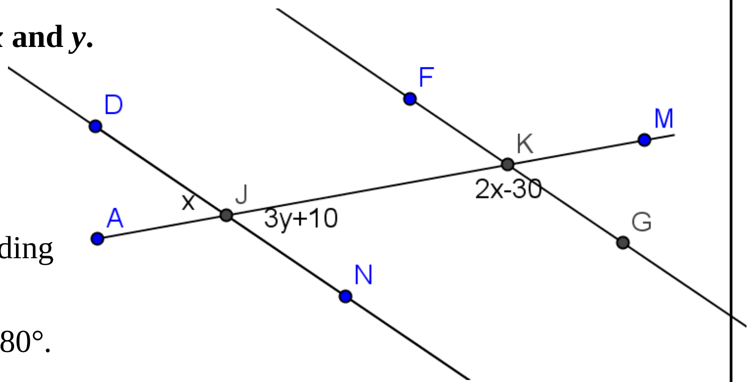
b. If (8,10) satisfies the equation of the line through A and B, then D is on the line. Plug the coordinates in: Is it true that $10 = \frac{3}{4}(8) + \frac{19}{4}$? No, $10 \neq \frac{43}{4}$, so D is not collinear with A and B. *Note: you could have also checked to see if the slope of $\overline{AD}$ or $\overline{BD}$ is the same as the slope of $\overline{AB}$. D is collinear only if the slopes are the same.*

c. The midpoint's *x*-coordinate is the average of the endpoints' *x*-coordinates. So (3+(-1))/2 = 1. Its *y*-coordinate is the average of the *y*-coordinates, so (7+4)/2=5.5. So the midpoint is (1,5.5)

d. The length can be found using the distance formula or the Pythagorean Theorem.

$$D = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(3 - -1)^2 + (7-4)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

2. Given that $\overline{DJ} \parallel \overline{KF}$ below, find the values of *x* and *y*.



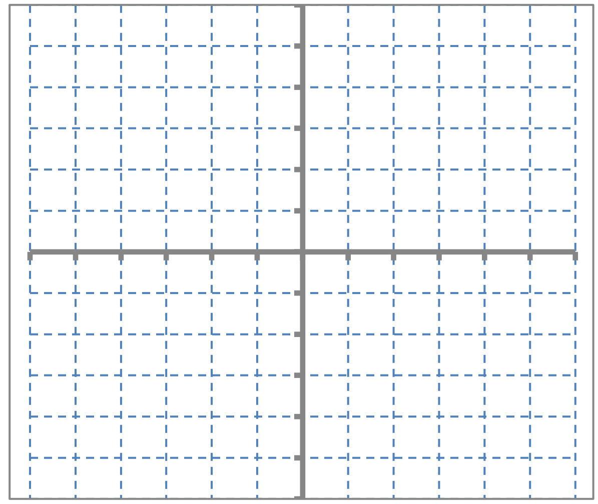
Solution:

Since angles DJA and KJN are vertical, they are congruent. So $x = 3y + 10$.

Since $\overline{DJ} \parallel \overline{KF}$, angles DJA and FKJ are corresponding and thus congruent, so angle FKJ measures *x*.

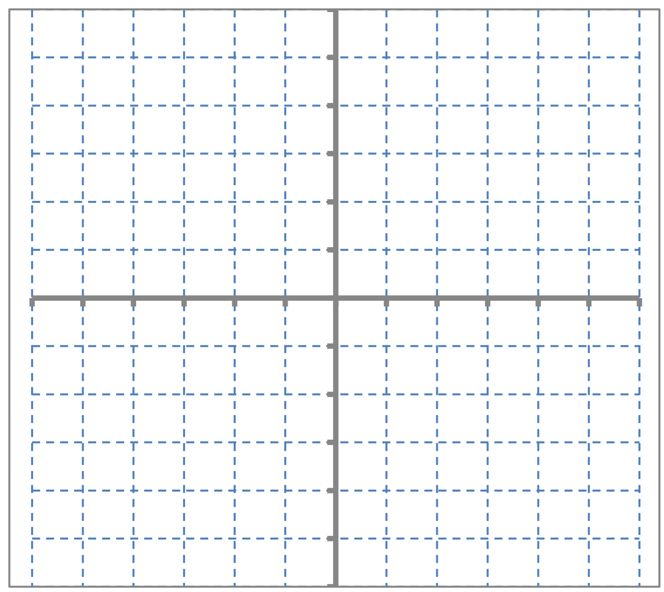
Angles FKJ and JKG form a linear pair, so sum to 180°. Thus $x + 2x - 30 = 180$ and $x = 70$. And because $x = 3y + 10$, we know $70 = 3y + 10$ so $3y = 60$ *and* $y = 20$.

1. Using the grid below, plot the points A (1,-3), C(-5,6), and D(2,-5). Are A, C, and D collinear? Support your answer with calculations.



2. Given points A (2,3), B(5,1), and C (-1,-3), find the following:

a. The midpoint of $\overline{BC}$.



b. The coordinates of points that trisect segment $\overline{AC}$.

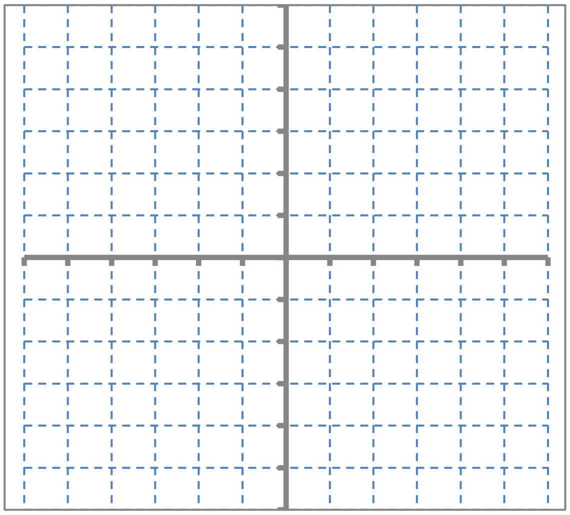
c. Assume that B is the midpoint of $\overline{AD}$. What are the coordinates of D?

d. Find the length of segment $\overline{AB}$. Hint: use the Pythagorean Theorem.

e. $\overline{AE}$ bisects $\overline{BC}$. Find any possible coordinates of E.

3. Given points A(-2,1), B(4,3), and C(2,-2), find the following:

a. The midpoint of $\overline{AB}$.



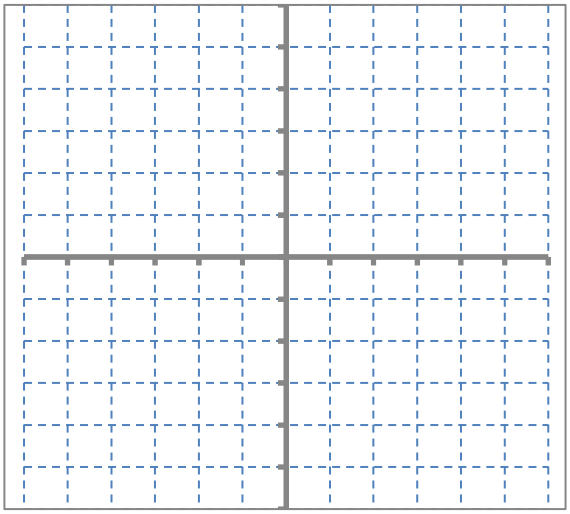
b. The length of segment $\overline{AC}$.

c. A point whose *x* coordinate is 6 and is collinear with points A and B.

d. If C is the midpoint of $\overline{AD}$, then what are the coordinates of point D?

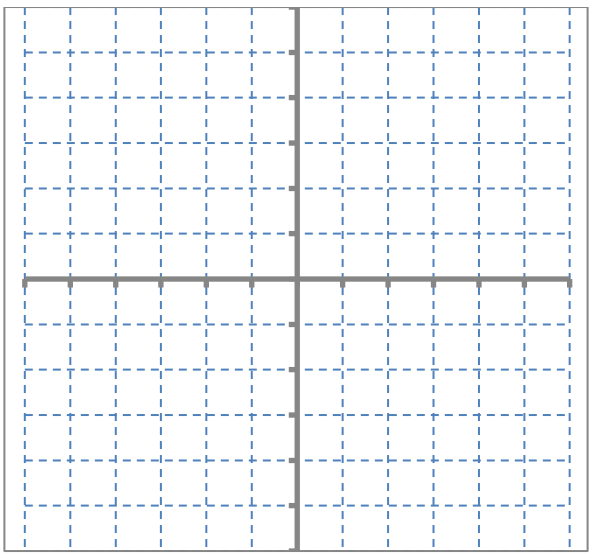
e. What is the equation of the horizontal line that bisects $\overline{AC}$?

4. Given points A(-4,5), B(5,2), C(4,-3), and D(-3,-4), use algebra to find the exact coordinates where $\overline{AC}$ and $\overline{BD}$ intersect. Leave your answer as a reduced fraction.



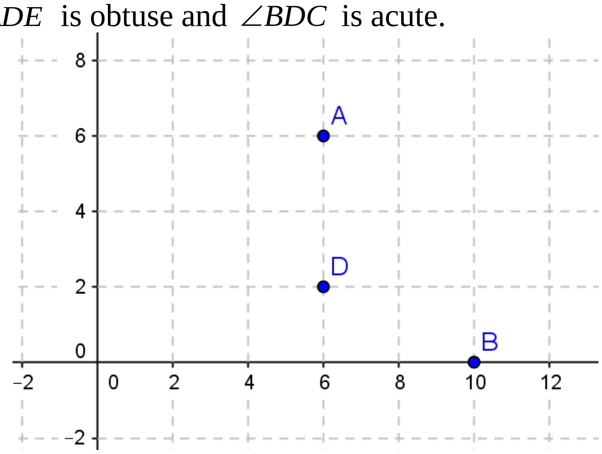
5. The points (5,3), (1,-3), (2,*a*), and (*b*,5) are all collinear. Find the values of *a* and *b*.

6. Given point A (1,-3); the length of segment $\overline{AB}$ is 5 units. Find five different possible coordinates of B.

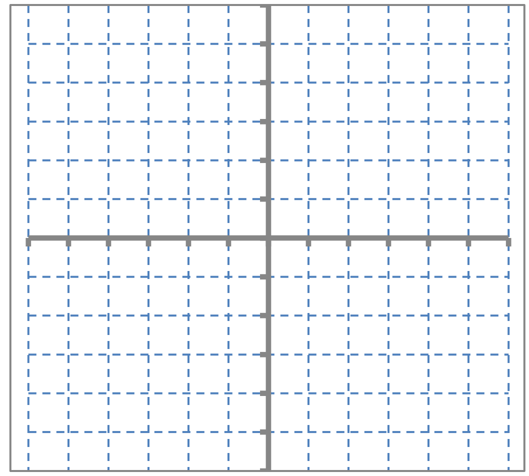


7a. Place points C and E on the plane on the right such that $\angle ADE$ is obtuse and $\angle BDC$ is acute.

b. Is it possible that $\angle CDE$ is a right angle?

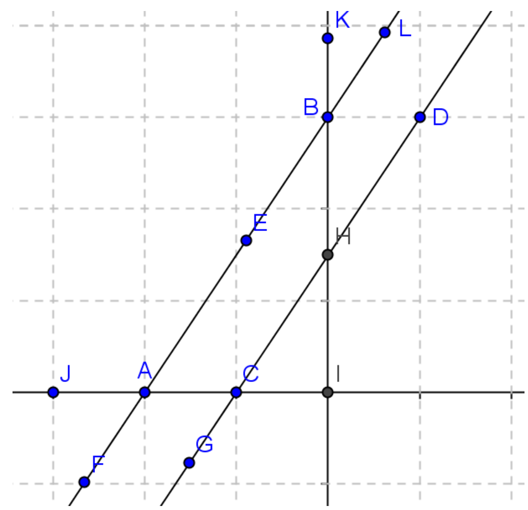


8. Given points J(-5,1) and K (1,4). If L is on the *x*-axis and angle JKL is a right angle, then use algebra to find the coordinates of L.



9. Lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ below are parallel.

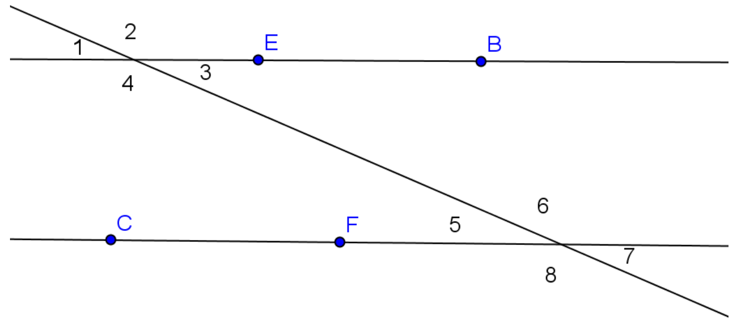
a. Name all angles congruent to $\angle KBL$. Explain why.



b. Name all angles congruent to $\angle FAC$.

c. Must angles JAF and KBL be congruent? Explain.

10. In the diagram below, $\overleftrightarrow{EB} \,//\, \overleftrightarrow{CF}$.



a. Name all angles congruent to angle 1.

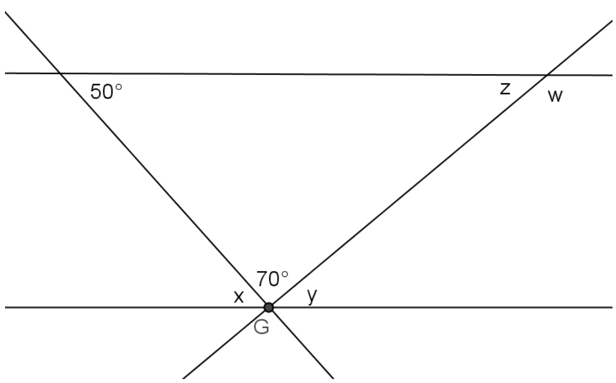
b. What angle(s) are corresponding to angle 7?

c. What angle(s) are alternate interior to angle 5?

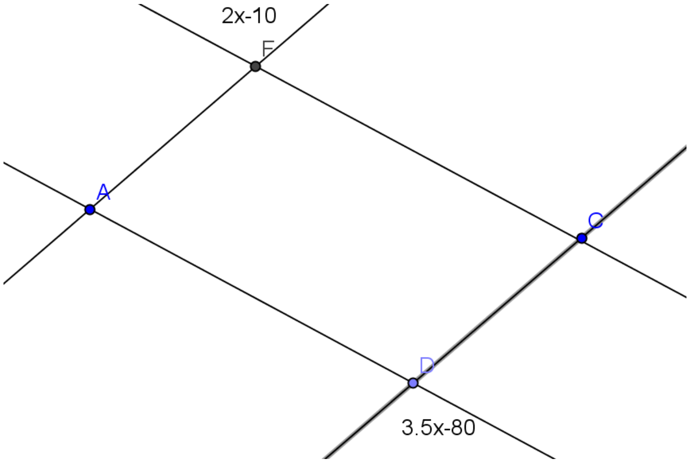
d. Name all angles supplementary to angle 5.

e. If the measure of angle 3 is *x*°, then find the measure of all other numbered angles (in terms of *x*).

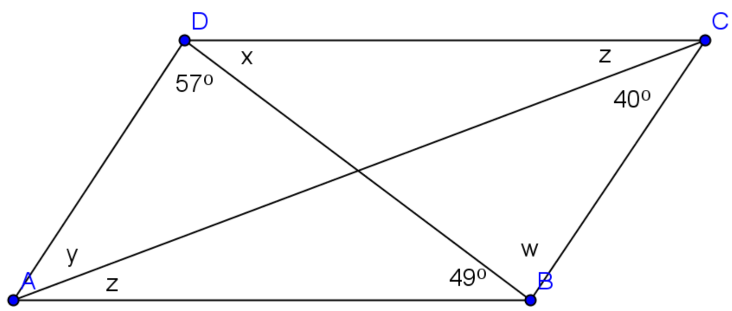
11. Two parallel lines are cut by transversals that intersect at point G. Find the values of *x*, *y*, *z*, and *w*.



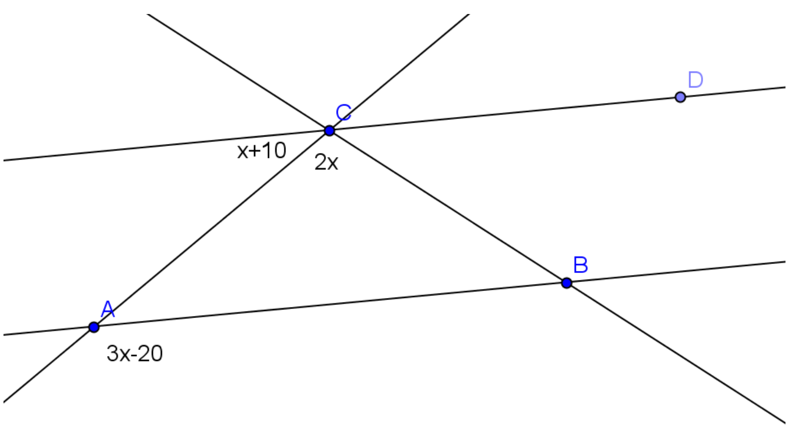
12. Two pairs of lines below are parallel. Find the value of *x*.



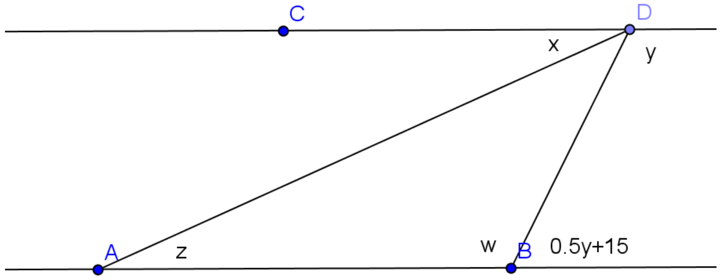
13. Given that $\overline{AB} \parallel \overline{CD}$ *and* $\overline{AD} \parallel \overline{BC}$ below, find the values of *w*, *x*, *y*, and z. Hint: extend some of the segments to make it look more familiar, with parallel lines and transversals!



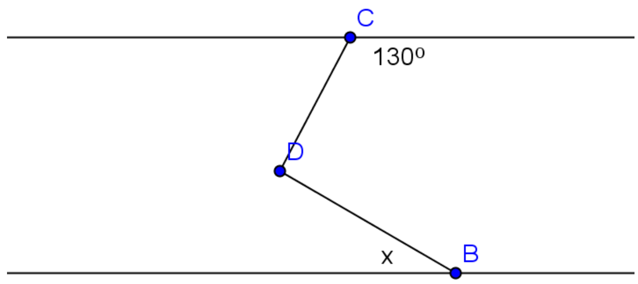
14. If $\overline{AB} \parallel \overline{CD}$, then find the measure of $\angle ACB$.



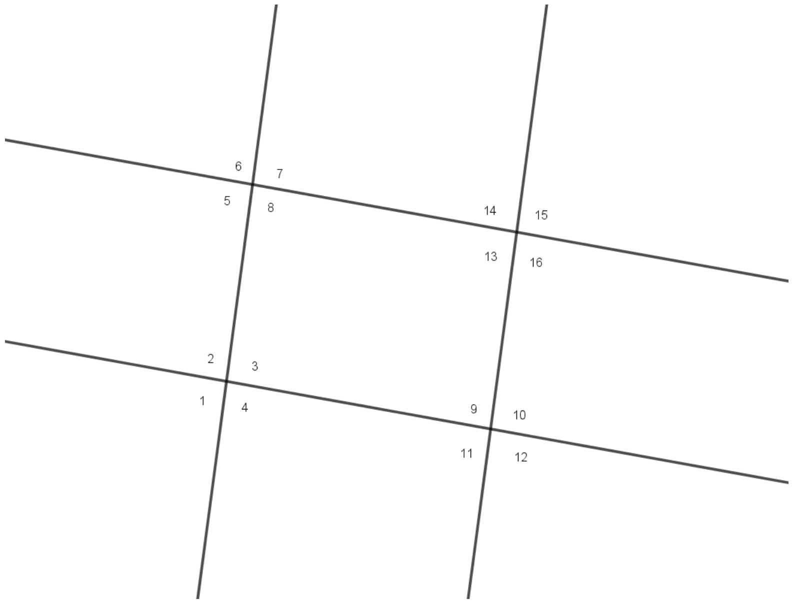
15. Given that $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and $\overline{AD}$ bisects $\angle BDC$, find the values of *w*, *x*, *y*, and *z*.



16. The horizontal lines below are parallel and angle D is a right angle. Find *x*. Hint: draw another horizontal line.

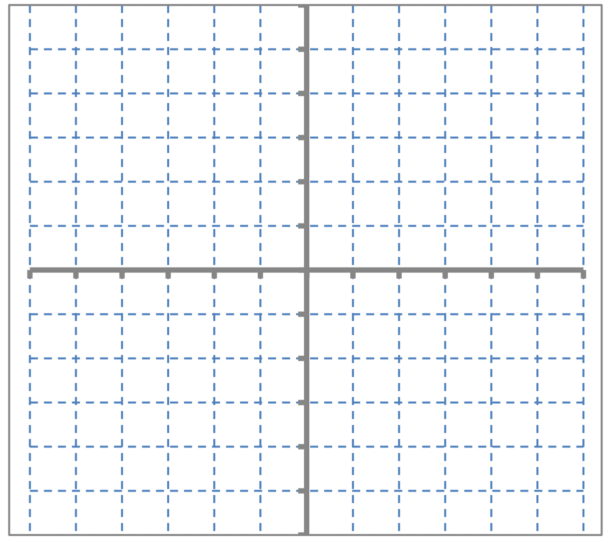


17. The diagram below has two pairs of parallel lines. What angles are alternate interior angles to angle 3 Note: this does not mean what angles are congruent, but what ones are alternate interior!



18. Given the points A(-2,6), B(6,4), C(2,-4), and D(-5,-2) do the following:

a. Show that the line connecting the midpoints of $\overline{AB}$ and $\overline{BC}$ is parallel to $\overline{AC}$.



b. Does it appear that $\angle ABD \cong \angle BDC$? Do some computations to determine whether they are. Hint: alternate interior.

c. Given that $\overline{AB}$ and $\overline{DE}$ bisect each other, what are E's coordinates?

d. Point F's coordinates are (8,1). Does $\overline{DF}$ bisect $\overline{BC}$?

Answers

1. No; slope of CD=-11/7 and slope of AC=9/6

2a. (2,-1) b. (0,-1) and (1,1) c. (8,-1) d. $\sqrt{13}$ e. anywhere where x=2 and $y \le -1$

3a. (1,2) b. 5 c. (6,11/3) d. (6,-5) e. y=-0.5 4. $y = -x + 1$ *and* $y = (3/4)x - 7/4$ meet at (11/7, -4/7)

5. a=-1.5 b= 19/3 6. some are (1,-8), (1,2), (6,-3), (-4,-3)… you've seen a 3-4-5 triangle you can also (4,1), (-2,1), (5,0), (-3,0), (5,-6), (-3,-6), (4,-7), (-2,-7)

7b. yes 8a. line through K perp to JK is $y = -2x + 6$; hits x-axis when y=0 so x=3

9a. angles EBH, BHD, CHI b. angles JAE, GCI, and ACH c. no

10a. 3, 5, and 7 b. 3 c.3 d. 6, 8, 2, and 4 e.angles 1, 5, and 7 area also x; angles 2, 4, 6, and 8 are 180-x

11. x=50; y=60, z=60, w=120 12. 140/3 13. w=57; x=49; y=40; z=34 14. 95°

15. y=110; w=110; x=35; z=35 16. 40° 17. Angles 5 and 11

18a. line thru (2,5) and (4,0) has slope=-5/2 as does AC

18b. need AB//CD so they'd be alternate interior angles; but -1/4 is not the same as -2/7 c. (9,12)

d. no; midpoint of BC (4,0) is not on DF.

Unit 1 Handout #4: Properties; Moving Towards Proof; Rotations

Some properties from algebra that we will use to justify conclusions in geometry are below.

-***Addition Property***: adding the same amount to two equal quantities results in equal quantities.

-***Subtraction Property***: subtracting the same amount to two equal quantities results in equal quantities.

-***Multiplication Property***: multiplying two equal quantities by the same thing results in equal quantities.

-***Division Property***: dividing two equal quantities by the same (non-zero) thing results in equal quantities.

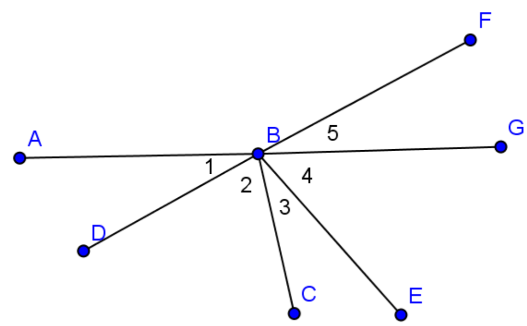
-***Transitive Property***: two quantities both equal to a third quantity are equal to each other.

-***Substitution Property***: if two quantities are equal, we can replace one with the other in an equation.

Rotations: Points and sets of points can be rotated around other points in a plane. Imagine tracing the figure to be rotated and holding the tracing paper at the point it is rotated about. Rotations will not change the shape or size of figures. For example, a line segment rotated around any point should have the same length after rotation as it did beforehand.

Also, the distance from any point to the point it is rotated about should not change. So if point A is rotated about point B to become point A', the distance AB should be equal to the distance A'B.

Example #1: In the diagram to the right, $\angle ABC \cong \angle DBE$ **and** $\overline{AG}$ *and* $\overline{DF}$ **intersect at B. Explain why** $\angle 3 \cong \angle 5$.



Solution:

Since $\angle 1$ and $\angle 5$ are vertical angles, $\angle 1 \cong \angle 5$.

Since $\angle ABC \cong \angle DBE$, we know $\angle 1 + \angle 2 = \angle 2 + \angle 3$ ("the whole equals the sum of the parts" or "substitution")

Using the subtraction property, we know $\angle 1 \cong \angle 3$.

By the transitive property, and the first and third steps above, $\angle 3 \cong \angle 5$.

Example #2: Point B (5,5) is rotated 90° counter-clockwise around A (2,1) What are its new coordinates?

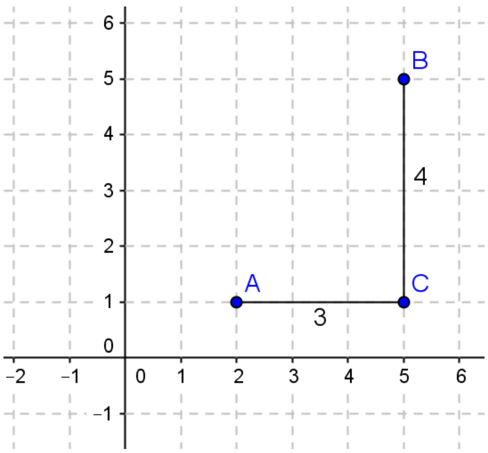
Solution:

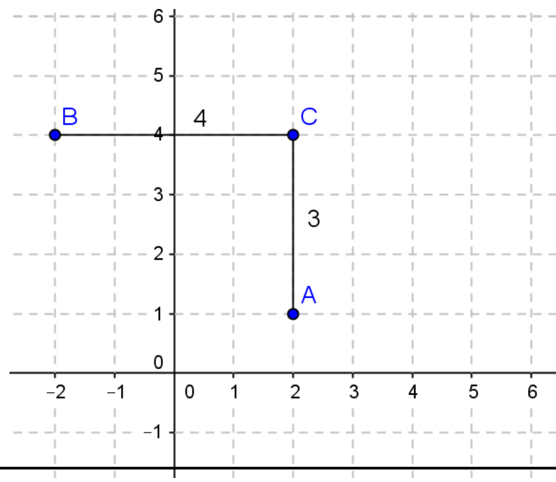
First draw a horizontal and a vertical segment to connect A with B and note their lengths (diagram on left below). The length of $\overline{AC}$ is 3 because it is B's *x*-coordinate minus A's *x*-coordinate, and the 4 represents the difference in the *y*-coordinates. *Note: one could also put C at (2,5).*

Next, rotate those segments 90° counterclockwise, as in the diagram on the right. The segment of length 3 now points north instead of east; the right angle– "left turn" – at C is maintained.

Now compute B's new coordinates. Its *x*-coordinate is 4 ***lower*** than A's, so it is -2. And its *y*-coordinate is 3 greater than A's, so 5. The answer is thus (-2,5).

Note that the distance from A to B is preserved. In both cases it is $\sqrt{(3)^2+(4)^2}$, or 5.



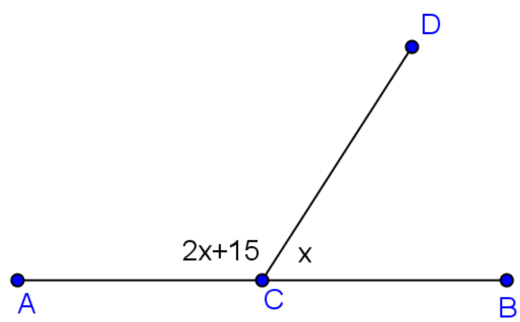


In questions #1-3 below, chose from the list of properties below fill in the missing justifications:

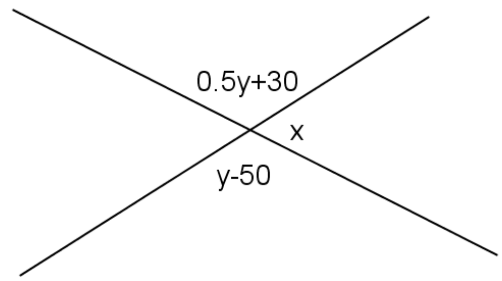
Addition property	Subtraction property	Multiplication property
Division Property	Transitive Property	Substitution Property

1. Find *x* below.

Statement	***Justification***
2x+15 + x = 180	straight angle
3x + 15 = 180	combine like terms
3x=165	________
x = 55	________

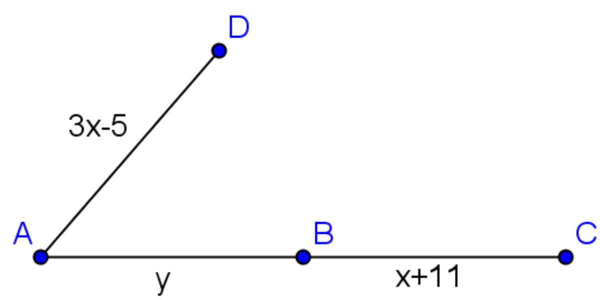


2. Find *x* and *y* below



Statement	***Justification***
0.5y+30 = y-50	vertical angles are congruent
0.5y+80 = y	______
80=0.5y	_______
160=y	_______
(y-50)+x=180	supplementary angles
110+x=180	______
x=70	________

3. Find *x* and *y* below, given that AB=BC and AB=AD.



Statement	***Justification***
AD=BC	_______
3x-5 = x+11	________
2x-5=11	_____
2x=16	______
x=8	_______
y=x+11	given
y=19	__________

4. Which of the following are transitive? For example, "is taller than" is transitive, because if Jane is taller than Steve and Steve is taller than Bob then Jane is taller than Bob.

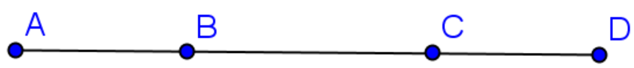
a. Has the same color hair

b. Has different color hair

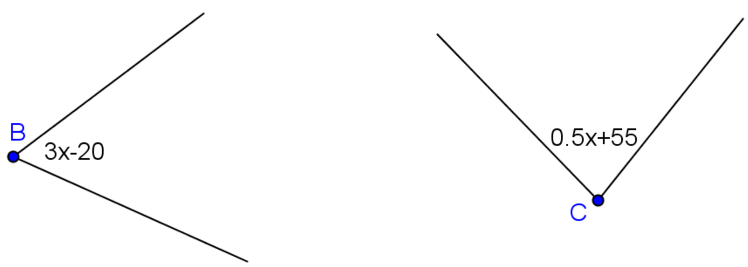
c. Is the same age as

d. Is friendly with

5. Given segment AD below where $\overline{AC} = \overline{BD}$. Must $\overline{AB} = \overline{CD}$? Explain carefully; refer to at least one property.

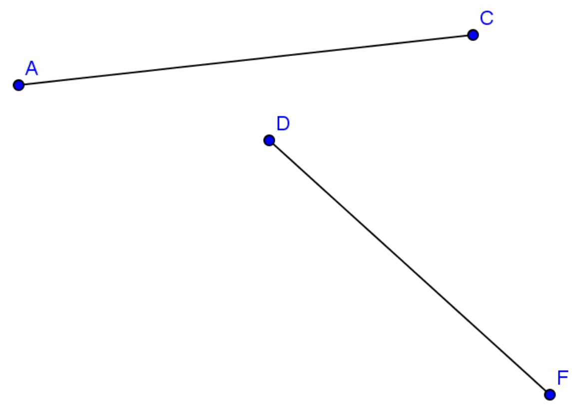


6. Both angles B and C below are complementary to angle *W*. What is the measure of angle *W*?



7. B is a point on line segment $\overline{AC}$ and E is a point on line segment $\overline{DF}$. What can you conclude if: (consider each separately)

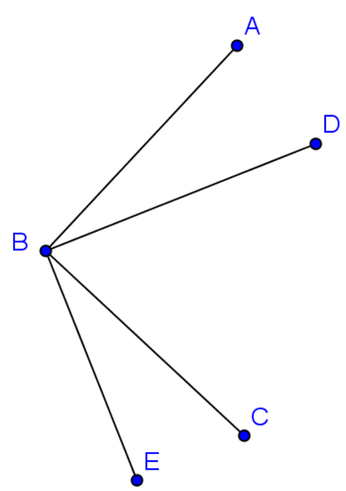
a. $\overline{AC} \cong \overline{DF}$ and $\overline{AB} \cong \overline{DE}$?



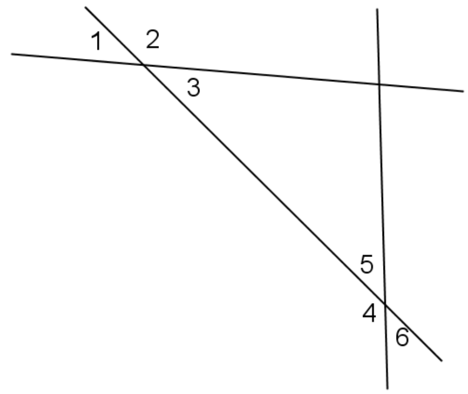
b. $\overline{AB} \cong \overline{DE}$ and $\overline{BC} \cong \overline{EF}$

c. $\overline{AC} \cong \overline{DF}$ and $\overline{BC} \cong \overline{EF}$

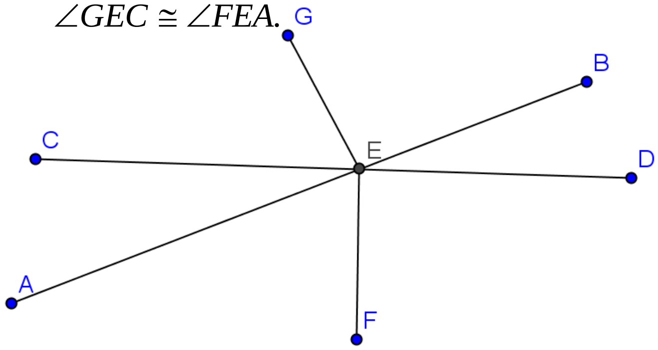
8. In the diagram below, $\overline{AB} \perp \overline{BC}$ *and* $\overline{DB} \perp \overline{BE}$. Is it true that $\angle ABD \cong \angle EBC$? Explain.



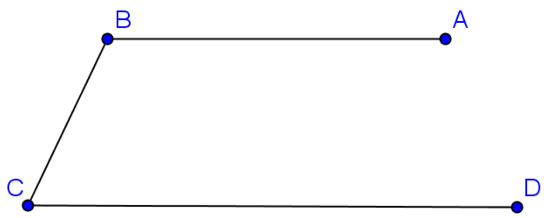
9. Given that $\angle 2 \cong \angle 4$, carefully explain why $\angle 1 \cong \angle 6$.



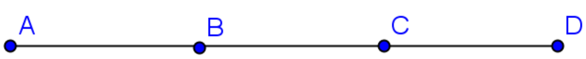
10. Given that $\overline{CD}$ *and* $\overline{AB}$ intersect at point E and FED and GEB are right angles, explain why $\angle GEC \cong \angle FEA$.



11. Given that $\overline{AB} // \overline{CD}$ in the diagram below, explain why angles ABC and DCB must be supplementary. Hint: you may want to extend some of the segments to get a more familiar diagram!

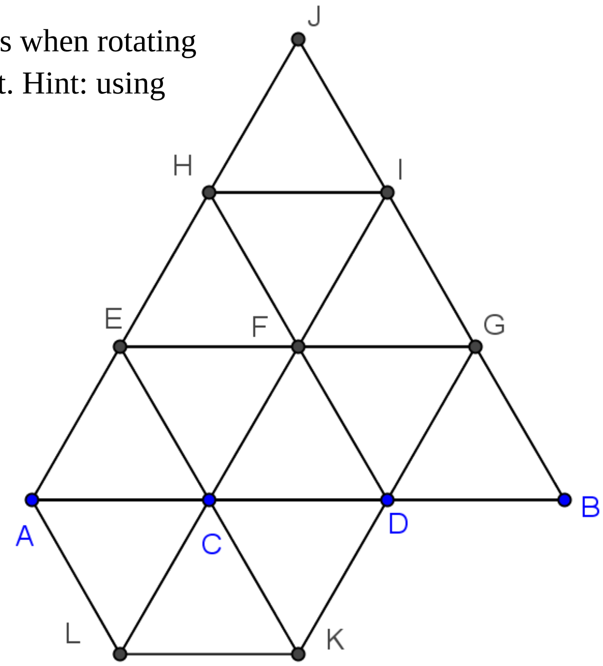


12. Below, B is the midpoint of $\overline{AC}$ and C is the midpoint of $\overline{BD}$. Must B and C trisect $\overline{AD}$? Justify.



13. In the diagram below, all segments are equal lengths and all angles (inside triangles) measure 60°.

Give the name of the point, segment, or triangle that results when rotating the given item the given amount around the specified point. Hint: using tracing paper if this is difficult!



a. E 60° counter-clockwise around G.

b. $\overline{HI}$ 120° clockwise around H.

c. ΔCDK 180° around C.

d. J 120° clockwise around E.

e. J 120° clockwise around F.

f. ΔCDK 60° clockwise around F.

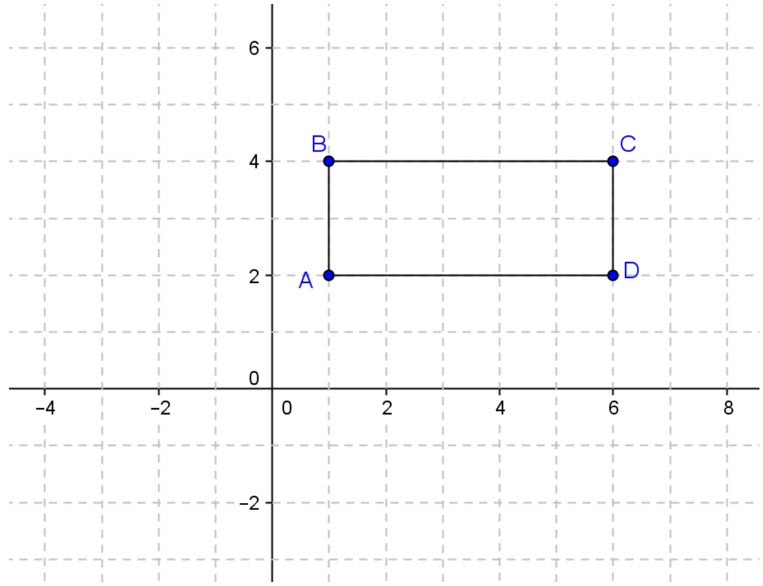
g. $\overline{HE}$ 60° clockwise around A.

h. ΔCEA 120° counter-clockwise around F.

i. $\overline{IG}$ 60° counter-clockwise around D.

j. $\overline{LK}$ 120° counter-clockwise around E.

14. Given rectangle ABCD below, find the new coordinates of the four vertices when the rectangle is…



a. Rotated 90° clockwise around B.

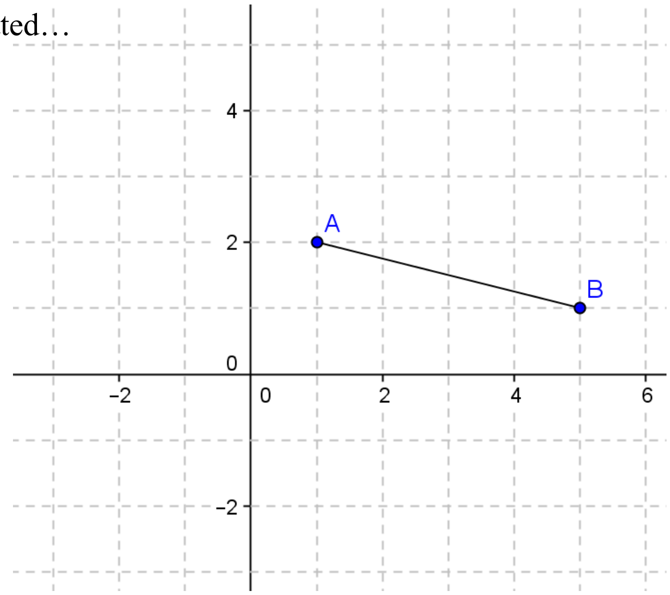
b. Rotated 180° around A

c. Rotated 90° clockwise around the origin.

d. Rotated 90° clockwise around the point (2,3)

e. Rotated 90° counter-clockwise around the point (5,4)

15. Find the coordinates of A and B when they are rotated…



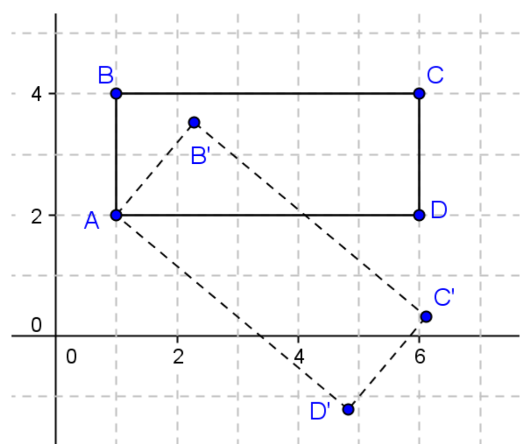
a. 90° clockwise around A.

b. 180° around the origin.

c. 90° clockwise around (3,0).

d. 90° clockwise around (6,1)

16. Rectangle ABCD is rotated 40° clockwise around point A, creating the dashed rectangle AB'C'D'. Note: the 40° means that angle BAB' measures 40°. Find the measure of the following:

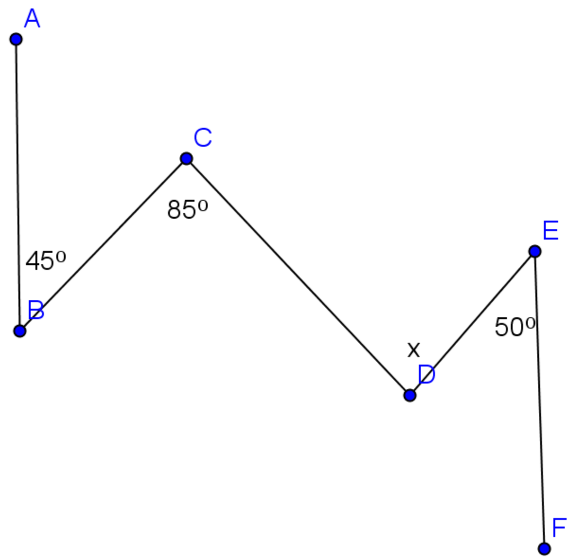


a. ∠B'AD

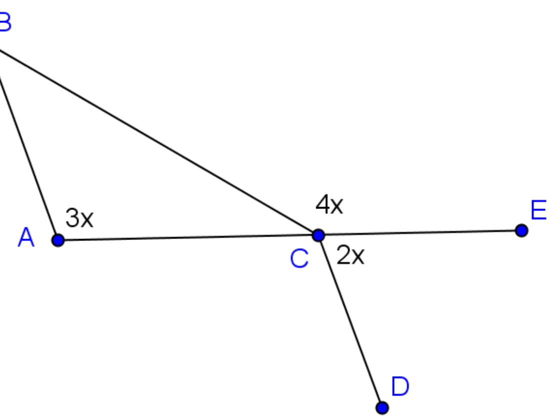
b. ∠DAD'

c. The acute angle where $\overline{AD'}$ hits the *x*-axis.

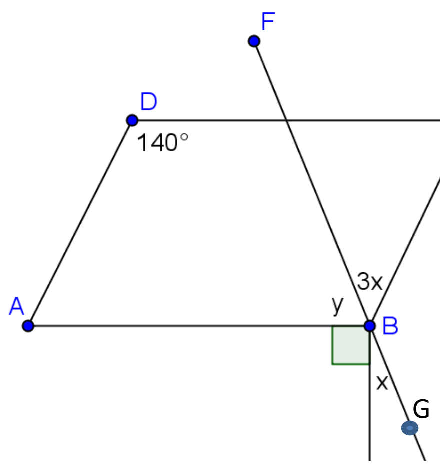
17. Given that $\overline{AB} \parallel \overline{EF}$ find x. hint: draw a few extra line segments.



18. If $\overline{AB} \parallel \overline{CD}$, then find the measure of $\angle BCE$. You may assume that A, C, and E are collinear.



19. In the diagram below, $\overline{AB} \parallel \overline{CD}$, $\overline{BC} \parallel \overline{AD}$, and F, B, and G are collinear. Find the value of x.



20. The point A(0,0) is rotated 90° counter-clockwise around point *W* and it ends up at A'(6,3). What are the coordinates of point *W*? Instead, if point (*a*,*b*) is rotated 90° counter-clockwise around *W* and ends up at (*c*,*d*), then what are *W*'s coordinates?

Answers

1. subtraction; division 2. addition; subtraction; multiplication; substitution; subtraction

3. transitive; substitution; subtraction; addition; division; substitution 4. a and c

5. yes; subtract BC from both of them (subtraction property) 6. 20°

7a. $\overline{BC} \cong \overline{EF}$ b. $\overline{AC} \cong \overline{DF}$ c. $\overline{AB} \cong \overline{DE}$ 8. Yes, subtract DBC from 2 equal angles…

9. Angles 1 & 6 have congruent supplements, so they are congruent

10. Add BED to both right angles so GED=FEB; now their supplements must also be congruent so GEC is congruent to FEA

11. Extend CD past C to E.. ABC and BCE are congruent; BCD and BCE are supplementary so ABC and BCD are also supplementary

12. AB=BC=CD by transitivity so, by the definition of trisect, they do.

13a. K b. HE c. ΔACE d. K e. B f. ΔACE g. CD h. ΔDGB i. EF j. IJ

14a. A(-1,4), B(1,4), C(1,-1), (-1,-1) b. (1,2), (1,0), (-4,0), (-4,2) c. (2,-1), (4,-1), (4,-6), (2,-6)
d. (1,4), (3,4), (3,-1), (1,-1) e. (7,0), (5,0), (5,5), (7,5)

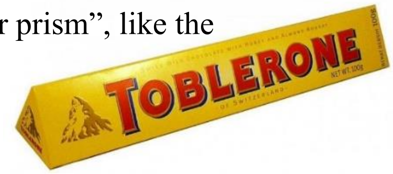
15a. A(1,2); B(0,-2) b. (-1,-2); (-5,-1) c. (5,2); (4,-2) d. (7,6); (6,2)

16a. 50° b. 40° c. 40° 17. 90° 18. 144° 19. 25°

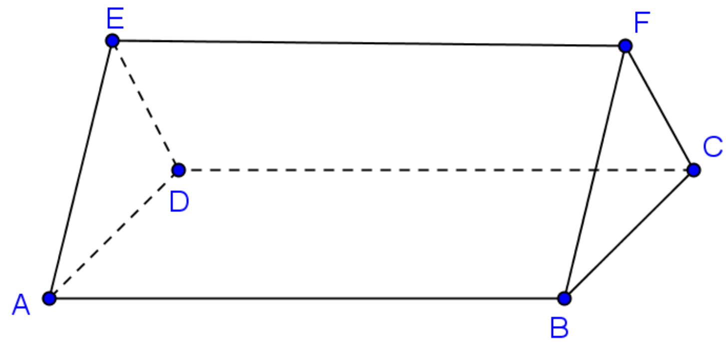
20. (1.5,4.5); ((b+c+a-d)/2 , (d+b+c-a)/2)

Unit 1 Handout #5: Practice Problems for Unit 1

1. Answer the following questions about the shape below. It is a "right triangular prism", like the Toblerone bar next to it!



a. What is $\overline{EF} \cap \overline{AE}$?



b. What is the intersection of planes ABF and DEC?

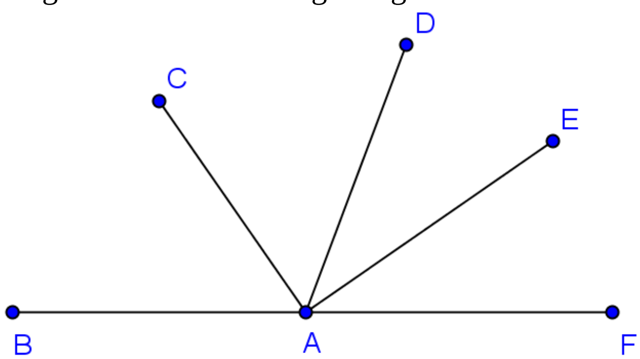
c. Name all drawn segments perpendicular to $\overline{BC}$.

d. Name all drawn segments skew to $\overline{DE}$.

e. Are A, F, and C coplanar?

f. Name a point coplanar with C, D, and F.

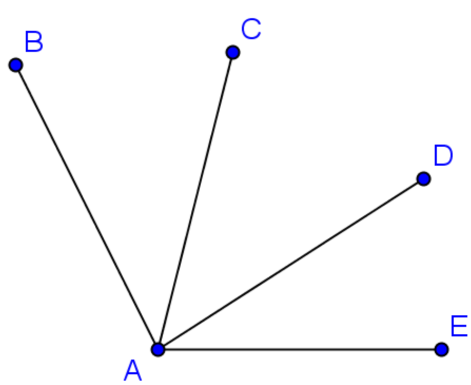
2. Given: $\overline{BF}$ where $\overrightarrow{AC}$ and $\overrightarrow{AE}$ bisect angles BAD and DAF respectively. Explain carefully why angle CAE must be a right angle.



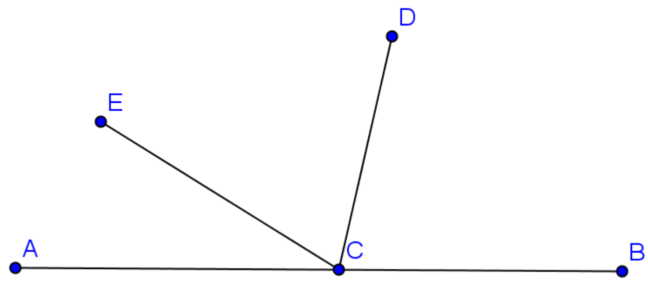
3. Two supplementary angles differ by 40 degrees. What is the measure of the larger one? Two supplementary angles differ by *n* degrees. What is the measure of the larger one?

4. The complement of angle A is 65 degrees less than the supplement of angle B. The supplement of angle A is ten degrees more than eight times complement of angle B. What are the measures of angles A and B?

5. Angle BAE is trisected by $\overrightarrow{AD}$ and $\overrightarrow{AC}$. The complement of angle BAC is ten degrees less than the supplement of angle BAE. Find the measure of angle DAC.

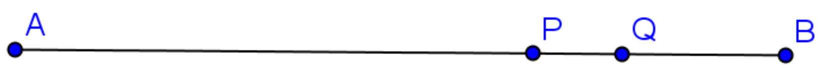


6. Given segment $\overline{AB}$ where angles ACE, ECD, and DCB are in the ratio 2:3:4. What is the measure of angle BCD?

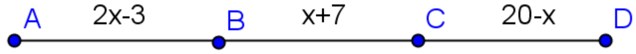


7. A, B, C, and D are in that order on a line. If AB=BD and BC=CD then find the ratio of $\overline{AC}$ *to* $\overline{CD}$..

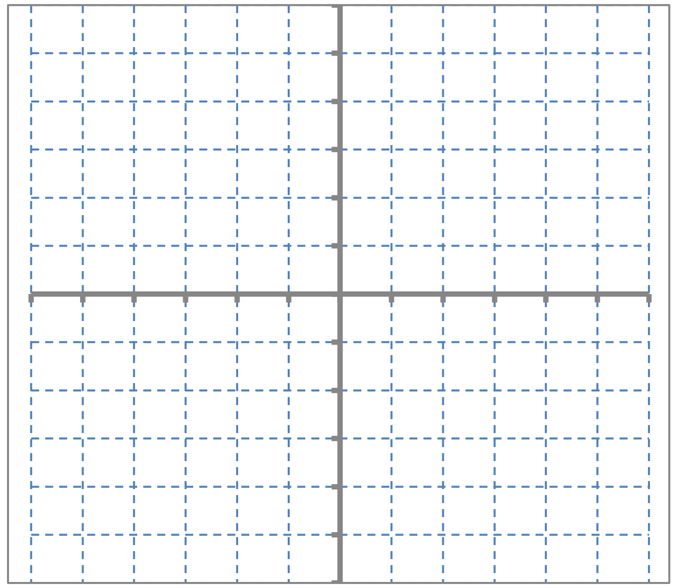
8. On segment AB below, AP/AB = 3/5 and PQ/QB=1/2. Find AQ/AB and AQ/QB. Hint: you can make up numbers for any one length and use the ratios to find the other lengths.



9. Do points B and C trisect $\overline{AD}$ in the diagram below? Justify your answer.



10. Given points A(-4,-3), B(2,5), and C(4,-1), do the following:



a. Find the length of segment $\overline{AB}$.

b. Find the midpoint of $\overline{BC}$.

c. A vertical line bisects $\overline{AB}$; what is its equation?

d. If C is the midpoint of $\overline{BD}$, then what are D's coordinates?

e. E's coordinates are (5,*w*). If E is collinear with B and C, then what is the value of *w*?

11. Given that $\overline{AB}$ *and* $\overline{CD}$ are line segments (not necessarily collinear), draw a diagram for each part below.

a. $\overline{AB} \cap \overline{CD} = \overline{AB}$

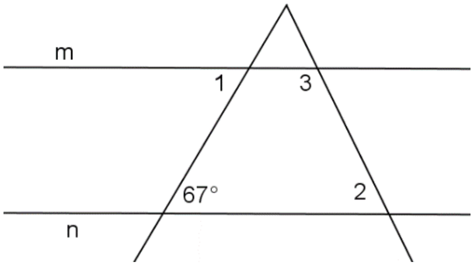
b. $\overline{AB} \cap \overline{CD} = D$

c. $\overline{AB} \cap \overline{CD} = \overline{AC}$

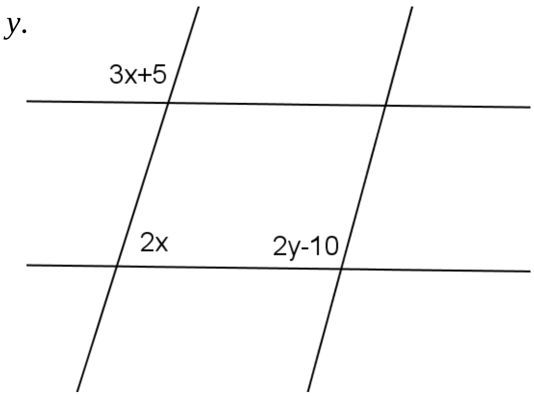
d. $\overline{AB} \cup \overline{CD} = \overline{AB}$

e. $\overline{AB} \cup \overline{CD} = \overline{AC}$

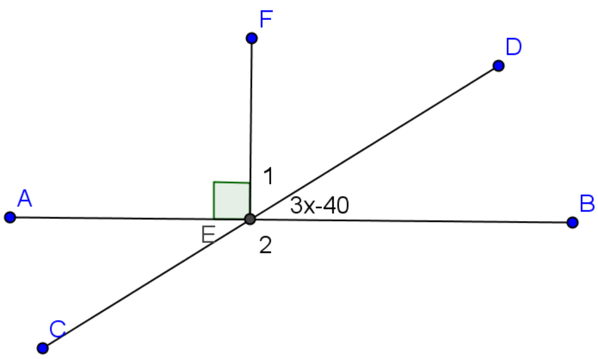
12. In the diagram below, lines *m* and *n* are parallel and $\angle 1 \cong \angle 2$. Find the measure of $\angle 3$.



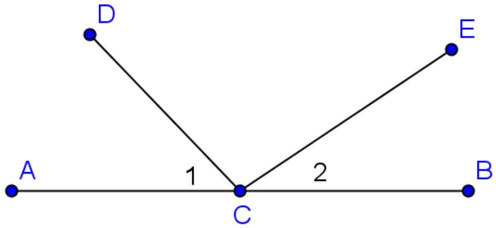
13. Given two pairs of parallel lines below, find the values of *x* and *y*.



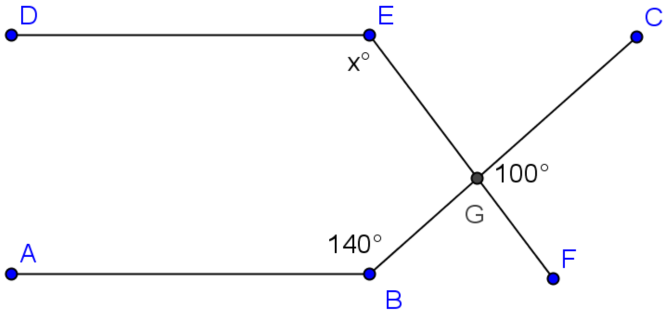
14. Find the measure of angles 1 and 2 in terms of *x* given that $\overline{AB}$ *and* $\overline{CD}$ intersect at point E.



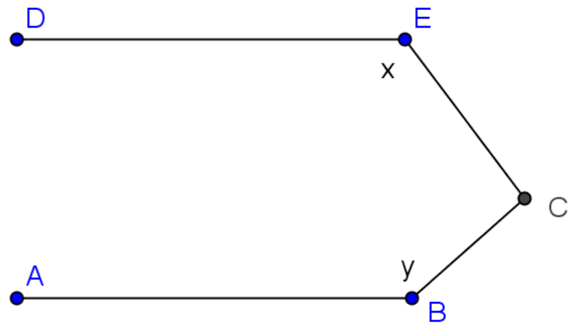
15. Find the measures of angles 1 and 2 in terms of *x* given that angle DCE measures 3*x*-28 and angle DCB measures 5*x*-10.



16. $\overline{AB} \parallel \overline{DE}$. Find *x*. (From *The Art of Problem Solving*)



17. Find C in terms of *x* and *y* given that $\overline{AB} \parallel \overline{DE}$.

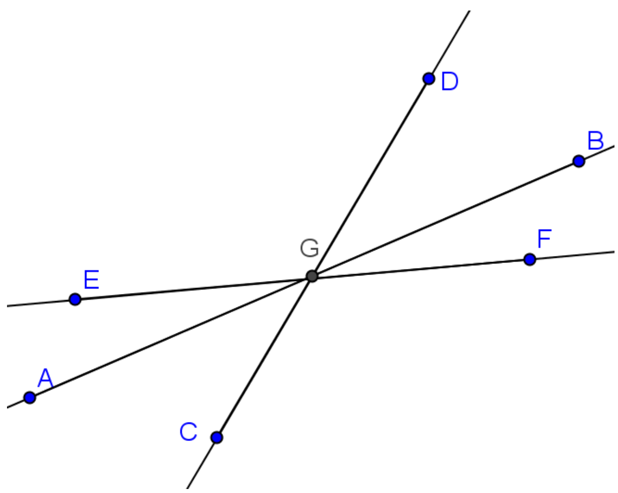


18. Lines $\overleftrightarrow{AB}, \overleftrightarrow{CD}, and\ \overleftrightarrow{EF}$ meet at G. Angle CGB measures 155° and angle EGD measures 142°.

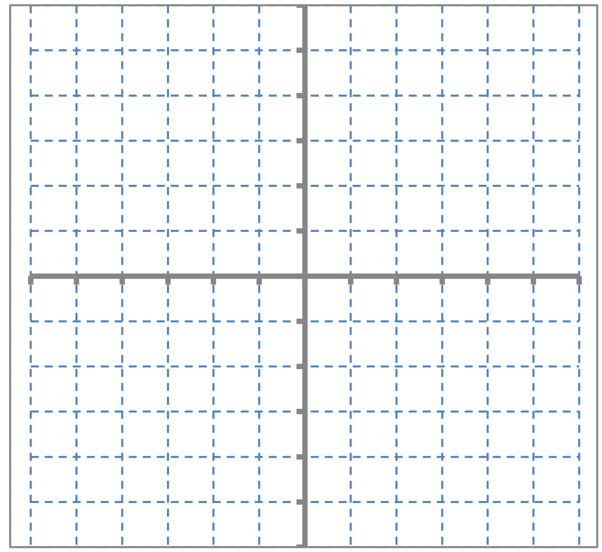
a. Find the measure of angle CGE

b. What angle must be congruent to CGE? CGB?

c. Find the measure of angle AGF.



19. Given points A(-2,-1), B(6,3), and C(1,4)



a. If $\overline{AB} \parallel \overline{CD}$ then where could D be?

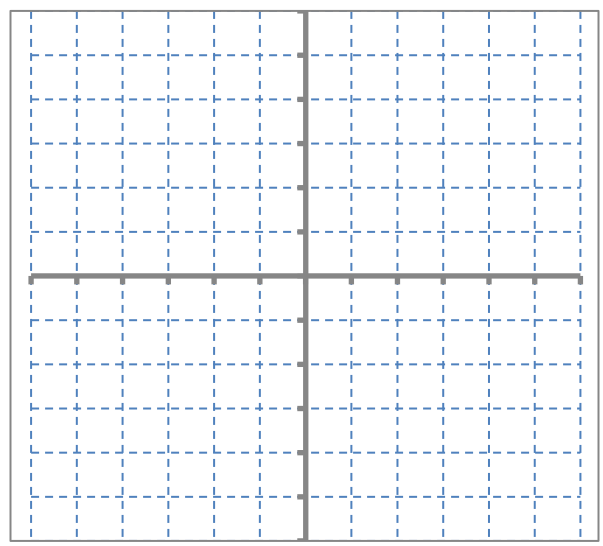
b. Where could point E be if $\angle ABC \cong \angle BAE$?

c. Is $\overline{AC} \perp \overline{BC}$? Support your answer with calculations.

d. Let $\overline{AF} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{CF}$. Where is F?

e. Where could G be if $\overline{AC} \parallel \overline{BG}$ and $\overline{AB} \perp \overline{CG}$

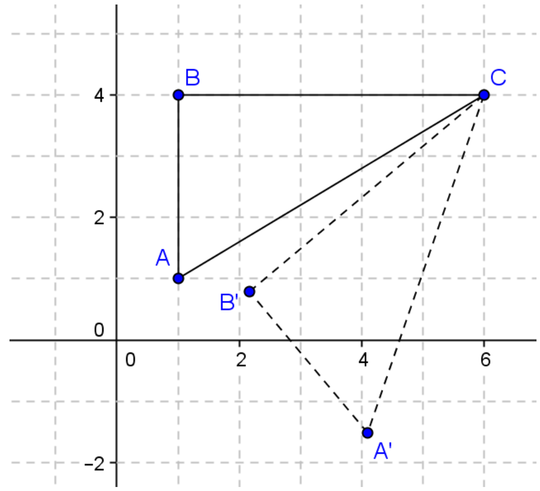
20. Given $\overline{AB}$ where A's coordinates are (-3,2) and B's are (1,4). Do the following:



a. Find the coordinates of A and B when they are rotated 90° counter-clockwise around the origin.

b. Find the coordinates of A and B when they are rotated 90° clockwise around (2,1).

21. Angle A measures 60°, B measures 90°, and C measures 30°.The triangle is rotated 40° counter-clockwise about point C. Find the measure of the following angles.



a. A'

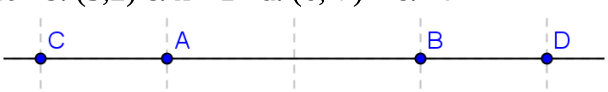
b. ACB'

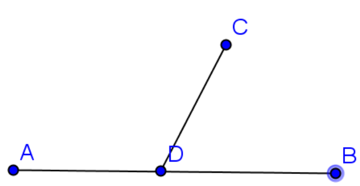
c. The acute angle A'C makes with the *x*-axis.

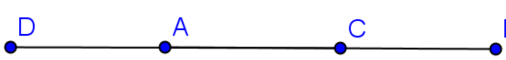
22. Given a rectangular prism (a box) with its eight vertices. How many different planes go through exactly four of the vertices?

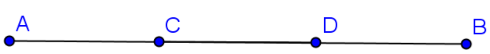
Answers

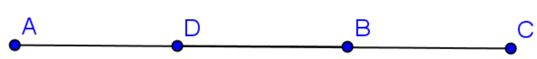
1a. point E b. segment EF c. AB, CD d. BF, BC AB e. yes; any 3 points are! f. E
2. CAD is ½ of BAD and DAE is ½ of DAF so CAE is ½ of BAE by addition; which is 90°
3. 110; 90+0.5n 4. 90-A=(180-B)-65 and 180-A=8(90-B)+10 so B=75 and A=50
5. 40° 6. 80° 7. 3:1 8. 11/15 and 11/4 9. No; if AB=BC then x=10 and then BC is not equal to CD
10a 10 b. (3,2) c. x=-1 d. (6,-7) e. -4

11a. 

b. 

c. 

d. 

e. 

12. 113° 13. x=35; y=60 14. $\angle 1 = 130 - 3x$; $\angle 2 = 220 - 3x$
15. $\angle 1 = 190 - 5x$; $\angle 2 = 2x + 18$ 16. 120° 17. 360-(x+y) 18a. 38° b. DGF; AGD c. 167°
19a. anywhere on y=0.5x+3.5 b. anywhere on y=-(1/5)x-7/5 where x>-2 c. slopes are not negative reciprocals, no so d. (-7,0) e. (39/11, -12/11)
20a. A(-2,-3) and B(-4,1) b. A(3,6) and B(5,2) 21a. 60° b. 10° c. 70°
22. 12; 6 are faces of the box and the other six go through diagonals of the box's faces

Unit 2 Handout #1: Triangle Basics

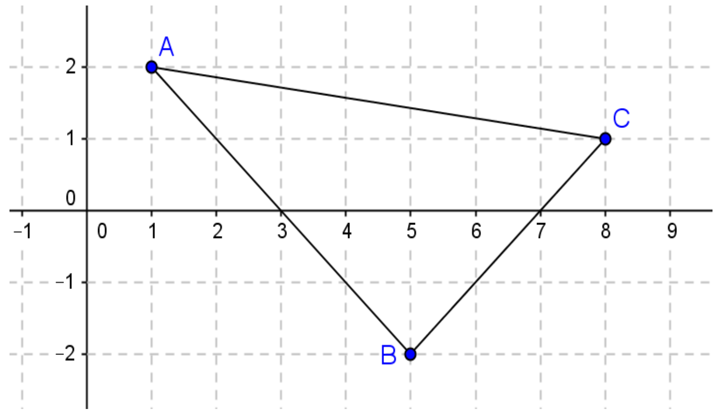
Triangles have three sides and three vertices. They can be scalene, isosceles, or equilateral. They can also be right, if they have an angle that measures 90°. It can be shown that the sum of the measures of the angles in any triangle is 180°.

Example #1: Given the points A(1,2), B(5,-2), and C(8,1), answer the following:

a. Is ΔABC scalene, isosceles, or equilateral?

b. Is ΔABC a right triangle?

c. Triangle ACD is a right triangle with a right angle at C. Give any set of coordinates for point D.



Solution

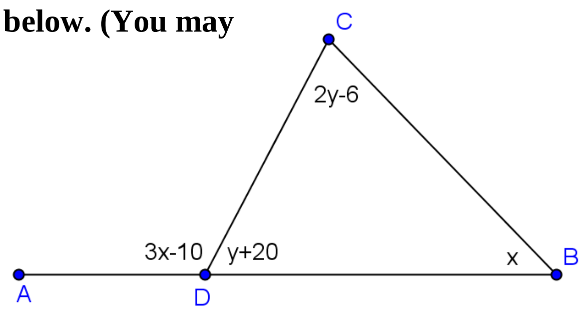
a. Find the lengths of the sides: AB= $\sqrt{4^2+4^2}=\sqrt{32}$

AC= $\sqrt{7^2+1^2}=\sqrt{50}$; BC= $\sqrt{3^2+3^2}=\sqrt{18}$. Since no two are the same, ΔABC is scalene.

b. Use slopes; the slopes are 1, -1, and -1/7. Since 1 and -1 are negative reciprocals, $\overline{AB}\perp\overline{BC}$ and ΔABC is a right triangle.

c. For C to be a right angle, $\overline{CD}$ must have a slope of 7. So go 7 up and one over and (9,8) works, as does any other point on the line $y=7x-55$.

Example #2: Find the values of *x* and *y* in the diagram below. (You may assume that points A, D, and B are collinear.)



Solution:

-Angles ADC and CDB form a linear pair, so $3x-10+y+20=180$.

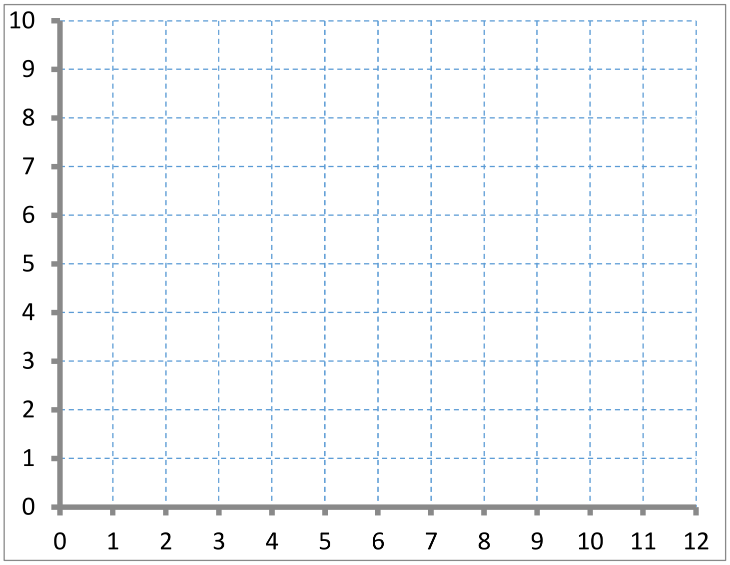
-The sum of the angles in ΔBCD must be 180, so $y+20+2y-6+x=180$

-Simplifying those yields $3x+y=170$ and $x+3y=166$.

-Use linear combination (elimination). Subtract the second equation from three times the first equation and the result is $8x=344\ so\ x=43$. Plug this *x* into the first equation and solve for *y*: $y=41$. Of course, one could also solve the system of equations with substitution.

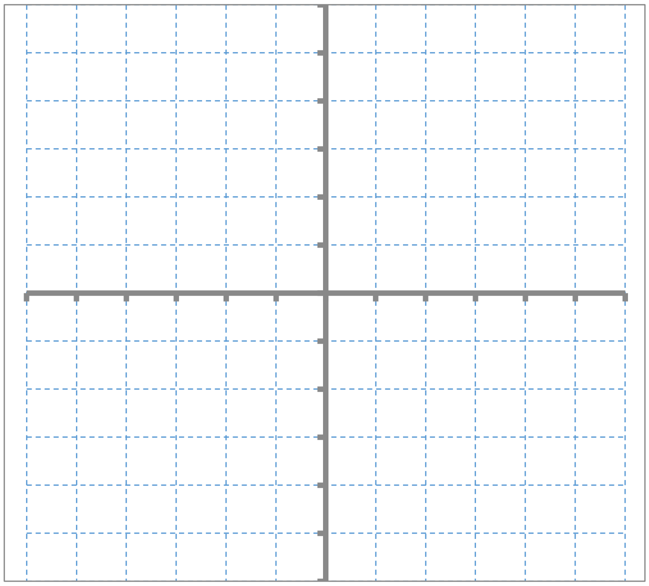
1. Given A(0,2), B(8,3), and C(4,9) do the following:

a. Is ΔABC equilateral, isosceles, or scalene? Justify your answer.



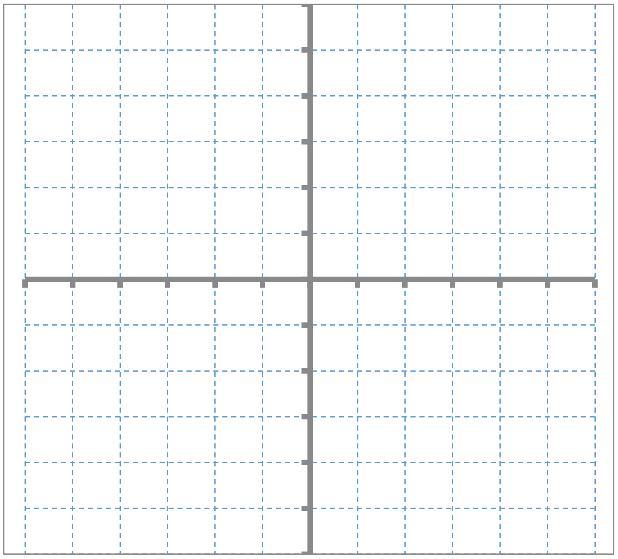
b. Is ΔABC a right triangle? Justify your answer.

2. The lines $y = 2,\ y = x - 4,\ and\ y = -\frac{1}{2}x - 1$ create a triangle. Is it scalene, isosceles, or equilateral? Support your answers with numbers.



3. Points A (1,4) and B (5,-4) and point C (somewhere on the *y*-axis) are the vertices of a right triangle.

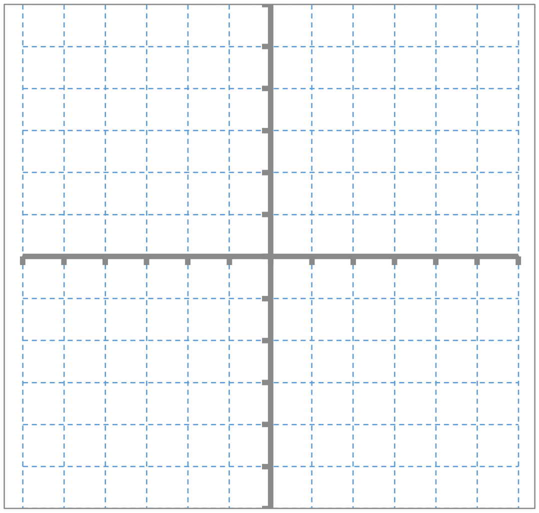
a. If angle A is a right angle, where on the *y*-axis can point C be?



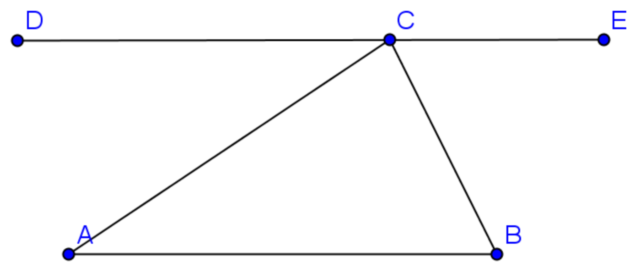
b. If angle B is a right angle, where on the *y*-axis can point C be?

c. If point C is a right angle, find the two places on the *y*-axis where it can be.

4. Given points A (2,-3) and B(-1,0), find the coordinates of all points C such that ΔABC is an isosceles right triangle. Hint: there are six of them!

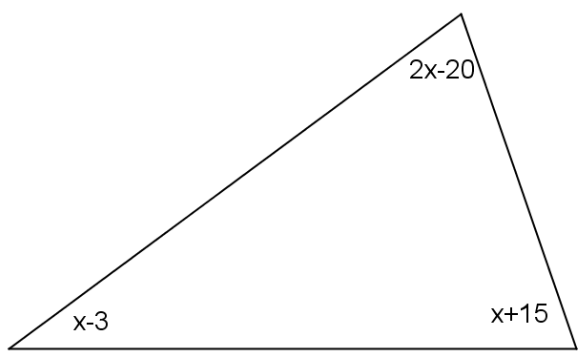


5. Why is the sum of the measures of the angles of a triangle always 180°? Use this diagram to explain. Segment $\overline{CD}$ is drawn to be parallel to $\overline{AB}$.

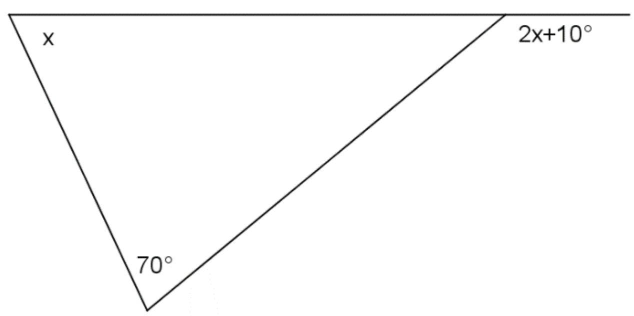


6. Find the value of *x* in each part below:

a.

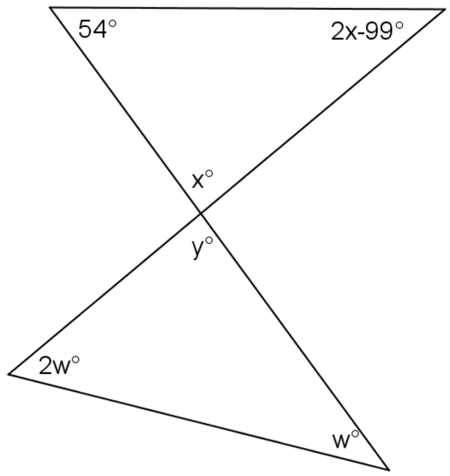


b.

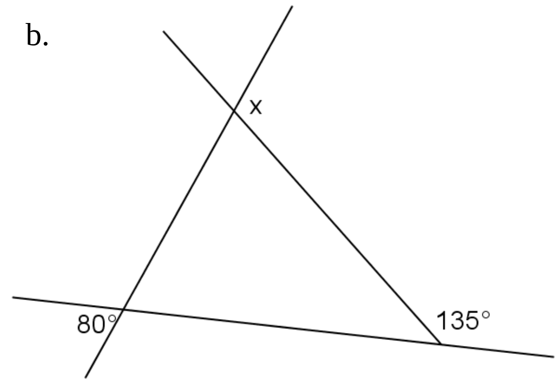


7. Find the values of all variables in the diagrams below:

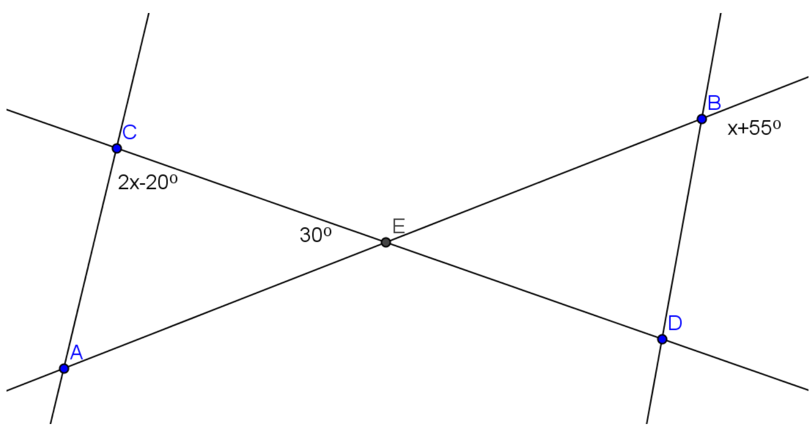
a.



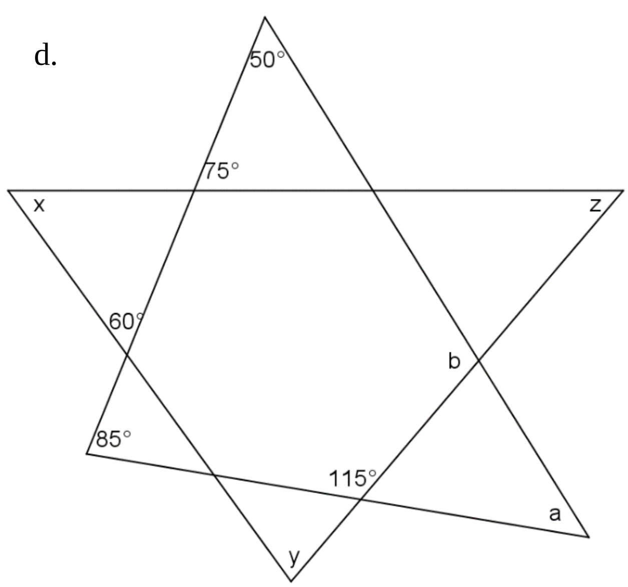
b.



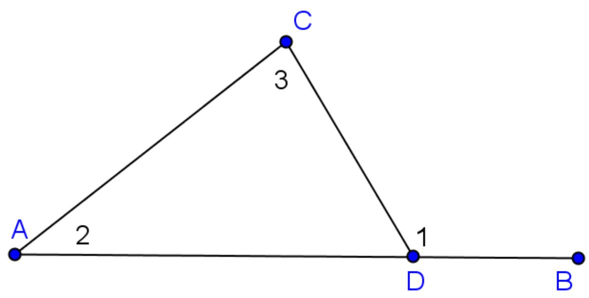
c. $\overline{AC} \parallel \overline{BD}$



d.

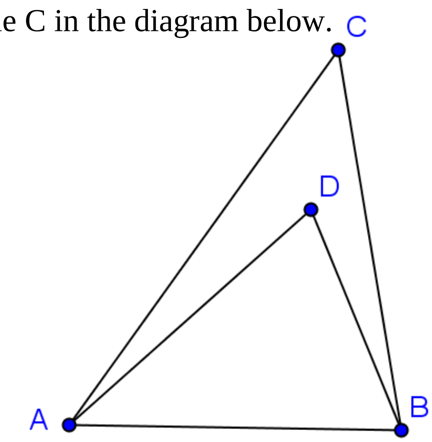


8. Carefully explain why an exterior angle of a triangle (angle 1 below) is equal to the sum of the ***two remote interior angles*** (angles 2 and 3 below).

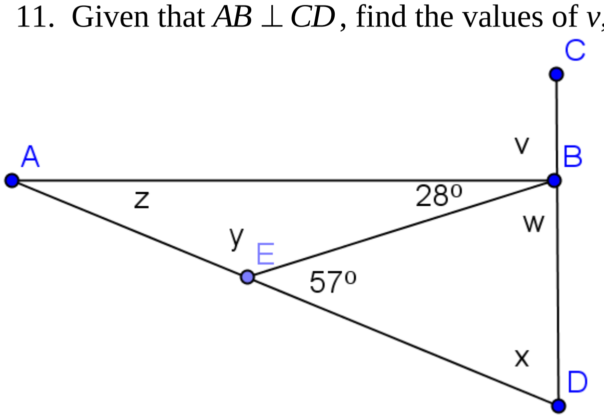


9. Find sum of the exterior angles of a triangle (one at each vertex).

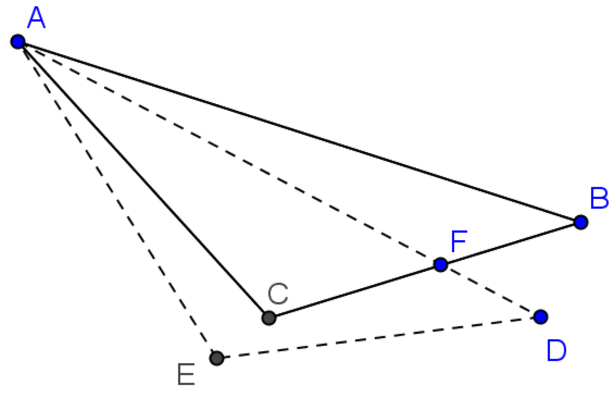
10. Explain why the measure of angle D must exceed the measure of angle C in the diagram below.



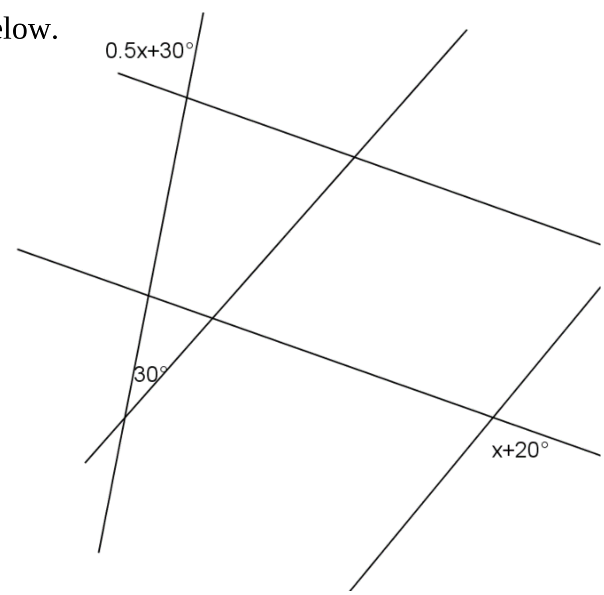
11. Given that $\overline{AB} \perp \overline{CD}$, find the values of *v*, *w*, *x*, *y*, and *z*.



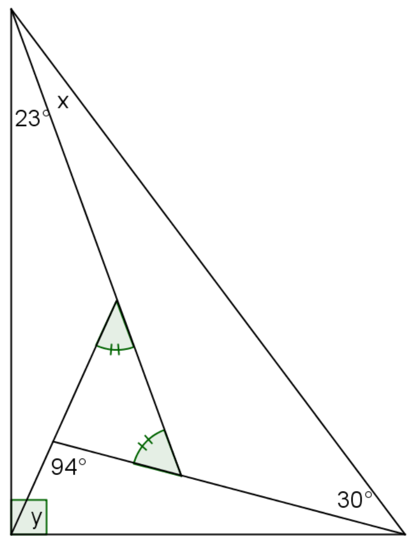
12. Triangle ABC's angles are 30°, 35°, and 115° respectively. It is rotated 10° clockwise around point A, forming triangle ADE. Find the measure of angle AFC. Note: the 10-degree rotation means that angles EAC and FAB measure 10°.



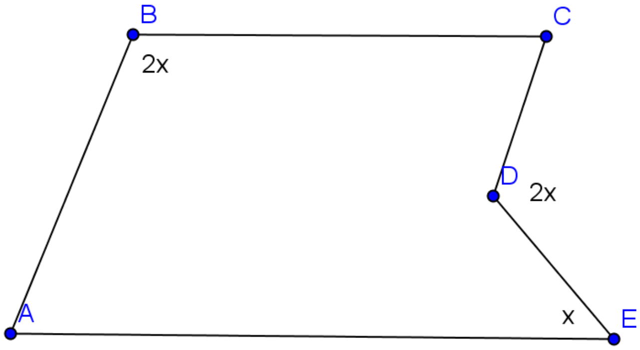
13. Find *x* given that there are two pairs of parallel lines below.



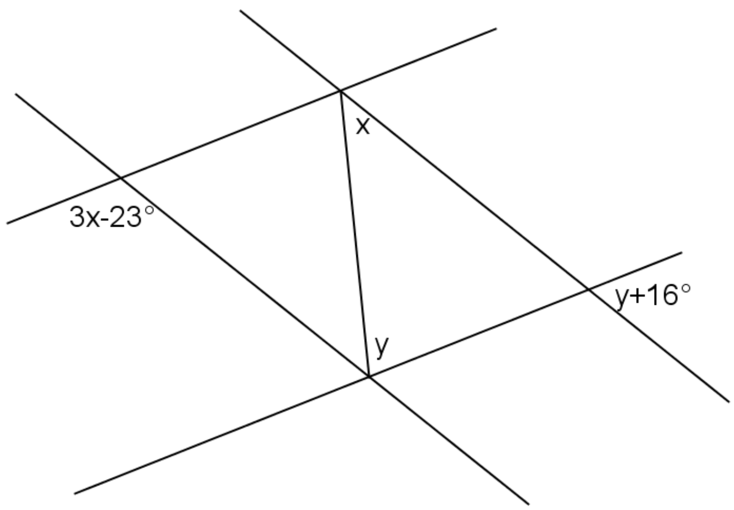
14. Find the values of *x* and *y* (acute angle) in the diagram below.



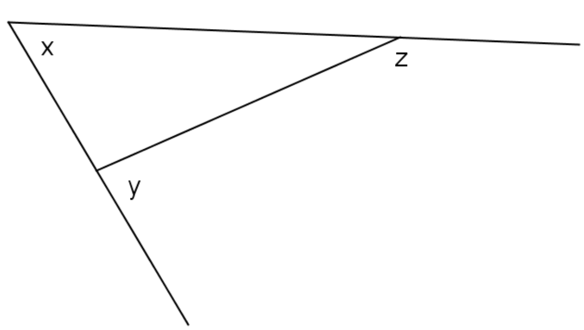
15. $\overline{AE} // \overline{BC}$ *amd* $\overline{AB} // \overline{CD}$. Find *x*.



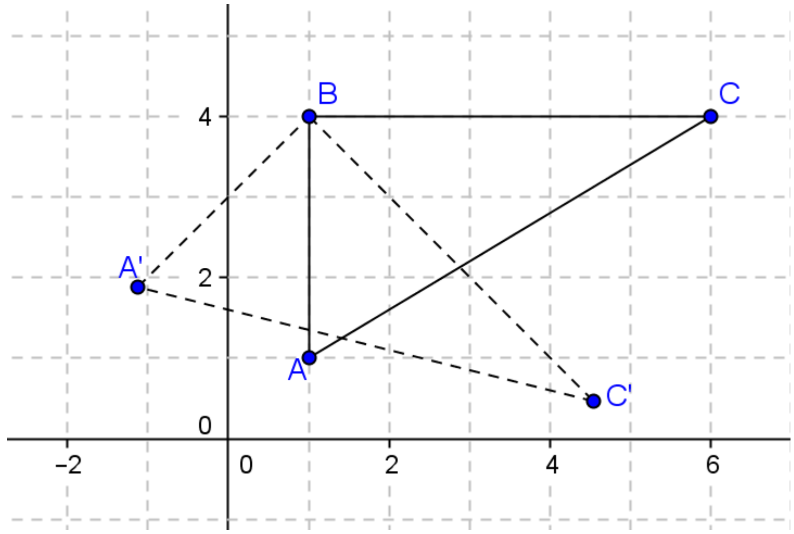
16. Given two pairs of parallel lines find *x* and *y*.



17. Find *x* in terms of *y* and *z*.



18. The measures of angles A, B, and C below are 60°, 90°, and 30° respectively. The triangle is rotated 45° clockwise about point B. Find the following:



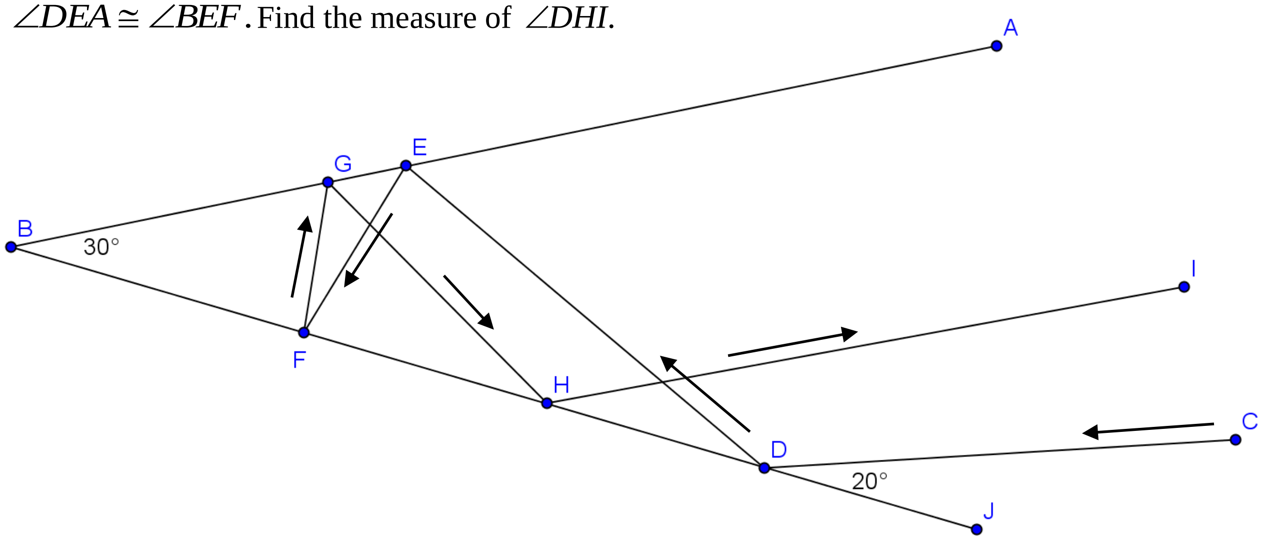
a. Angle ABA'

b. The acute angle $\overline{BC'}$ makes with $\overline{AC}$

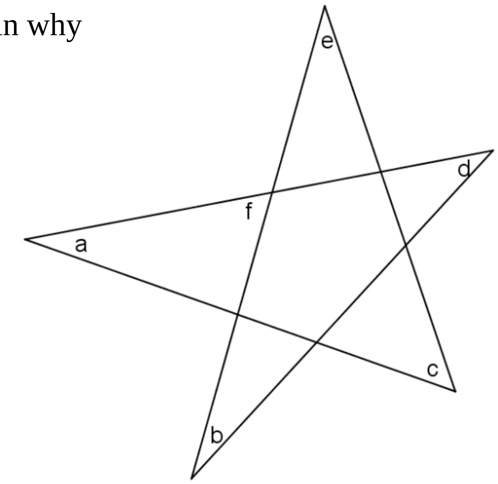
c. The acute angle $\overline{A'C'}$ makes with $\overline{AC}$

d. The acute angle $\overline{A'C'}$ makes with the y-axis

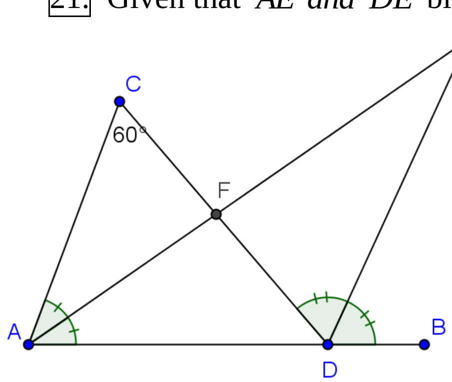
19. In the diagram below, imagine a ball starting at point C and bouncing around between "walls" $\overline{AB}$ *and* $\overline{BJ}$. At each bounce, the "angle in" equals the "angle out", so, for example $\angle CDJ \cong \angle EDF$ and $\angle DEA \cong \angle BEF$. Find the measure of $\angle DHI$.



20. In the diagram below, explain why $f = b + d$. Use this to explain why the sum of the vertex angles $(a + b + c + d + e)$ must equal 180°.



21. Given that $\overline{AE}$ *and* $\overline{DE}$ bisect angles CAB and CDB respectively, find the measure of angle E.



Answers

1a. isos since AB= $\sqrt{65}$, AC= $\sqrt{65}$; BC= $\sqrt{52}$ b. no; slopes are 1/8; 7/4, and -3/2 → no negative recips

2. meet at (-6,2), (6,2), and (2,-2) so side lengths are 12, $4\sqrt{5}$, *and* $4\sqrt{2}$ so scalene

3. A is a right angle if slope of AC=0.5 so C is (0,3.5); B is a right angle if slope of BC=0.5 so C is (0,-6.5); C is a right angle if AC is perp to BC so if C is at (0,y) then $\frac{y-4}{-1} = -1/\frac{y+4}{-5}$ and y= $\pm\sqrt{11}$

4. (2,0), (-1,-3), (2,3), (-4,-3), (5,0), (-1,-6)

5. $\angle BAC \cong \angle DCA$ and $\angle ABC \cong \angle ECB$ b/c alt interior; since $\angle DCA + \angle ACB + \angle BCE = 180^\circ$, can substitute to show that $\angle BAC + \angle ACB + \angle ABC = 180^\circ$ 6a. 47 b. 60

7a. x=75; y=75; w=35 b. 125 c. 45 d. x=45 y=80 z=55 a=45 b=110

8. $\angle 1 + \angle ADC = 180^\circ$; $\angle 2 + \angle 3 + \angle ADC = 180^\circ$; so by transitivity we know $\angle 1 + \angle ADC = \angle 2 + \angle 3 + \angle ADC$ and subtracting angle ADC from both sides yields $\angle 1 = \angle 2 + \angle 3$

9. 360° 10. DAB<CAB and DBA<CBA and both triangles' angles sum to 180…

11a. v=90° w=62° x=61° y=123° z=29° 12. 45° 13. 80 14. x=17° and y=66°

15. 60° 16. x=42° ; y=61° 17. 180-(180-y)-(180-z)=y+z-180 18a.45° b. 75° c.45° d.75° 19. 40°

20. The third angle in the small triangles with angles a and f must be equal to e+c (also by exterior angles). So, in this triangle, the sum of the angles is 180 is $(a + b + c + d + e)$ 21. 30°

Unit 2 Handout #2: Triangle Congruence

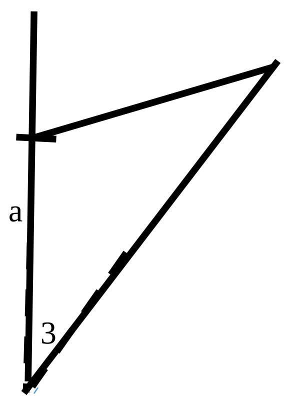
We know that triangles are congruent when corresponding sides and angles are all congruent. Now we want to look for short-cuts. Do we need to show all three corresponding sides and three corresponding angles of two triangles are congruent to establish triangle congruency, or are there some smaller sets of congruent parts that can establish triangle congruency.

Part I: Activity

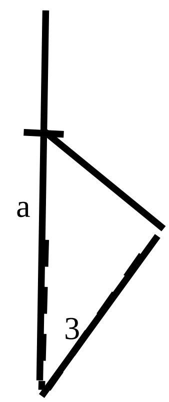
You have transparencies with three line segments and three angles. The line segments have portions marked *a*, *b*, and *c*. But they extend beyond those lengths. Your job is to create as many different triangles as possible with each set of restrictions below. The goal of this activity is to see what characteristics are necessary to conclude that two triangles are congruent.

Example: create triangles with a side of length *a* adjacent to an angle of measure 3.

One person may create this triangle:



While another creates this:



So we should conclude that there are potentially many different triangles that have an angle of 3 adjacent to a side of length *a*. And that one side next to one angle (nicknames "side-angle" or "SA") is not enough to uniquely identify a triangle.

Here are some to try. For each one, determine whether there are no possible triangles, only one possible triangle, two possible triangles, or many possible triangles.

1. One side is length of length *a*, one of length *b*, and one of length *c*. (with any angles—angles 1, 2, and 3 do not need to be included at all)

2. One side is length *a*, one is length *b*, and the angle between them is 3. (angles 1 and 2 do not need to be included, nor does a side of length *c*)

3. One angle of measure 1, one of measure 2, and one of measure 3. (the sides can be any lengths, not necessarily *a*, *b*, and *c*)

4. One side of length *a* the angle opposite this side equal to angle 3.

5. A side of length *b* between angles of measure 1 and 2.

6. A side of length *b* adjacent to an angle of measure 2 and opposite an angle of measure 3.

7. Sides of lengths *a*, *b*, and *c* and an angle of measure 1 somewhere in the triangle.

8. One side of length *a*, one side of length *b*, and an angle opposite the first side equal to angle 3.

9. One angle of 3 and one angle of 2.

Summarize:

List a few ways that triangles can be proven congruent.

Part II: Practice Naming and Identifying Congruent Triangles

Some ways of establishing triangle congruence are:

-SSS: three sides of one triangle are each congruent to the corresponding sides of another triangle.

-SAS: two sides of one triangle are each congruent to the corresponding sides of another triangle and the angles between these two sides in the two triangles are also congruent.

-ASA: two angles of one triangle are each congruent to the corresponding angles of another triangle and the sides between these two angles in the two triangles are also congruent.

-AAS: any two angles of one triangle are each congruent to the corresponding angles of another triangle and a side of one triangle is congruent to the corresponding side of the other triangle.

Note: we will add one more to this list at the end of this unit—it is called Hypotenuse-Leg and it only works in right triangles.

1. Which of the following are true? More than one may be correct.

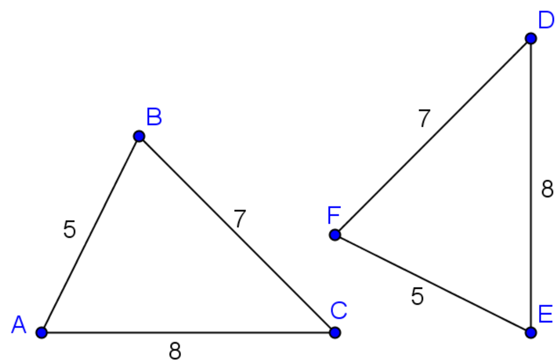
a. $\Delta ABC \cong \Delta DFE$

b. $\Delta ABC \cong \Delta EFD$

c. $\Delta BCA \cong \Delta FDE$

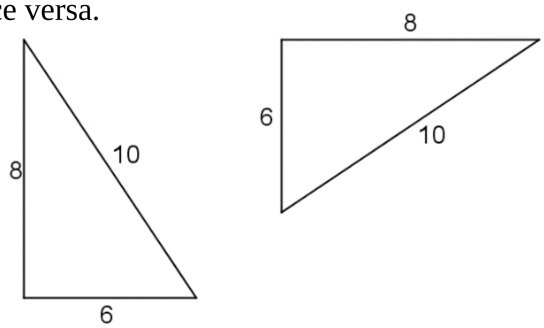
d. $\Delta CAB \cong \Delta EDF$

e. $\Delta CBA \cong \Delta DFE$

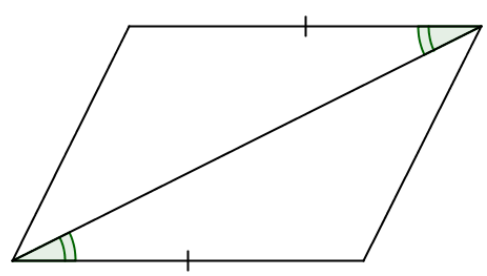


2. For each part, determine whether the two triangle MUST be congruent. If so, what justifies it (ie, SAS, SSS…). Note: the diagrams are not necessarily to scale; angles that appear acute may actually be obtuse, and vice versa.

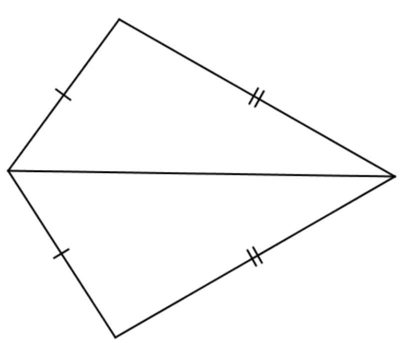
a.



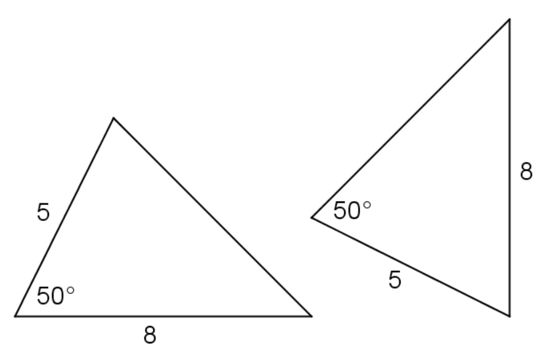
b.



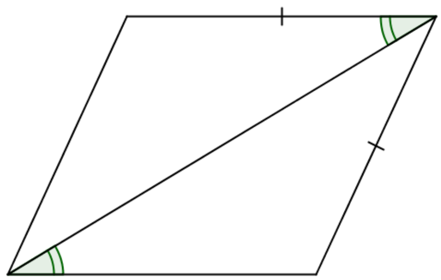
c.



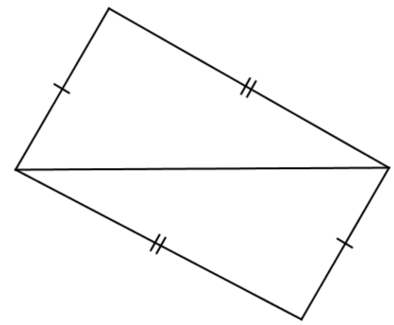
d.



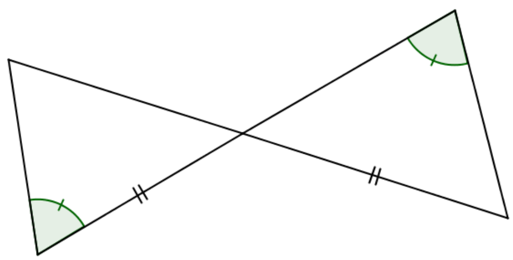
e.



f.



g.



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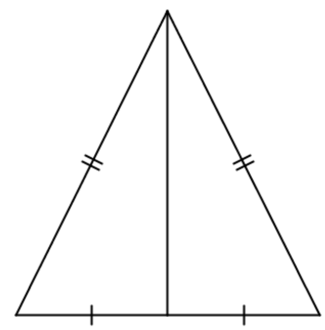
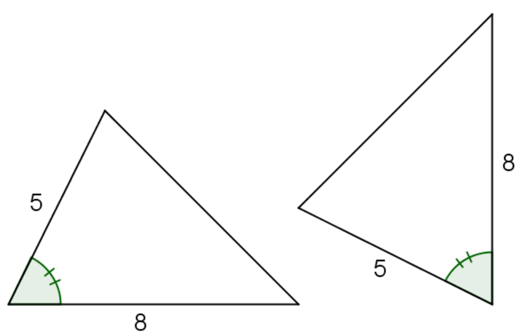
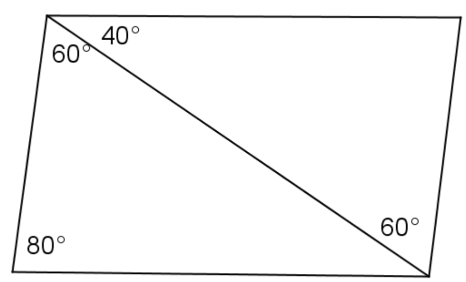


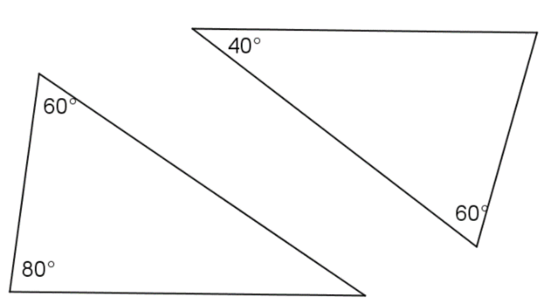
i.



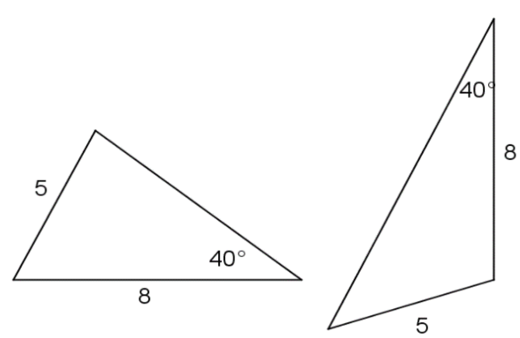
j.



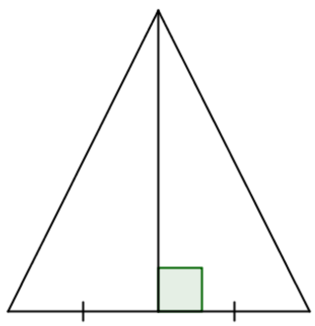
k.



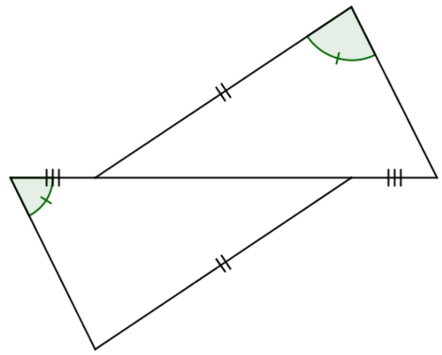
l.



m.



n.



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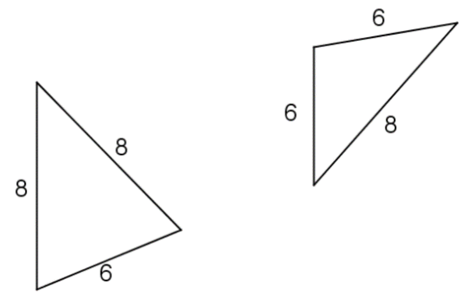

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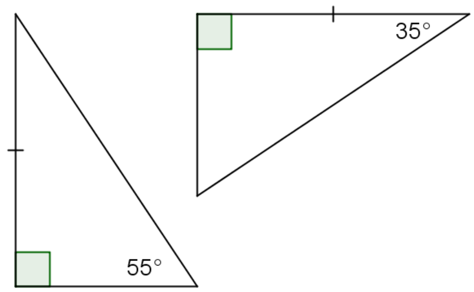

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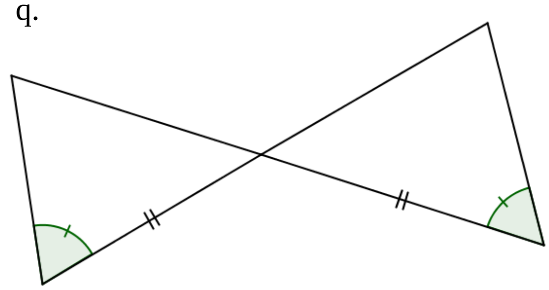

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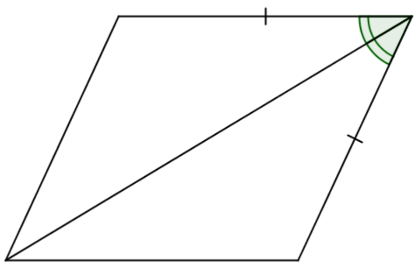

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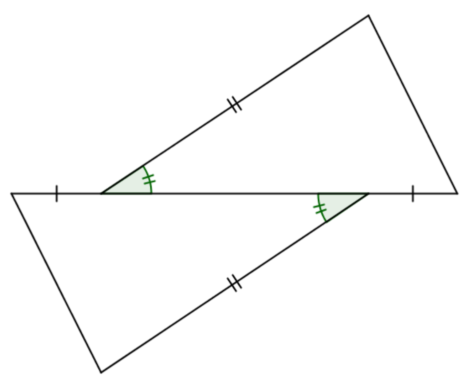

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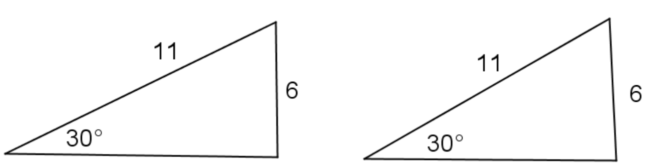

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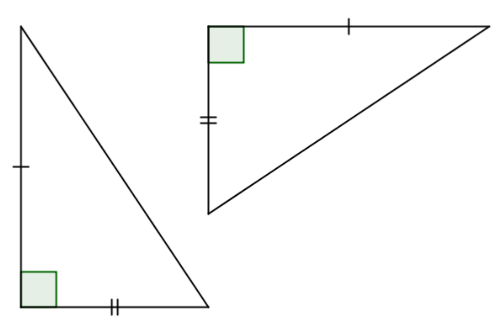

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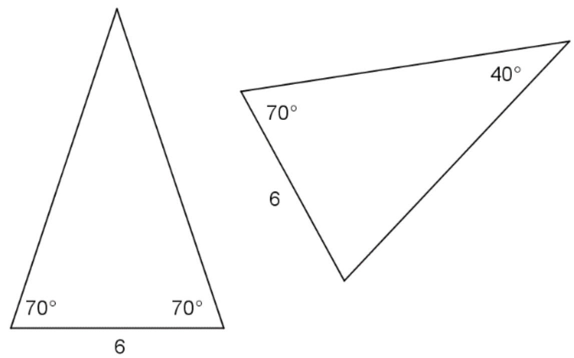

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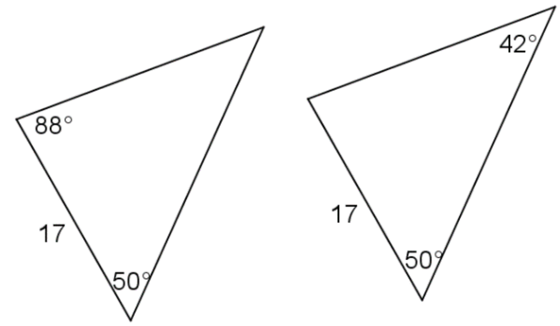


Answers

1. b, c, and e are correct

2a. SSS b. SAS c. SSS d. no e. no f. SSS g. no h. SSS i. SAS j. ASA k. no l. no

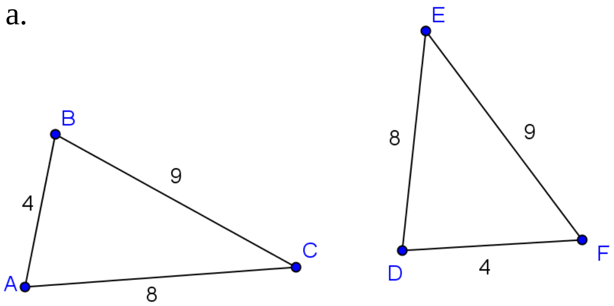
m. SAS n. no o. no p. AAS or ASA q. ASA r. SAS s. SAS t. no u. SAS v. AAS or ASA

w. AAS or ASA

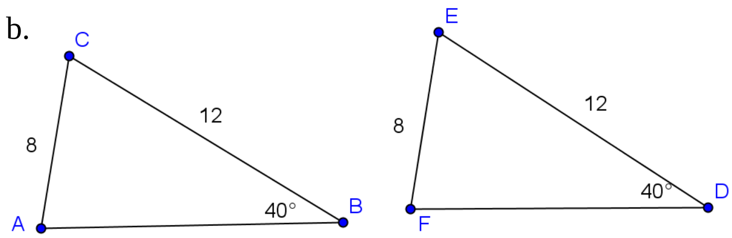
Unit 2 Handout #3: Triangle Congruence

Example #1: In each part below, determine whether the two triangles must be congruent. If so, write any valid congruence statement. Diagrams are not necessarily to scale.

a.



b.

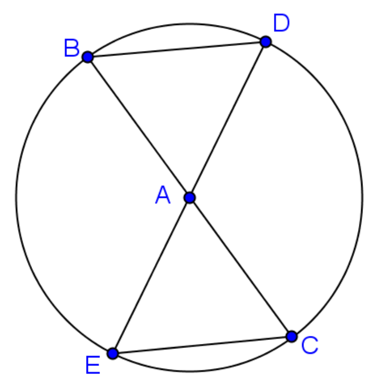


Solution:

a. Yes, the two triangles are congruent by SSS. One relation is $\Delta ABC \cong \Delta DFE$.

b. No; SSA is not a valid congruence relation, so we do not know if the two triangles are congruent. Note: it may be the case that $\angle A$ is acute and $\angle F$ is obtuse, or vice versa. That is why the triangles may not be congruent.

Example #2: Diameters $\overline{BC}$ *and* $\overline{DE}$ meet at A, the center of the circle. Must it be the case that $\Delta ABD \cong \Delta AEC$? Justify your answer.



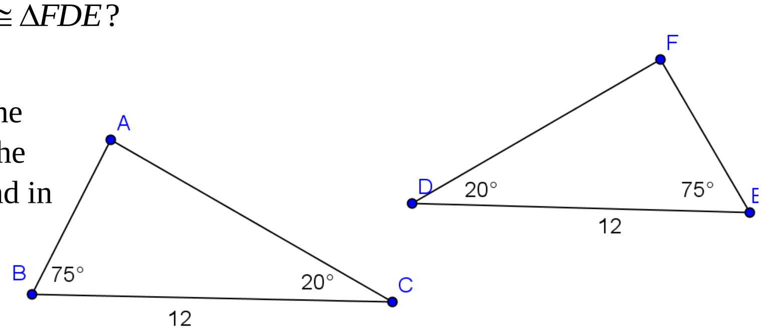
Solution:

Yes, it must be. The triangles are congruent by SAS. Angles BAD and EAC are congruent because they are vertical angles. And sides $\overline{AD}, \overline{AB}, \overline{AE}$, *and* $\overline{AC}$ are all congruent because they are radii of circle A. Note: in this case, one could also state that $\Delta ABD \cong \Delta ACE$.

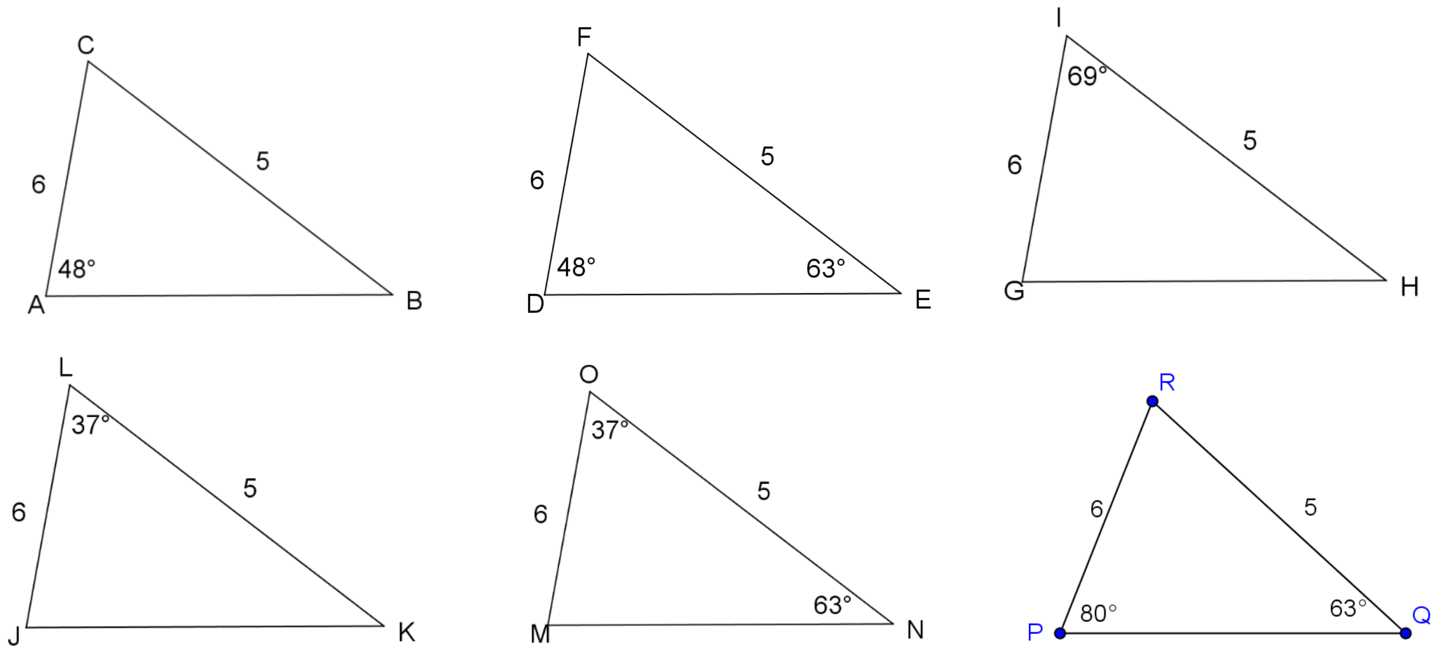
Example #3: Must it be the case that $\Delta ABC \cong \Delta FDE$?

Solution:

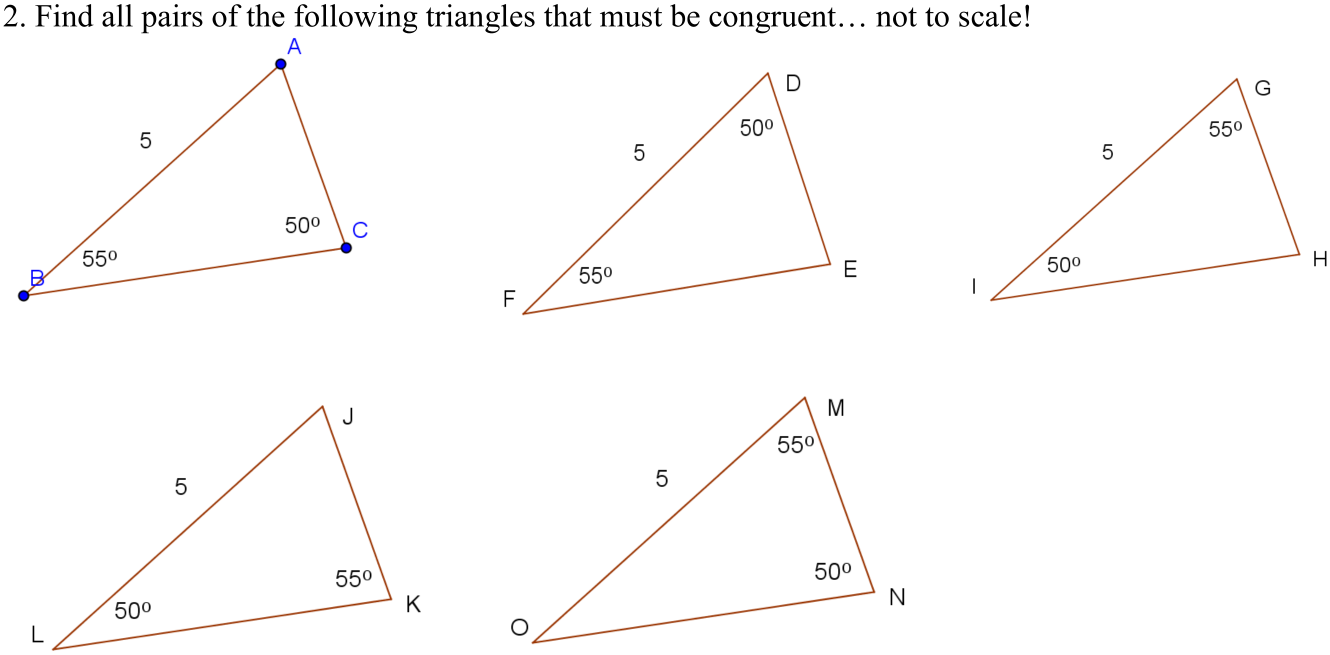
The two triangles are congruent by ASA. But the statement $\Delta ABC \cong \Delta FDE$ is not true because the congruent parts of the triangle do not correspond in the statement. Instead, one should say that $\Delta ABC \cong \Delta FED$.



1. Find all sets of the following triangles that must be congruent… not to scale!

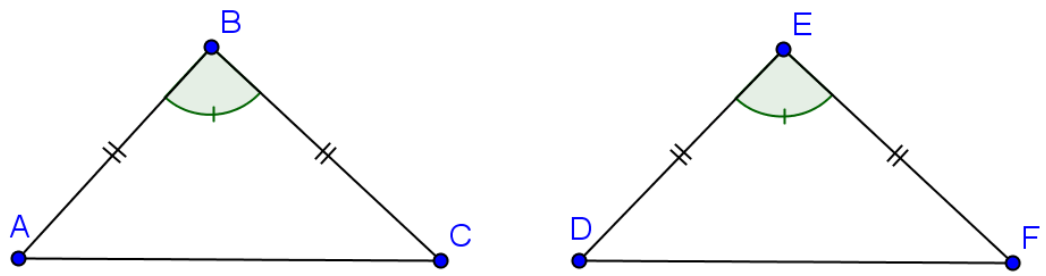


2. Find all pairs of the following triangles that must be congruent… not to scale!

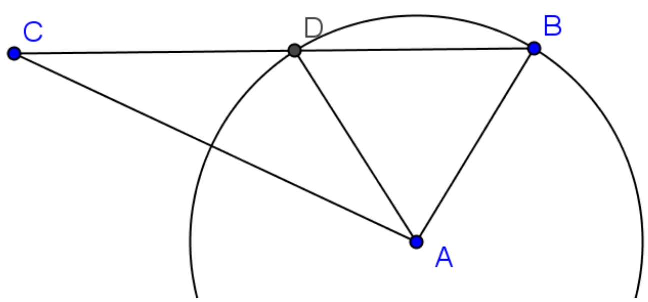


3. In the following diagram, which are true? Circle all that apply.

a. $\Delta ABC \cong \Delta DEF$ b. $\Delta ABC \cong \Delta FED$ c. $\Delta ABC \cong \Delta DFE$ d. $\Delta ABC \cong \Delta CBA$

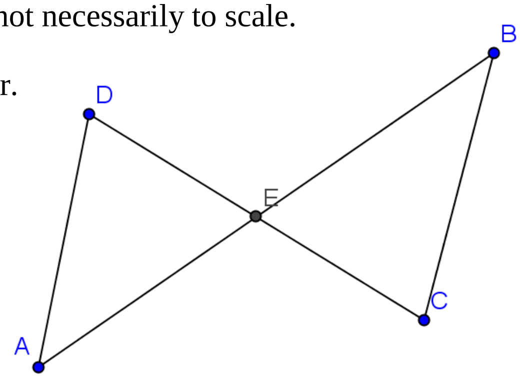


4. Because A is a circle, we know that $\overline{AB} \cong \overline{AD}$. Look at triangles ABC and ADC. What does this diagram tell us about whether SSA establishes triangle congruence?

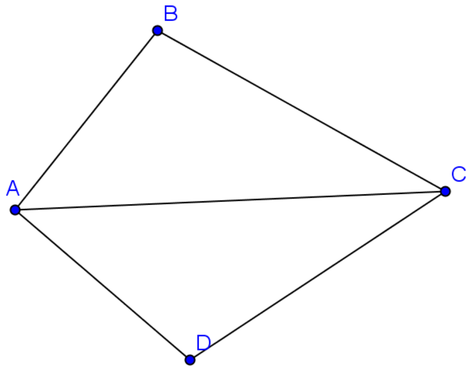


5. In each part below, determine whether the pairs of triangles are congruent. Justify your answers. Be careful with the way the triangles are named! Note: figures are not necessarily to scale.

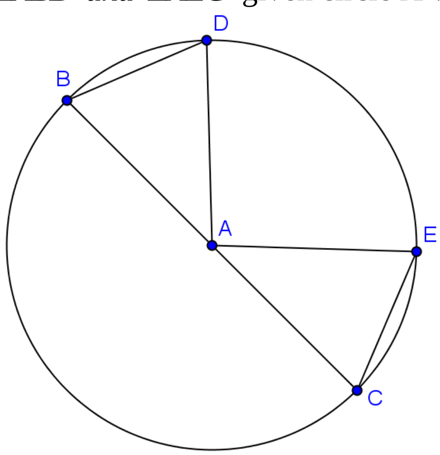
a. ΔAED *and* ΔBEC given that $\overline{AB}$ *and* $\overline{CD}$ bisect each other.



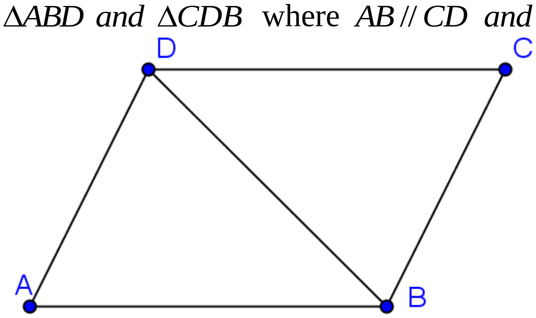
b. ΔABC *and* ΔADC given that $\overline{AC}$ bisects $\angle DAB$ *and* $\overline{AB} \cong \overline{AD}$



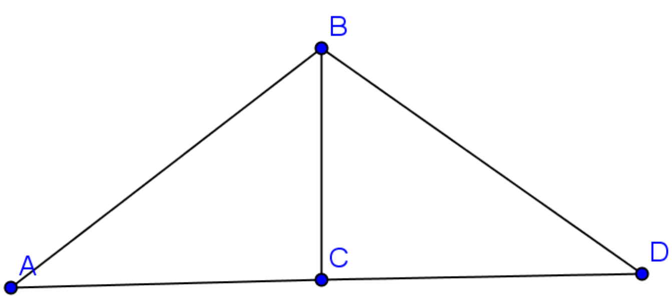
c. ΔABD *and* ΔAEC given circle A where $\overline{BD} \cong \overline{CE}$



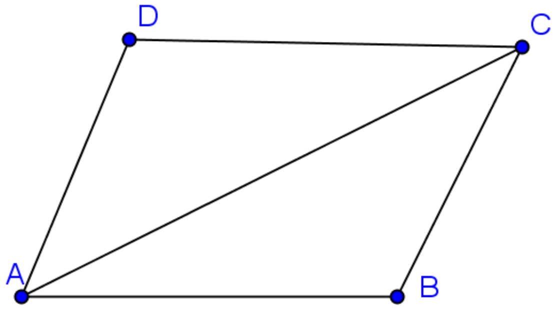
d. ΔABD *and* ΔCDB where $\overline{AB} // \overline{CD}$ *and* $\overline{AD} // \overline{CB}$



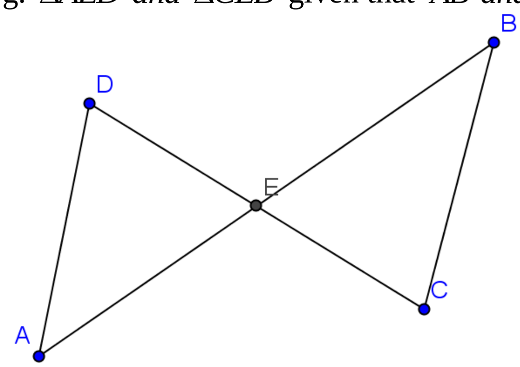
e. ΔABC *and* ΔDBC given $\overline{AB} \cong \overline{DB}$ and $\angle A \cong \angle D$



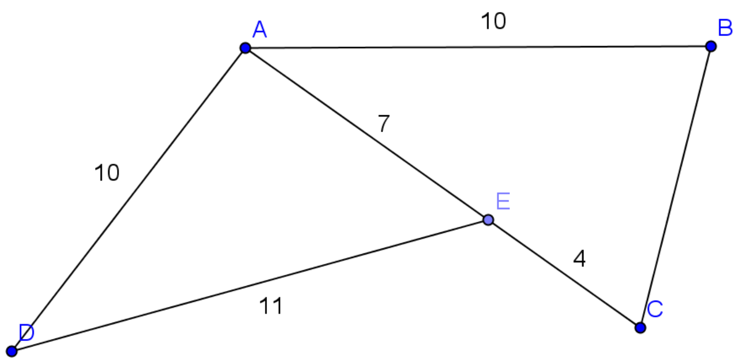
f. ΔABC *and* ΔCDA where $\overline{CD} \cong \overline{AB}$ and $\overline{CD} // \overline{AB}$



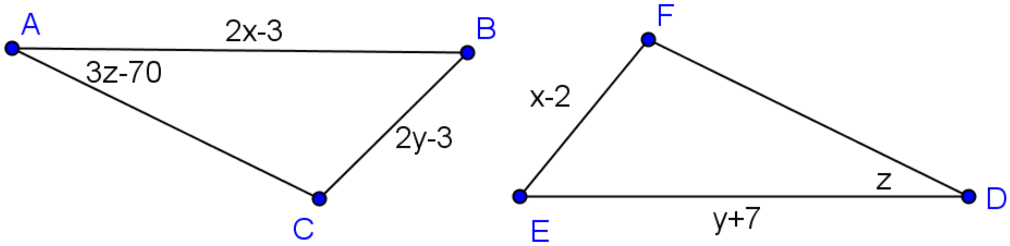
g. ΔAED *and* ΔCEB given that $\overline{AB}$ *and* $\overline{CD}$ bisect each other.



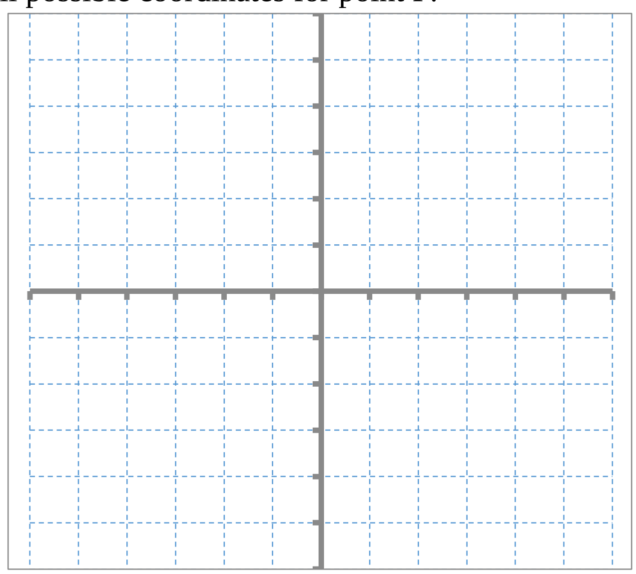
6. Given that $\angle D \cong \angle BAC$, find the length BC – justify your answer.



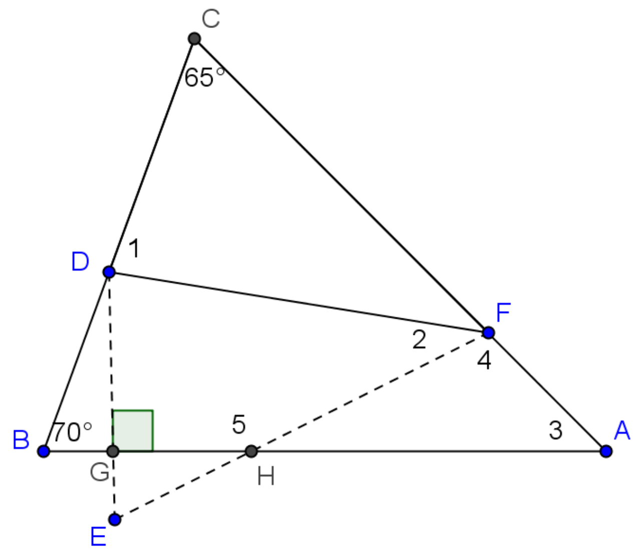
7. $\Delta ABC \cong \Delta DEF$ in the diagram below. Find the values of *x*, *y*, and *z*.



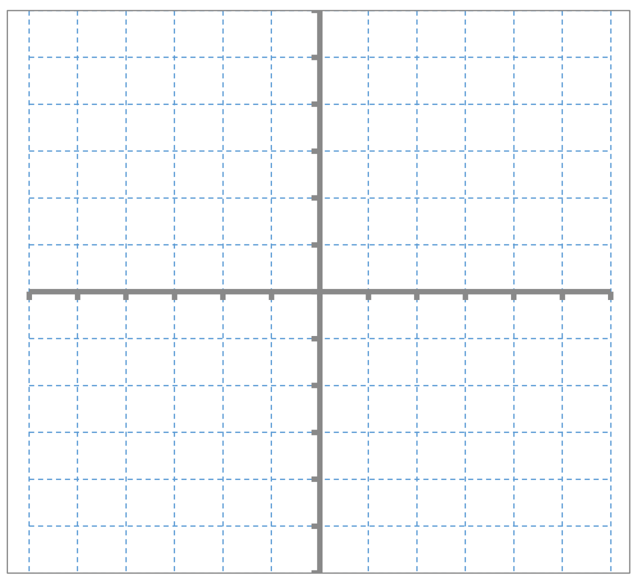
8. The vertices of triangle ABC are A(-1,2), B(5,2), and C(3,5). D's coordinates are (-3,2) and E's coordinates are (-3,-4). Given that ΔDEF ≅ ΔABC, find all possible coordinates for point F.



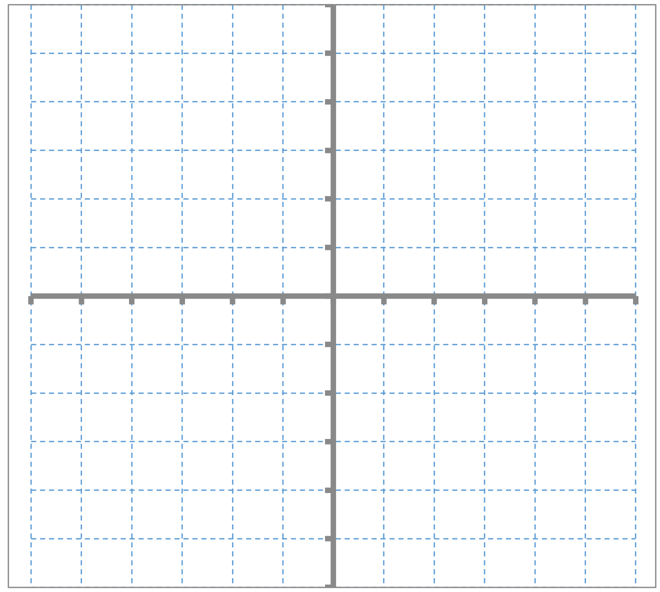
9. Given ΔABC. From vertex C, part of the triangle is folded over segment $\overline{DF}$, resulting in ΔEDF (which must be congruent to ΔCDF since it is folder over). The segment $\overline{DF}$ is chosen such that the folded-over side $\overline{DE}$ is perpendicular to the base of the original triangle $\overline{AB}$. Find the measure of angles 1, 2, 3, 4, and 5.



10. Given the points A (-5,0), B (5,0), C (2,6), D (-2,-3), E (6,3), and F (0,6). Are triangles ABC and DEF congruent? Support your answer with computations.

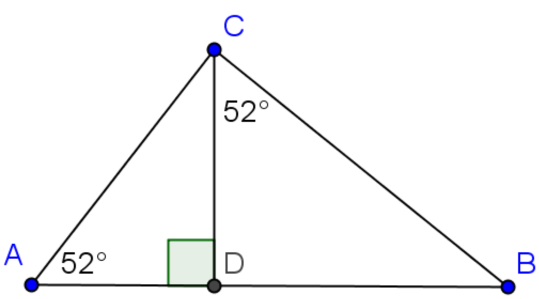


11. Given the points A(5,2), B(-1,3), C(-5,-2), D(1,-3), and E(0,0), find as many pairs of congruent triangles as you can. All of the vertices of all triangles must be among these five points. And don't bother saying things like ABC is congruent to ABC- the ones you find should not have the same vertices as each other! (From *Exeter Math 2*)

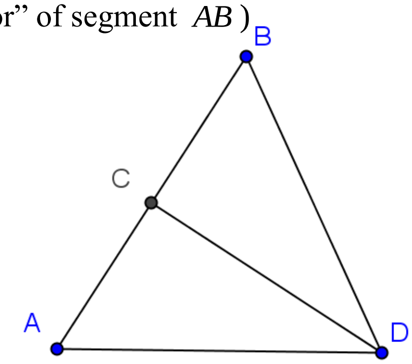


12. In each part below, determine whether the pairs of triangles are congruent. Justify your answers. Be careful with the way the triangles are named! Note: figures are not necessarily to scale.

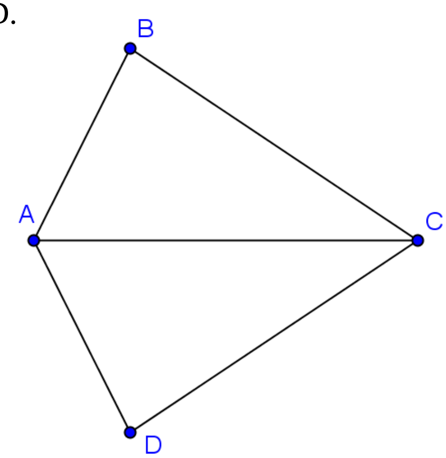
a. ΔADC *and* ΔCDB



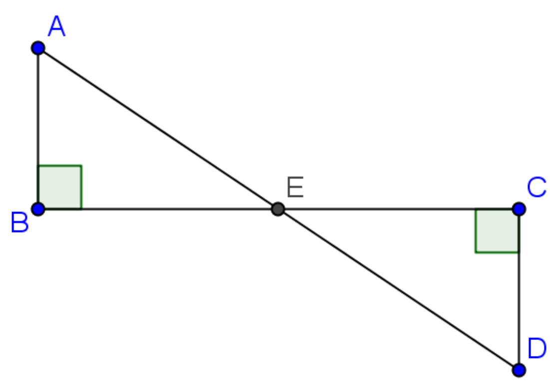
b. ΔADC *and* ΔBDC where $\overline{DC} \perp \overline{AB}$ and $\overline{DC}$ bisects $\overline{AB}$ (note: $\overline{DC}$ is called the "perpendicular bisector" of segment $\overline{AB}$)



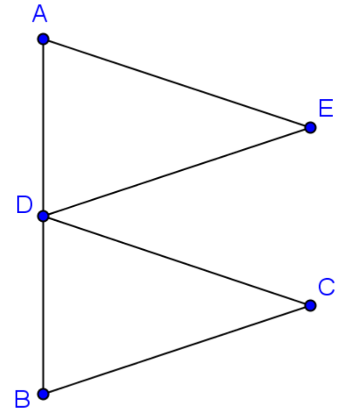
c. ΔACB *and* ΔACD given that $\overline{AC}$ bisects angles BAD and BCD.



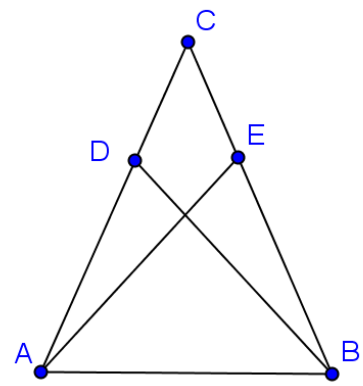
d. ΔABE *and* ΔDCE given that $\overline{DC} \cong \overline{BA}$



e. ΔADE *and* ΔDBC given that D is the midpoint of $\overline{AB}$ and $\overline{DE} // \overline{BC}$ *and* $\overline{AE} // \overline{DC}$.



f. ΔCDB *and* ΔCEA given that $\angle CDB \cong \angle CEA$ and $\overline{CE} \cong \overline{CD}$



Answers

1. $\Delta DEF \cong \Delta GHI$ and $\Delta JKL \cong \Delta MNO \cong \Delta PQR$ 2. $\Delta DEF \cong \Delta IHG$ and $\Delta ABC \cong \Delta OMN$

3. a, b, and d are true 4. SSA does not work (still!)

5a. yes; by SAS b. yes, by SAS c. yes, by SSS

5d. yes by ASA, since $\angle ADB \cong \angle CBD$ *and* $\angle ABD \cong \angle CDB$ by alternate interiors

5e. no; SSA does not establish congruence

5f. Yes by SAS since $\angle BAC \cong \angle ACD$ as they are alt interior angles

5g. no; names are off. $\Delta AED \cong \Delta BEC$ not ΔCEB

6. ΔABC congr to ΔADE by SAS so BC=7 7. x=7 and y=4 and z=35

8. (0,-2), and (-6,-2) 9. 1=80°; 2=35°; 3=45°; 4=110°; 5=155°

10. yes by SSS: sides are $10, 3\sqrt{5},$ *and* $\sqrt{85}$

11. Lots! $\Delta EDA \cong \Delta EBC$, $\Delta EBA \cong \Delta EDC$; $\Delta BDA \cong \Delta DBC$ *and* $\Delta BAC \cong \Delta DCA$

12a. no; they share angles and a side but it is not in the same place (opp 52 in one; 38 in other)

12b. yes by SAS c. yes by ASA d. yes by AAS e. yes by ASA.. $\angle A \cong \angle BDC$ and $\angle B \cong \angle ADE$

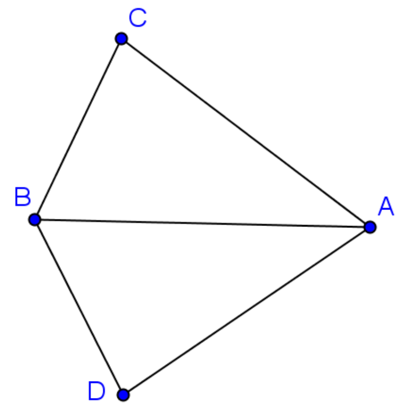
12f. yes by ASA since angle C is in both triangles (it can be hard to see, maybe draw the two triangles separately)

Unit 2 Handout #4: Triangle Congruence Proofs

In two-column proofs one proceeds methodically, using the "givens" along with theorems and properties in order to prove something. Each step needs to be carefully justified. Conclusions are in the left column, and justifications are in the right column. Justifications often include prior lines in the proof. Different teachers may have different ideas of the rigor required in proofs; your teacher should make his or her standards clear.

Example #1. Given $\overline{BC} \cong \overline{BD}$, $\overline{BA}$ **bisects** $\angle DBC$**. Prove** $\Delta ABC \cong \Delta ABD$

Solution



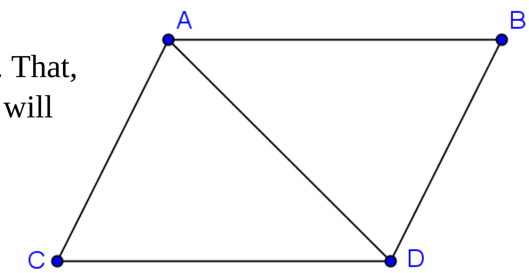
The first step in a triangle congruence proof is to determine which congruence short-cut to use. In this case, SAS looks appealing because the triangles share $\overline{AB}$ and the givens enable us to prove another side and the included angles congruent.

Once this has been determined, one needs steps to show one pair of corresponding sides is congruent, another pair is congruent, and the included angles are congruent:

Statement	Justification
1. $\overline{BC} \cong \overline{BD}$	given
2. $\overline{BA}$ bisects $\angle DBC$	given
3. $\angle CBA \cong \angle DBA$	2; definition of bisect
4. $\overline{AB} \cong \overline{AB}$	reflexive property
5. $\Delta ABC \cong \Delta ABD$	1, 3, 4, SAS

Example #2. Given: $\angle B \cong \angle C$, $\overline{AB} // \overline{CD}$ **Prove** $\Delta ABD \cong \Delta DCA$

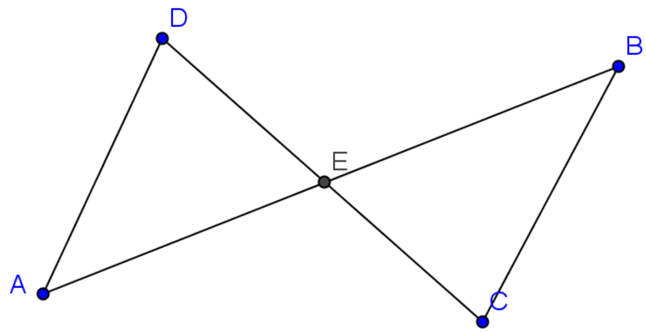
Solution:



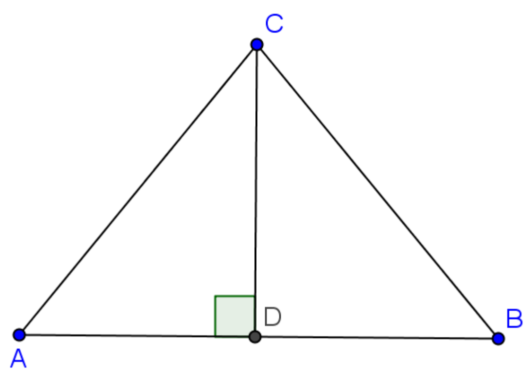
The parallel lines can enable us to show that $\angle CDA \cong \angle BAD$. That, with the other angle given and the side the two triangles share will let us prove them congruent using AAS.

Statement	Justification
1. $\angle B \cong \angle C$	given
2. $\overline{AB} // \overline{CD}$	given
3. $\angle CDA \cong \angle BAD$	2; alternate interior angles
4. $\overline{AD} \cong \overline{AD}$	reflexive property
5. $\Delta ABD \cong \Delta DCA$	1, 3, 4, AAS

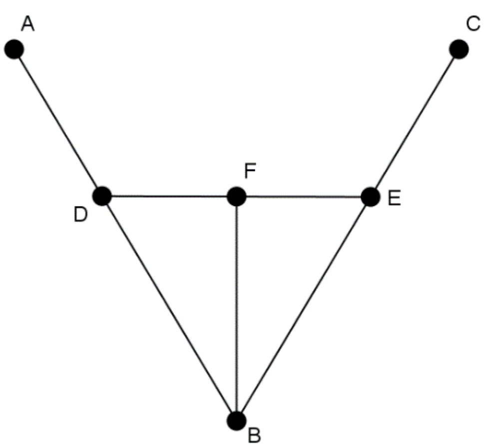
1. Given $\overline{AB}$ *and* $\overline{CD}$ bisect each other at point E. Prove: $\Delta AED \cong \Delta BEC$



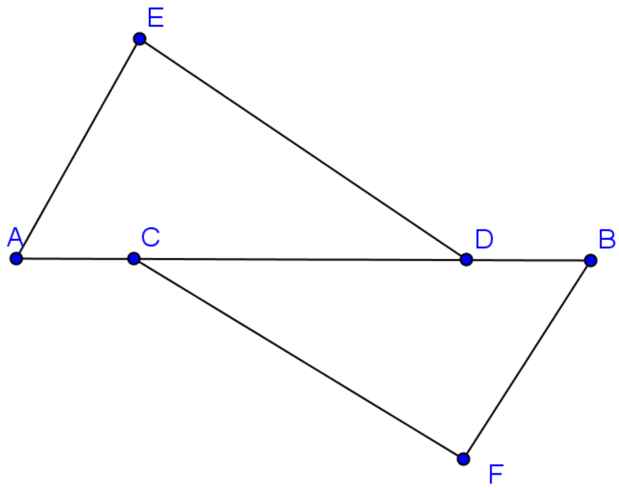
2. Given $\overline{CD}$ bisects $\angle ACB$; $\overline{CD} \perp \overline{AB}$. Prove: $\Delta ACD \cong \Delta BCD$



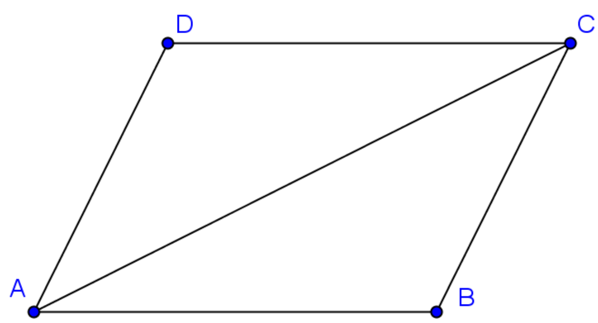
3. Given: F is the midpoint of $\overline{ED}$; $\overline{BD} \cong \overline{EB}$. Prove: $\Delta BDF \cong \Delta BEF$



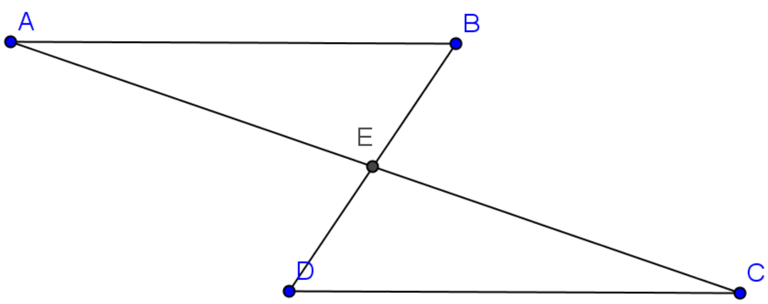
4. Given: $\overline{BD} \cong \overline{AC}$, $\overline{ED} \cong \overline{FC}$, $\overline{ED} \parallel \overline{FC}$ Prove: $\Delta CFB \cong \Delta DEA$



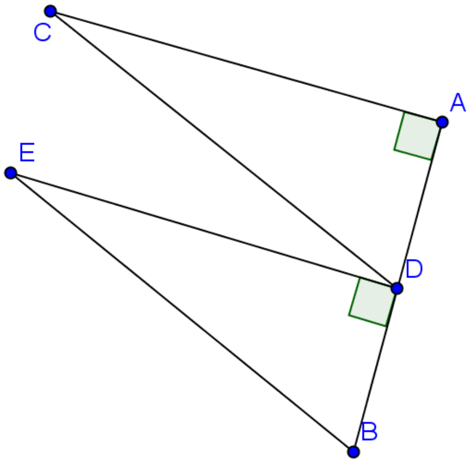
5. Given: $\overline{AD} \parallel \overline{BC}$, $\overline{CD} \parallel \overline{AB}$ Prove: $\Delta ADC \cong \Delta CBA$



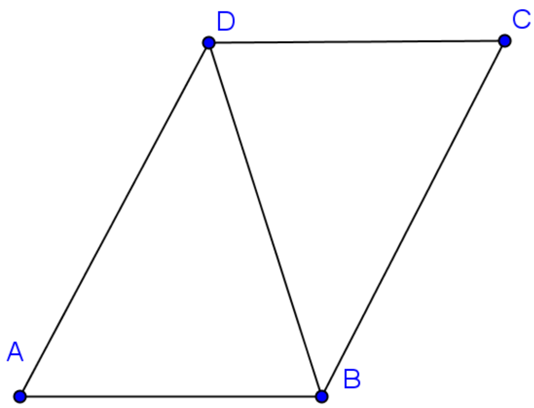
6. Given: $\overline{ED} \cong \overline{BE}$, $\overline{CD} \parallel \overline{AB}$ Prove: $\Delta ABE \cong \Delta CDE$



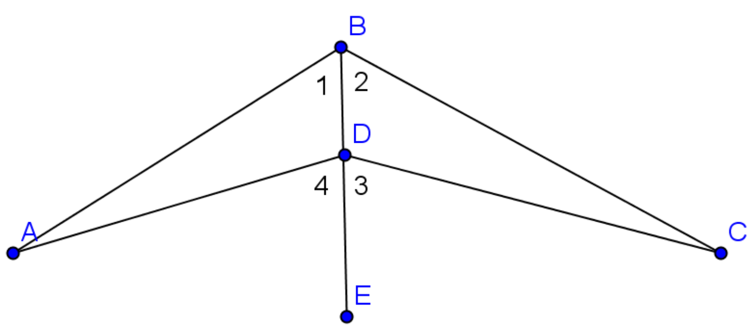
7. Given: $\overline{CD} \parallel \overline{EB}$; D is the midpoint of $\overline{AB}$ Prove: $\Delta ACD \cong \Delta DEB$
(note: you may use the fact that angles DAC and BDE are right angles, using a reason such as "given from diagram" or merely "given")



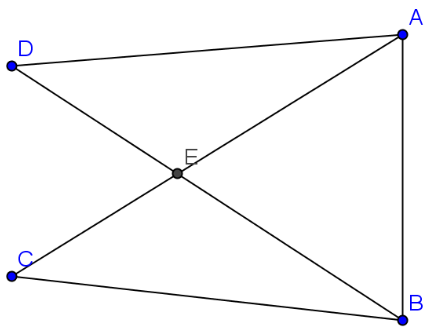
8. Given: $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$ Prove: $\Delta ADB \cong \Delta CBD$



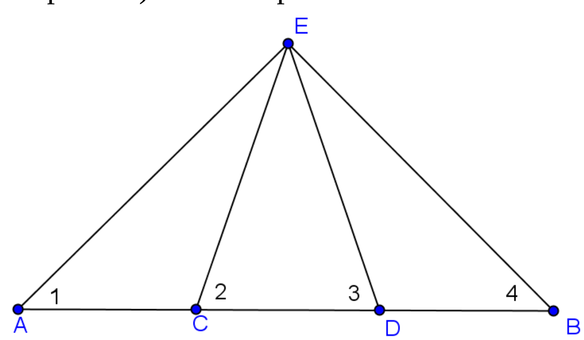
9. Given: $\angle 1 \cong \angle 2, \angle 3 \cong \angle 4$ Prove: $\Delta ADB \cong \Delta CDB$



10. Given: $\overline{ED} \cong \overline{CE}, \angle D \cong \angle C$ Prove: $\Delta ADE \cong \Delta BCE$



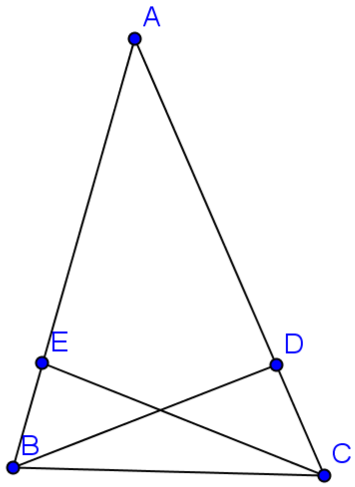
11 & 12 (one diagram for two proofs!) Given: points C and D trisect $\overline{AB}$; $\angle 1 \cong \angle 4, \angle 3 \cong \angle 2$



11. Prove $\Delta ACE \cong \Delta BDE$

12. Prove $\Delta ADE \cong \Delta BCE$

13. Given: $\overline{AD} \cong \overline{AE}$, $\angle ADB \cong \angle AEC$ Prove: $\Delta ADB \cong \Delta AEC$



Some answers. ***Note: there are many ways to do some of these proofs***. Think of these as representative answers, and certainly not the only acceptable ones. Your teacher should explain the level of detail he or she wants to see in your proofs.

1.

1. $\overline{AB}$ bisects $\overline{CD}$	given
2. $\overline{CD}$ bisects $\overline{AB}$	given
3. $\angle AED \cong \angle CEB$	vertical angles
4. $\overline{AE} \cong \overline{EB}$	2; definition of bisect
5. $\overline{DE} \cong \overline{EC}$	1; definition of bisect
6. $\Delta AED \cong \Delta BEC$	3, 4, 5, SAS

2.

1. $\overline{CD}$ bisects $\angle ACB$	given
2. $\overline{CD} \perp \overline{AB}$	given
3. $\angle ACD \cong \angle BCD$	1; definition of bisect
4. $\angle CDB$ *and* $\angle CDA$ are right angles	2; definition of perpendicular
5. $\angle CDB \cong \angle CDA$	4; all right angles are congruent
6. $\overline{CD} \cong \overline{CD}$	reflexive property
7. $\Delta ACD \cong \Delta BCD$	3, 5, 6, ASA

3.

1. F is the midpoint of $\overline{ED}$	given
2. $\overline{BD} \cong \overline{EB}$	given
3. $\overline{EF} \cong \overline{DF}$	1, def of bisect
4. $\overline{BF} \cong \overline{BF}$	reflexive property
5. $\Delta BDF \cong \Delta BEF$	2, 3, 4, SSS

4.

1. $\overline{BD} \cong \overline{AC}$	given
2. $\overline{ED} \cong \overline{FC}$	given
3. $\overline{ED} \parallel \overline{FC}$	given
4. $\overline{CD} \cong \overline{CD}$	reflexive property
5. $\overline{BC} \cong \overline{AD}$	1, 4 ,additive property
6. $\angle FCB \cong \angle EDC$	3, alternate interior angles
7. $\Delta CFB \cong \Delta DEA$	2, 5, 6, SAS

5.

1. $\overline{AD} \parallel \overline{BC}$	given
2. $\overline{CD} \parallel \overline{AB}$	given
3. $\angle BAC \cong \angle DCA$	2, alt int angles
4. $\angle BCA \cong \angle DAC$	1, alt int angles
5. $\overline{CA} \cong \overline{CA}$	reflexive prop
6. $\Delta ADC \cong \Delta CBA$	3, 4, 5, ASA

8.

1. $\overline{AB} \cong \overline{CD}$	given
2. $\overline{AD} \cong \overline{CB}$	given
3. $\overline{DB} \cong \overline{DB}$	reflexive prop
4. $\Delta ADB \cong \Delta CBD$	1, 2, 3, SSS

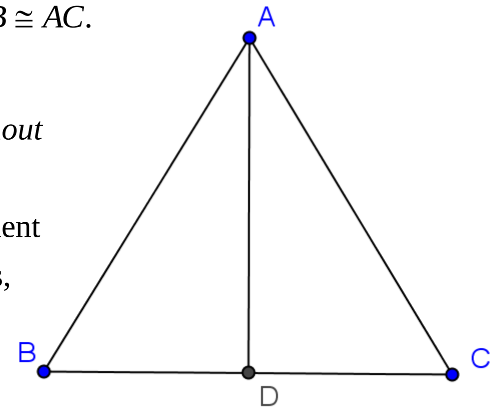
9.

1. $\angle 1 \cong \angle 2$	given
2. $\angle 3 \cong \angle 4$	given
3. $\angle BDA \cong \angle BDC$	2. Supplements of congruent angles are congruent
4. $\overline{DB} \cong \overline{DB}$	reflexive prop
5. $\Delta ADB \cong \Delta CDB$	1, 3, 4, ASA

Unit 2 Handout #5: CPCTC

CPCTC stands for "Corresponding Parts of Congruent Triangles are Congruent". It enables us to prove parts of triangles are congruent once we have proven the triangles congruent. This may be what the proof calls for. Sometimes we also need to prove parts of one pair of triangles are congruent in order to prove another pair of triangles congruent.

Example #1. Given $\overline{AD} \perp \overline{BC}$, D is the midpoint of $\overline{BC}$. Prove $\overline{AB} \cong \overline{AC}$.



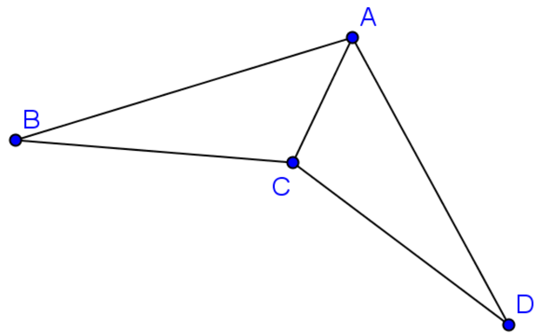
Solution

The key idea here is to prove triangles ABD and ACD congruent *without* using $\overline{AB} \cong \overline{AC}$, since it is not a given. Then we can use CPCTC to conclude that $\overline{AB} \cong \overline{AC}$, since they are corresponding parts of congruent triangles. With the givens above, and side $\overline{AD}$ shared by the triangles, we should use SAS.

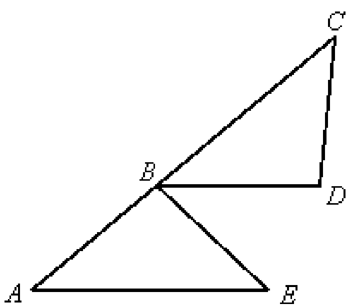
Statement	Justification
1. D is the midpoint of $\overline{BC}$	given
2. $\overline{BD} \cong \overline{CD}$	1; definition of midpoint
3. $\overline{AD} \perp \overline{BC}$	given
4. $\angle ADC, \angle ADB$ are right angles	3; definition of $\perp$
5. $\angle ADC \cong \angle ADB$	4; all right angles are congruent
6. $\overline{AD} \cong \overline{AD}$	reflexive property
7. $\Delta ABD \cong \Delta ACD$	2, 5, 6; SAS
8. $\overline{AB} \cong \overline{AC}$	7; CPCTC

1. Given: $\overline{AC}$ bisects $\angle BAD$ and $\angle BCA \cong \angle DCA$

Prove: $\overline{BC} \cong \overline{CD}$

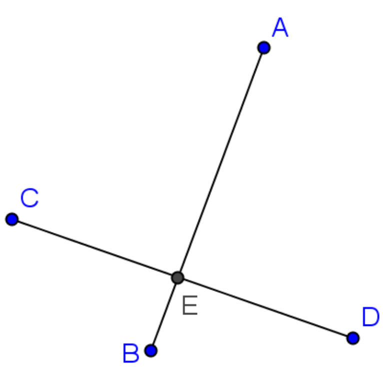


2. Given: $\angle C \cong \angle E$, $\overline{BC} \cong \overline{AE}$, $\overline{BE} \cong \overline{CD}$
Prove: $\angle A \cong \angle CBD$

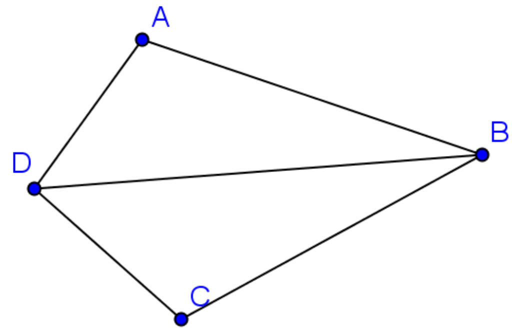


3. Given: $\overline{AB} \perp \overline{CD}$ and $\overline{AB}$ bisects $\overline{CD}$ (note: we can call $\overline{AB}$ the "perpendicular bisector" of $\overline{CD}$)

Prove: $\overline{AC} \cong \overline{AD}$

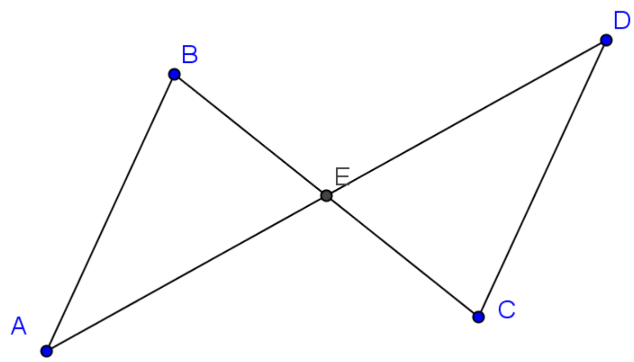


4. Given: $\overline{BD}$ bisects $\angle ABC$; $\overline{AB} \cong \overline{BC}$
Prove $\angle A \cong \angle C$



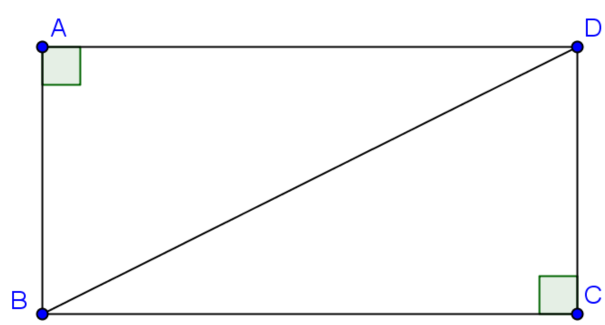
5. Given: $\overline{AB} \parallel \overline{CD}$ *and* $\overline{AE} \cong \overline{ED}$

Prove E is the midpoint of $\overline{BC}$

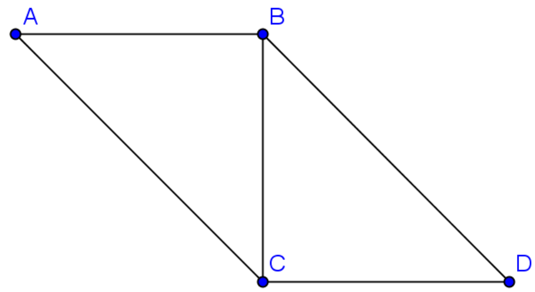


6. Given: $\angle CDB \cong \angle ABD$

Prove $\overline{AD} \cong \overline{CB}$

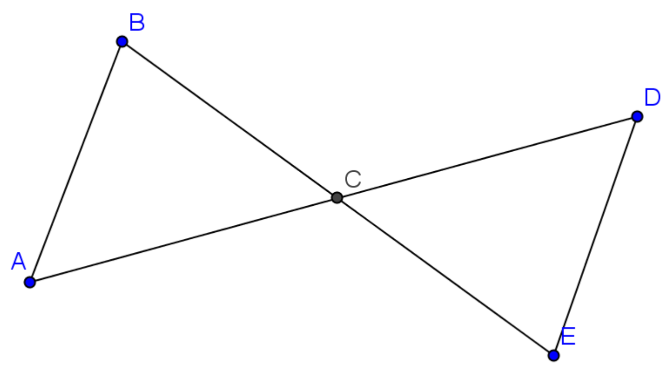


7. Given: $\overline{AB} \perp \overline{BC}$, $\overline{CD} \perp \overline{BC}$, $\overline{AB} \cong \overline{DC}$
Prove: $\angle A \cong \angle D$



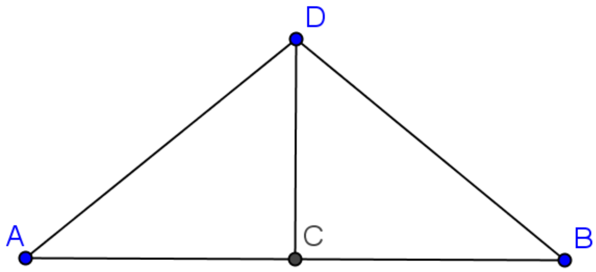
8. Given C is the midpoint of both $\overline{AD}$ *and* $\overline{BE}$

Prove: $\overline{AB} \cong \overline{DE}$



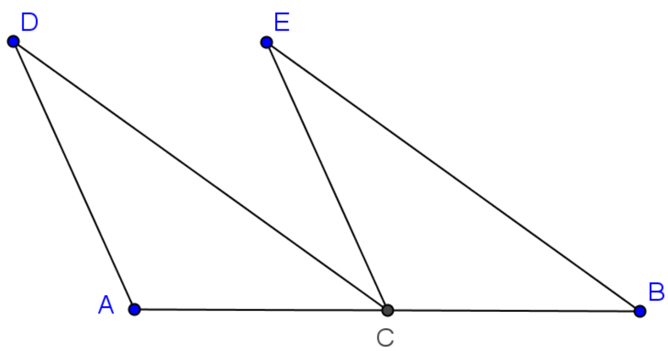
9. Prove that if two angles are congruent and supplementary then they must be right angles. (create a diagram, and write the givens)

10. Given C is the midpoint of $\overline{AB}$; $\overline{AD} \cong \overline{DB}$
Prove: $\angle DCB$ is a right angle
(use what you proved in the previous problem!)



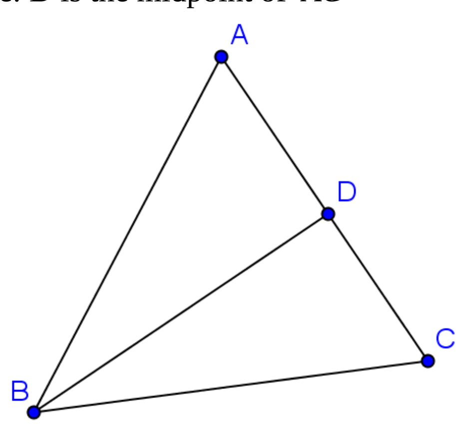
11. Given: $\overline{AD} // \overline{CE}$ *and* $\overline{AD} \cong \overline{EC}$ and C is the midpoint of $\overline{AB}$

Prove: $\overline{CD} \cong \overline{EB}$



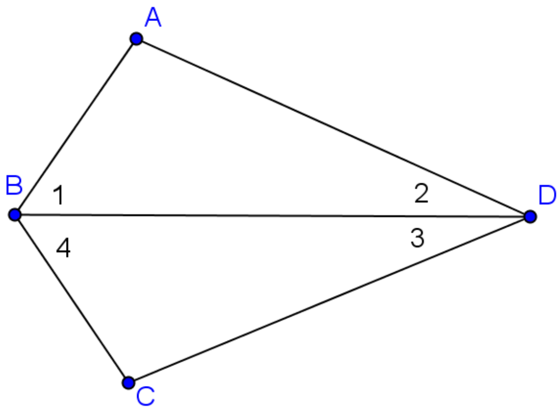
12. Given: $\overline{AC} \perp \overline{BD}$, $\overline{BD}$ bisects $\angle ABC$

Prove: D is the midpoint of $\overline{AC}$

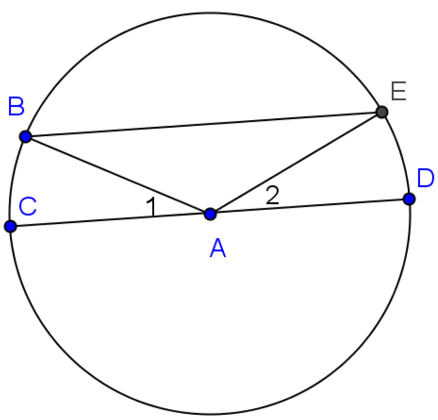


13. Given: $\overline{AD} \cong \overline{DC}$, $\angle 2 \cong \angle 3$

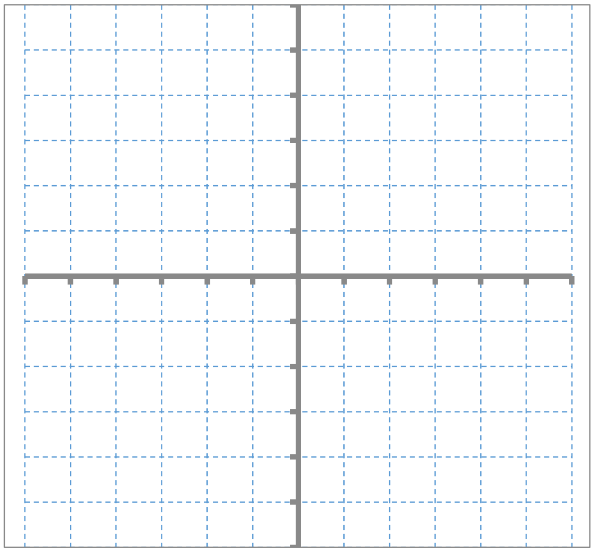
Prove: $\overline{BD}$ bisects $\angle ABC$



14. Given: $\overline{BE}$ is parallel to diameter $\overline{CD}$ in the circle below.
Prove: $\angle 1 \cong \angle 2$



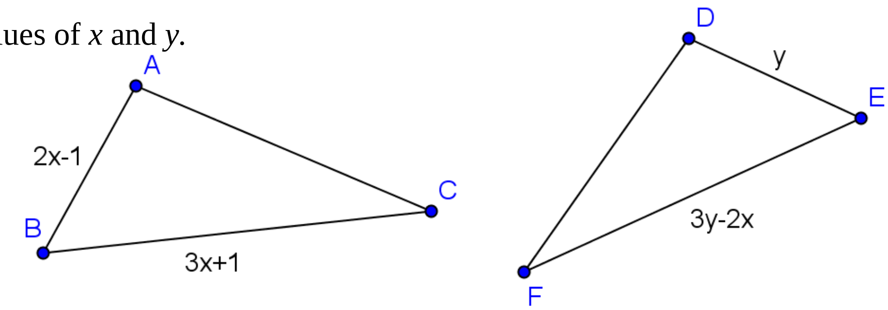
15a. Given the points A (5,0), B(5,5), C(-5,0), D(3,-2), E(0,-6), and F (-5,4) explain why $\angle BCA \cong \angle DFE$.



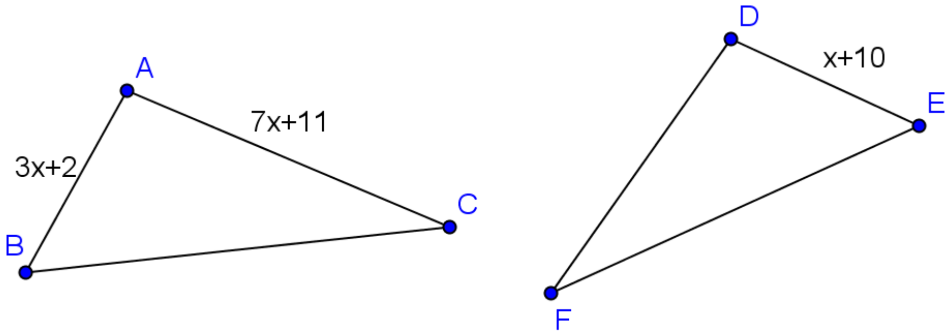
15b. Given the points A (0,1), B (-5, 1), C (5,-4), D (-3,5), and E (-1,-6). Explain why $\angle BAC \cong \angle DAE$.

15c. In 15b above, explain why $\angle BAD \cong \angle EAC$.

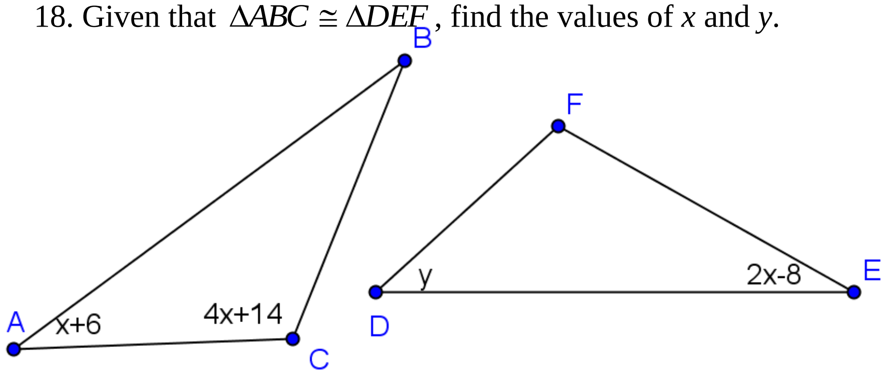
16. Given $\Delta ABC \cong \Delta DEF$, find the values of x and y.



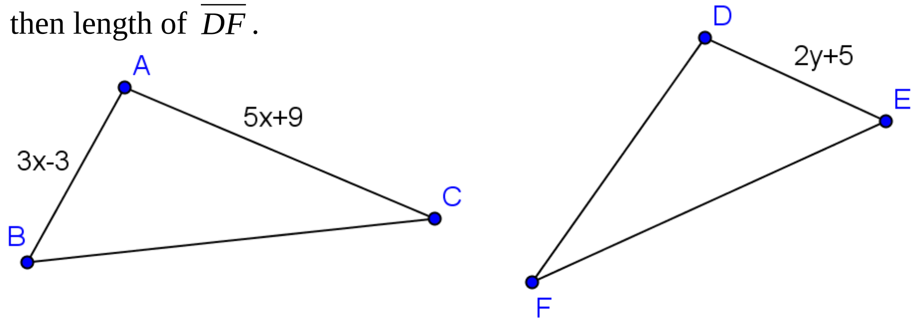
17. Given that $\Delta ABC \cong \Delta DEF$, find the length of $\overline{DF}$:



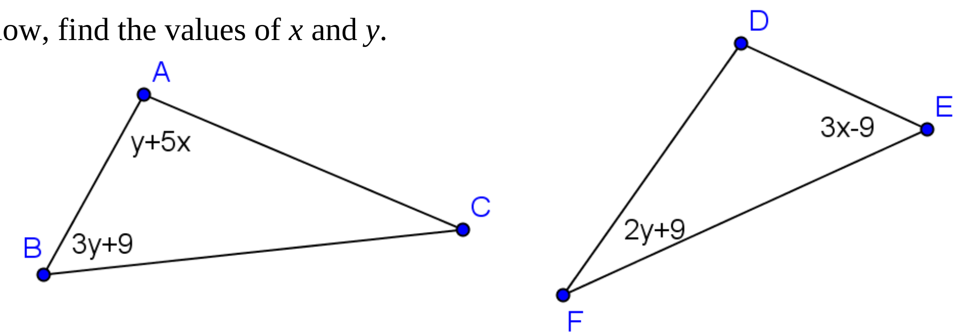
18. Given that $\Delta ABC \cong \Delta DEF$, find the values of x and y.



19. Given that $\Delta ABC \cong \Delta DEF$ and the ratio of the length of $\overline{AC}$ to $\overline{AB}$ is 7:3. Find the value of y and then length of $\overline{DF}$.



20. Given that $\Delta ABC \cong \Delta DEF$ below, find the values of x and y.



Some answers
15a. triangles are congr by SAS or SSS 15b. triangles ABC and DAE are congr by SSS c. subtrac prop
16. x=4 and y=7 17. 39 18. x=24 ; y=30 19. 8 and 49 20. x=18 and y=12

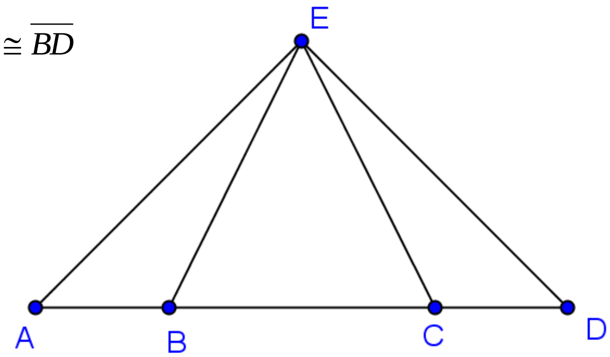
Unit 2: Proof Activity

"You Be the Judge"

Five Students were given the following proof. Their answers follow. How well did they do? Rank them from the best to the worst.

Given: $\overline{AE} \cong \overline{DE}$; $\angle AEB \cong \angle DEC$; $\angle A \cong \angle D$

Prove: $\overline{AC} \cong \overline{BD}$



Annie

Statement	Reason
1. $\overline{AE} \cong \overline{DE}$; $\angle AEB \cong \angle DEC$; $\angle A \cong \angle D$	Given
2. $\Delta AEB \cong \Delta DEC$	1, ASA
3. $\overline{AB} \cong \overline{CD}$	2, CPCTC
4. $\overline{AB} + \overline{BC} \cong \overline{CD} + \overline{BC}$	3, addition prop of equality
5. $\overline{AC} \cong \overline{BD}$	4, parts add to whole

Belle

Statement	Reason
1. $\overline{AE} \cong \overline{DE}$	Given
2. $\angle AEB \cong \angle DEC$	Given
3. $\angle A \cong \angle D$	Given
4. $\Delta AEB \cong \Delta DEC$	1, 2, 3; ASA
5. $\overline{BE} \cong \overline{CE}$	4, CPCTC
6. $\Delta AEC \cong \Delta DEB$	1, 3, 5; SAS
7. $\overline{AC} \cong \overline{BD}$	6, CPCTC

Chloe

Statement	Reason
1. $\Delta BEA \cong \Delta EDC$	SAS
2. $\overline{AB} \cong \overline{DC}$	1, CPCTC
3. $\overline{BC} \cong \overline{BC}$	reflexive property
4. $\overline{AC} \cong \overline{BD}$	2, 3, addition property

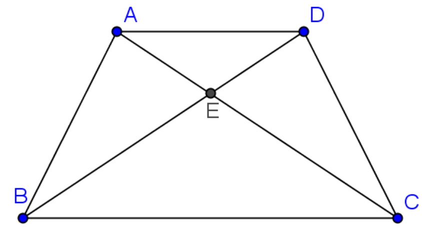
Daniel

Statement	Reason
1. $\overline{AE} \cong \overline{DE}$; $\angle AEB \cong \angle DEC$; $\angle A \cong \angle D$	Given
2. $\Delta AEB \cong \Delta DEC$	1, ASA
3. $\angle ABE \cong \angle CDE$	2, CPCTC
4. $\angle EBD \cong \angle ECA$	3, supplements of congruent angles
5. $\Delta AEC \cong \Delta DEB$	4, ASA
6. $\overline{AC} \cong \overline{BD}$	5, CPCTC

Edward

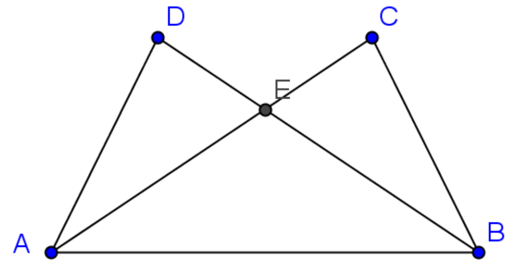
Statement	Reason
1. $\overline{AE} \cong \overline{DE}$; $\angle AEB \cong \angle DEC$; $\angle A \cong \angle D$	Given
2. $\overline{BE} \cong \overline{CE}$	1, opposite from congruent angles
3. $\Delta AEB \cong \Delta DEC$	1, 2, SAS
4. $\overline{AC} \cong \overline{BD}$	3, CPCTC

Unit 2 Handout #6: Overlapping Triangles; Angle Bisectors; Statements of Logic



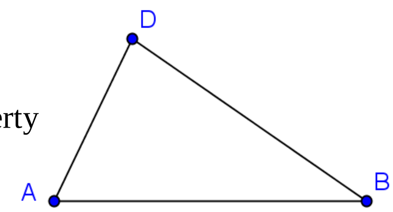
One way that proofs get more challenging is by having more complicated diagrams. Sometimes the triangles overlap, as in the diagrams to the right and below. One way to handle such proofs is to draw the triangles separately. Students often use different colored highlighters to help clarify the diagrams as well.

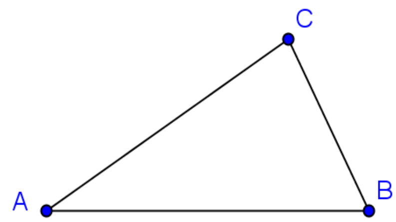
Example #1: Given $\overline{BC} \cong \overline{AD}$ **,** $\angle DAB \cong \angle CBA$ **Prove:** $\overline{BD} \cong \overline{AC}$



One good approach is to draw the triangles you want to establish as congruent separately. Looking at the givens and prove statements, a good approach is to try to prove that $\Delta ABD \cong \Delta BAC$ and then use CPCTC.

Statement	Justification
1. $\overline{BC} \cong \overline{AD}$	given
2. $\angle DAB \cong \angle CBA$	given
3. $\overline{AB} \cong \overline{AB}$	reflexive property
4. $\Delta ABC \cong \Delta BAD$	1, 2, 3, SAS
5. $\overline{BD} \cong \overline{AC}$	4; CPCTC





Logical statements: any logical statement ("if A then B") has three related statements: the ***inverse***, the ***converse***, and the ***contrapositive***. Just because a statement is true ***does not*** imply that the three related statements are also true!

Here's an example from <u>Alice in Wonderland</u> (Lewis Carroll, the author, was a mathematician):

> **'Then you should say what you mean,' the March Hare went on.**
> **'I do,' Alice hastily replied; 'at least — at least I mean what I say — that's the same thing, you know.'**

The March Hare disagrees, responding with the following example: "If I eat it then I see it."

The ***inverse*** of this statement is, "If I do not eat it then I do not see it." This may not always be true; he does not eat Alice, yet he sees her.

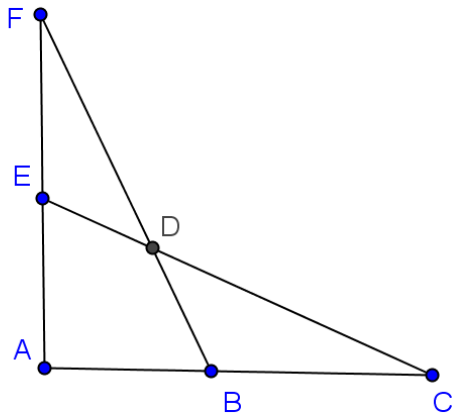
The ***converse*** of this statement is, "If I see it then I eat it." This also does not seem to be always true.

The ***contrapositive*** of this statement is, "If I do not see it then I do not eat it." This must be true.

The original statement and the contrapositive are always logically equivalent (meaning one cannot be true without the other also being true), as are the inverse and converse.

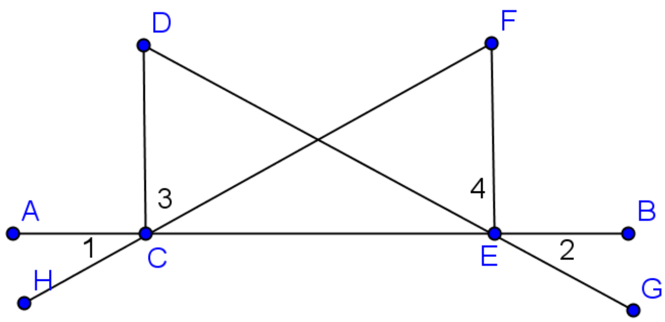
1. Given: $\overline{AE} \cong \overline{AB}$ and $\overline{EF} \cong \overline{BC}$ Prove: $\angle F \cong \angle C$

Hint: it may be worth drawing the overlapping triangles separately…



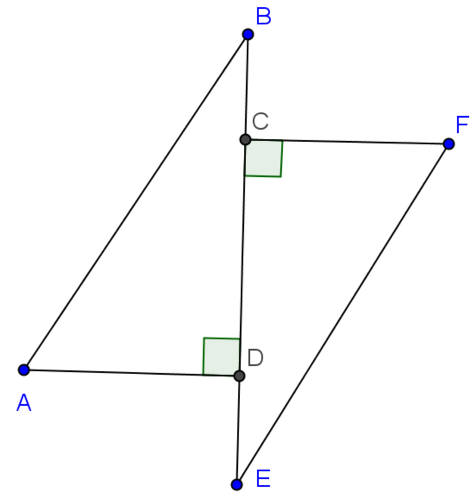
2. Given: $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

Prove: $\overline{CD} \cong \overline{EF}$



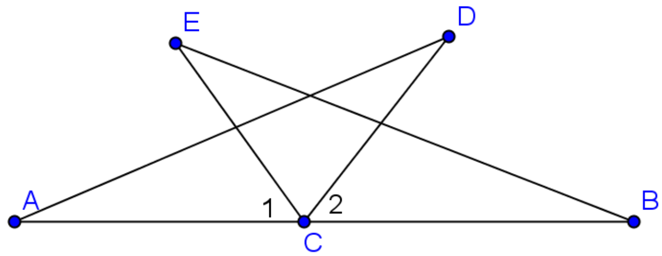
3. Given $\overline{AB} \parallel \overline{EF}$; $\overline{BC} \cong \overline{DE}$

Prove: $\overline{FC} \cong \overline{DA}$



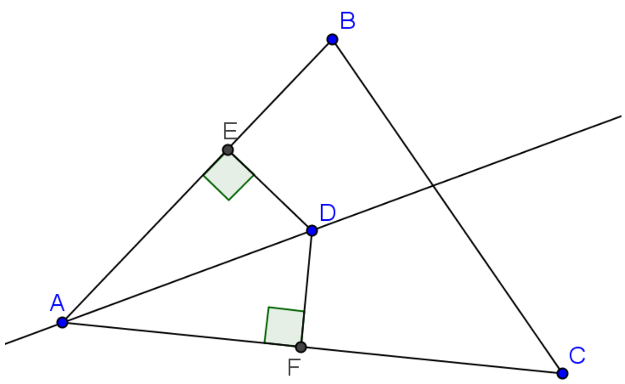
4. Given: $\angle A \cong \angle B$; $\angle 1 \cong \angle 2$; C is the midpoint of $\overline{AB}$

Prove: $\overline{CD} \cong \overline{EC}$

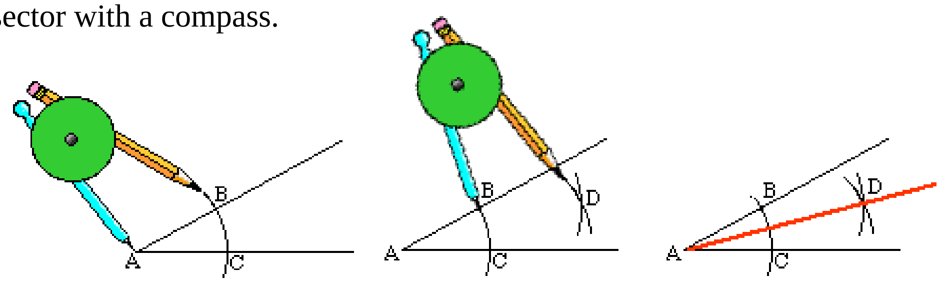


5. Given: $\overline{AD}$ bisects $\angle CAB$

Prove $\overline{DE} \cong \overline{DF}$



6. Constructing an angle bisector with a compass.



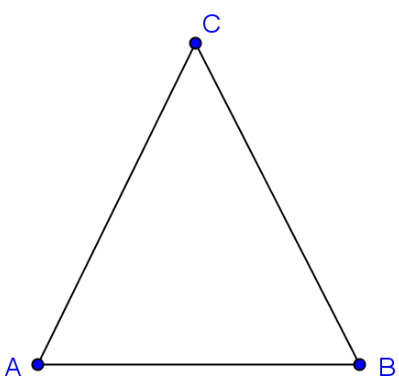
Left picture: Given an angle A, first pick an arbitrary length and draw a circular arc centered at A. What does this tell us about $\overline{AC}$ *and* $\overline{AB}$?

Middle picture: Then, using either the same arbitrary length or a different one, draw an arc of a circle centered at B and an arc of a circle (with that same radius) centered at C. The arcs intersect at D.

Right picture: Ray $\overline{AD}$ thus bisects angle BAC.

Why does this work? Hint: CPCTC plays a role!

7. In triangle ABC, $\overline{AC} \cong \overline{BC}$. Prove that $\angle A \cong \angle B$. Hint: draw the bisector of angle C.



8. In the previous problem, you showed that, if a triangle has two congruent sides, then the angles opposite those sides must be congruent. (this is often called "The Isosceles Triangle Theorem").

Annie has a triangle with two congruent angles. Does the Isosceles Triangle Theorem mean that the sides opposite must be congruent? Note: Do not focus on whether the sides ***actually*** are congruent (you'll examine that soon)! Justify focus on whether the Isosceles Triangle Theorem itself implies that this must be the case!

9. What, if anything, can we conclude from the following statements?

a. All rabbits have two ears. Sam is a rabbit.

b. All rabbits have two ears. Sally has two ears.

c. All rabbits have two ears. Erica has one ear.

10. Given that the statement, "If the light is purple then we stop" is true, which must also be true?

a. If the light is not purple then we do not stop.

b. If we do not stop then the light is not purple.

c. If we stop then the light is purple.

11. For each statement below, give the inverse, converse, contrapositive and state which ones are correct.

a. If x=3 then x^2= 9

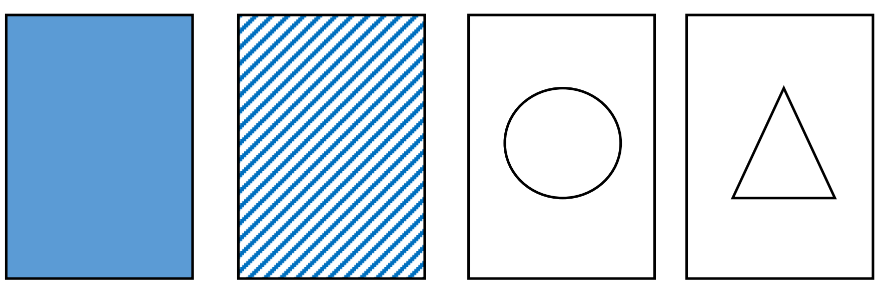
b. If Y is the head of house in Wheeler, then Y lives in Wheeler.

c. A pair of lines is cut by a transversal. If the two lines are parallel then the alternate interior angles are congruent.

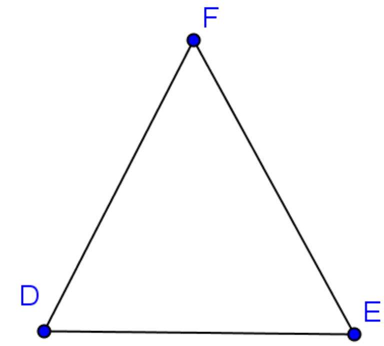
d. If a triangle is equilateral then it is also isosceles.

Note: Mathematicians often use the term "if and only if" when both the statement and the inverse are true. This is sometimes called the "biconditional".

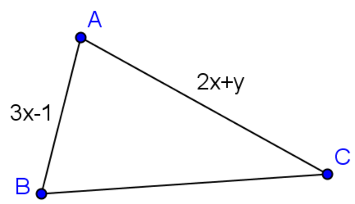
12. A set of cards has a shape on one side and a pattern (solid, striped, polka-dotted…) on the other. You want to test the validity of the statement "If one side of a card is striped, then the other side has a circle on it." There are four cards that you can see. Which ones do you need to turn over to test the validity of the statement? Use the fewest possible!

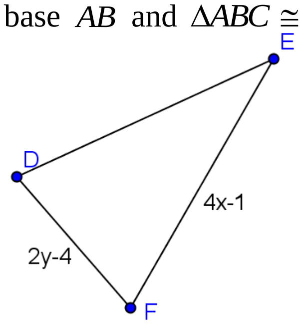


13. Now to prove the converse of the Isosceles Triangle Theorem! Given that angles D and E are congruent in the triangle below, explain why $\overline{DF} \cong \overline{EF}$. Hint: you may want to draw an angle bisector.



14. Triangle ABC is isosceles with base $\overline{AB}$ and $\Delta ABC \cong \Delta DFE$. Find the length of $\overline{BC}$.





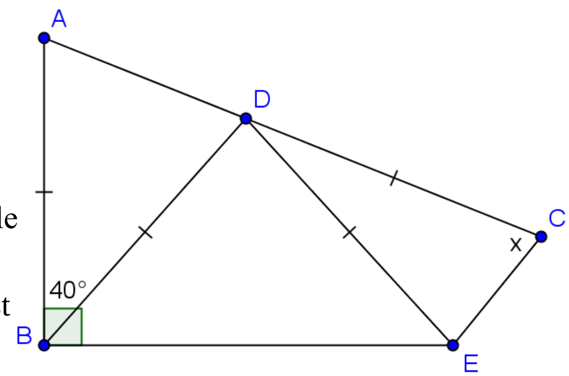
Answers
6. $\Delta ACD \cong \Delta ABD$ by SSS; so angles are congruent by CPCTC!
8. not necessarily (but it seems like a good bet…) 9a. Sam has 2 ears b. nothing c. Erica is not a rabbit
10. only b (we may also stop if the light is pink…)
1a. inverse: if x is not 3 then x^2 is not 9 converse: if $x^2=9$ then x=3 contrapositive: if x^2 is not 9 then x is not 3… only contrapositive must be true, since $(-3)^2$ is also 9
11b. inverse: if y is not the head of Wheeler then Y does not live in Wheeler converse: if Y lives in Wheeler then Y is the head of Wheeler contrapositive: if Y does not live in Wheeler then Y's not the head of Wheeler… only contrapositive must be true
11c. inverse: if the lines are not // then the alt int angles are not congruent converse: if alt int angles are congruent then lines are // contrapositive: if alt int angles are not congr then lines are not //… all are true
11d. inverse: if a triangle is not equilateral then it is not isos converse: if a triangle is isos then it is also equilateral contrapositive: if a triangle is not isos then it is not equilateral.. only contrap must be true
12. need to see that there's a circle on the back of the striped card and that the other side of the card with the triangle is not striped… 14. x=5 and y=9 so BC = 19

Unit 2 Handout #7: Isosceles and Equilateral Triangles

Isosceles triangles have two (or more) congruent sides. The ***Isosceles Triangle Theorem*** states that angles in a triangle opposite congruent sides must be congruent. The ***Converse Isosceles Triangle Theorem*** states that, if two angles of a triangle are congruent, then the sides opposite those angles must also be congruent.

Example #1: In the diagram below, points A, C, and D are collinear. There are also three isosceles triangles. Find the value of *x*.

Solution



Since ΔABD is isosceles with $\overline{AB} \cong \overline{DB}$, we know that $\angle A \cong \angle BDA$. If we call them both *w*, then we get the equation $w + w + 40 = 180$, so *w*=70.

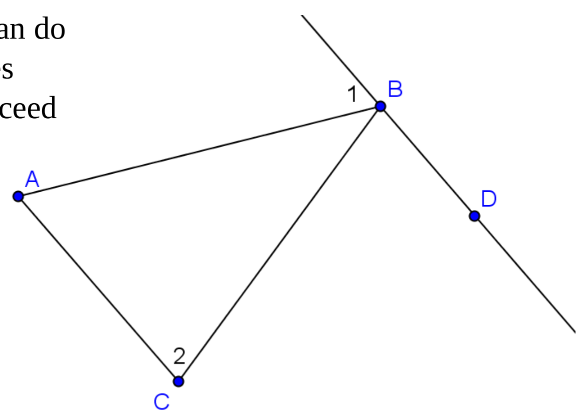
Look at ΔBDE. Since angle ABE is a right angle, we know angle DBE measures 50°. Angle BED must also measure 50° since ΔBDE is isosceles with $\overline{BD} \cong \overline{DE}$. That means angle BDE must measure 180-50-50=80°.

Since A, D, and C are collinear, the sum of angles ADB, BDE, and EDC must be 180. Thus angle EDC measures 30°. Since $\overline{CD} \cong \overline{DE}$, angles ECD and DEC are congruent. They are both *x*, so 2*x*+30=180 and *x*=75°.

Example #2: In the diagram below, $\angle 1 \cong \angle 2$ and $\overline{AC} \parallel \overline{BD}$. Prove that ΔABC is isosceles.

Solution

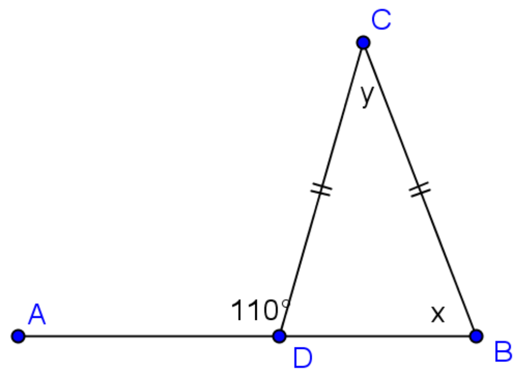
To prove a triangle is isosceles, we need to show two sides are congruent. Since we are given nothing about side lengths, we can do this by showing that two angles are congruent. The parallel lines mean that angle 1 is congruent to angle BAC, so we should proceed in that way.



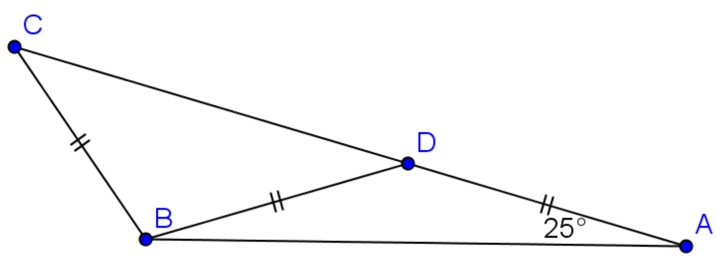
Statement	**Justification**
1. $\overline{AC} \parallel \overline{BD}$	given
2. $\angle 1 \cong \angle 2$	given
3. $\angle 1 \cong \angle BAC$	1; alternate interior angles
4. $\angle 2 \cong \angle BAC$	2, 3, transitive property
5. $\overline{AB} \cong \overline{BC}$	4; sides in a Δ opposite congruent angles are congruent
6. ΔABC is isosceles	5; definition of isosceles

1. Sally has an isosceles triangle with one side of length 3 and another side of length 4. Bob does too. Must their triangles be congruent? Explain

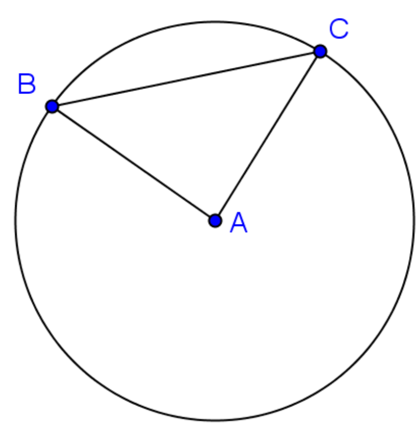
2. Find x and y in the diagram to the right:



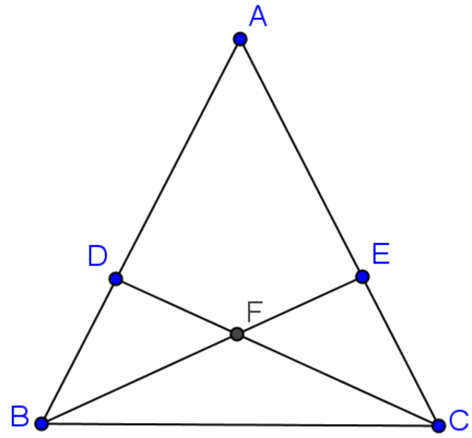
3. In the diagram below, D is on $\overline{AC}$. Find the measure of $\angle CBD$.



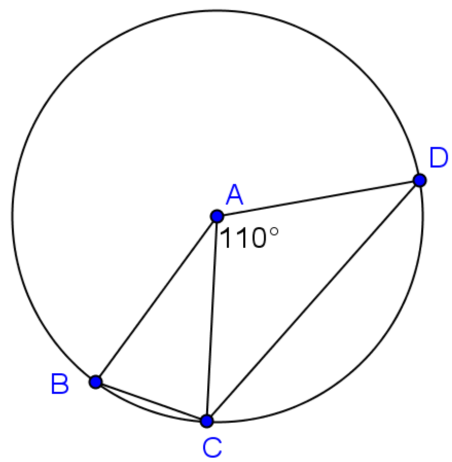
4. Given circle A
Prove $\angle C \cong \angle B$



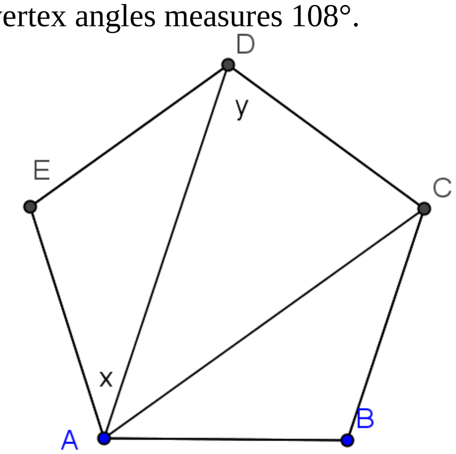
5. Given: $\overline{BE} \cong \overline{CD}$ and $\overline{BD} \cong \overline{CE}$ Prove: ΔBFC is isosceles



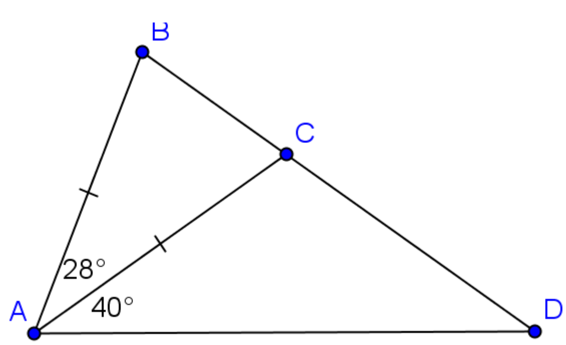
6. In circle A $\angle CAD \cong \angle DCB$. Find the measure of $\angle CAB$.



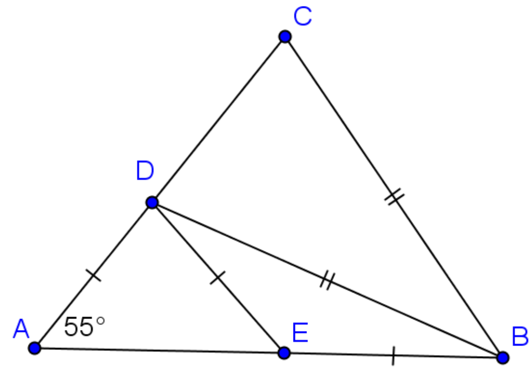
7. In the regular pentagon below, all sides are congruent and each of its vertex angles measures 108°. Find *x* and *y*.



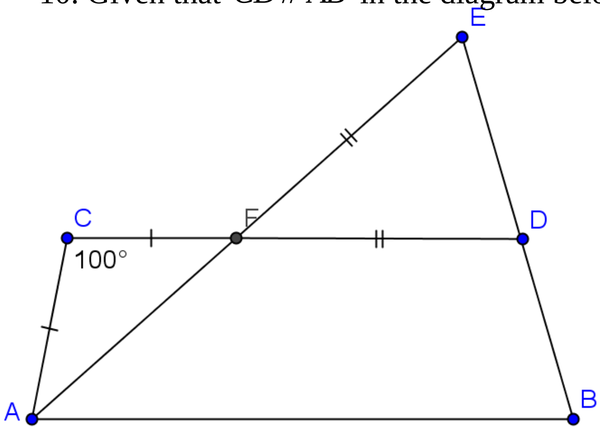
8. Find the measures of all angles below:



9. Find the measure of $\angle DBC$ in the diagram below.



10. Given that $\overline{CD} \parallel \overline{AB}$ in the diagram below, find the measure of $\angle B$

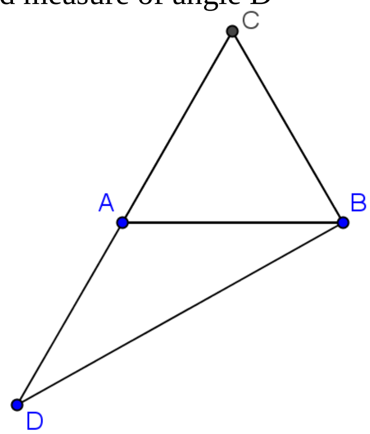


11. Given that one angle of an isosceles triangle is 40° and another is (3*x*-5)°, find all possible values of *x*. Note: consider cases where the 40° is the vertex angle and where it is a base angle.

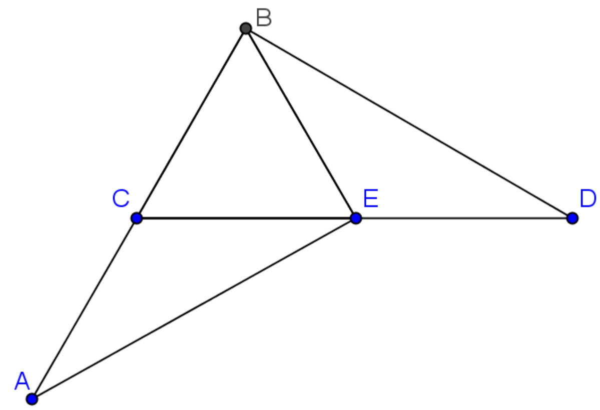
12. Given that two angles of an isosceles triangle measure $x°$ and $(2x+6)°$, find all possible values of x.

13. Equilateral Triangles: all sides are congruent. Use transitivity to show that all angles must also be congruent. Then determine the measure of each angle.

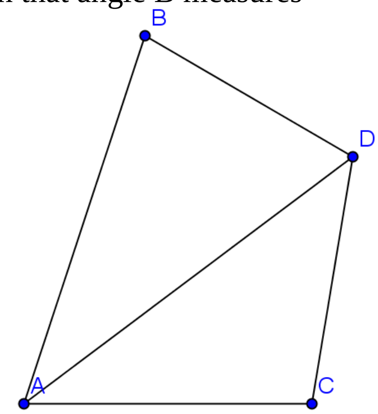
14. In the diagram below, A is the midpoint of $\overline{CD}$ and ΔABC is equilateral. Find measure of angle D



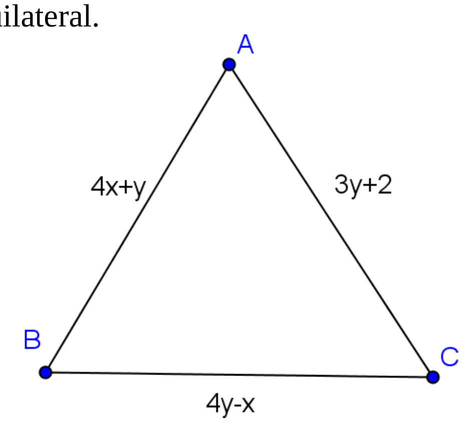
15. In the diagram below, C and E are the midpoints of $\overline{AB}$ *and* $\overline{CD}$ respectively, and $\overline{BE} \cong \overline{BC}$. Explain why $\overline{AE} \cong \overline{BD}$.



16. In the diagram below, $\overline{AD}$ bisects $\angle CAB$, $\overline{AB} \cong \overline{AD}$, and $\overline{CD} \cong \overline{CA}$. Given that angle B measures 75° find the measure of angle C.

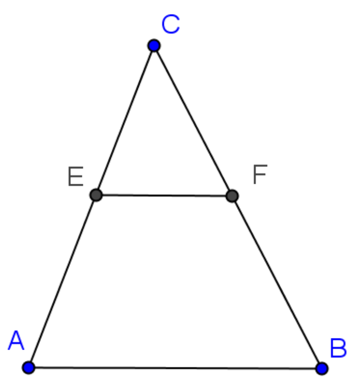


17. Find the values of *x* and *y* that make the triangle below equilateral.

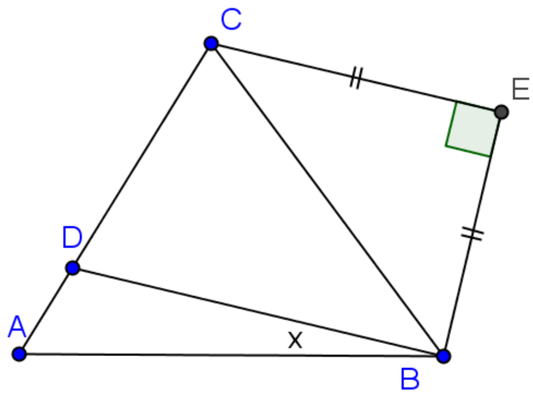


18. Given: $\overline{EF} \parallel \overline{AB}$ and $\overline{CE} \cong \overline{CF}$

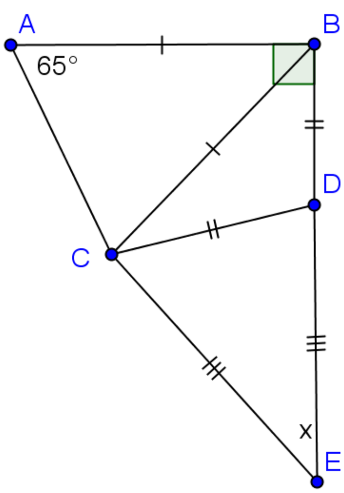
Prove: $\overline{AE} \cong \overline{BF}$



19. Given that ΔABC is equilateral and $\overline{CE} \parallel \overline{BD}$, find the value of x.



20. Find angle x. Now, instead of A measuring 65°, let it measure a°. Now what is x (in terms of a?) A good check on your answer is to let a=65°



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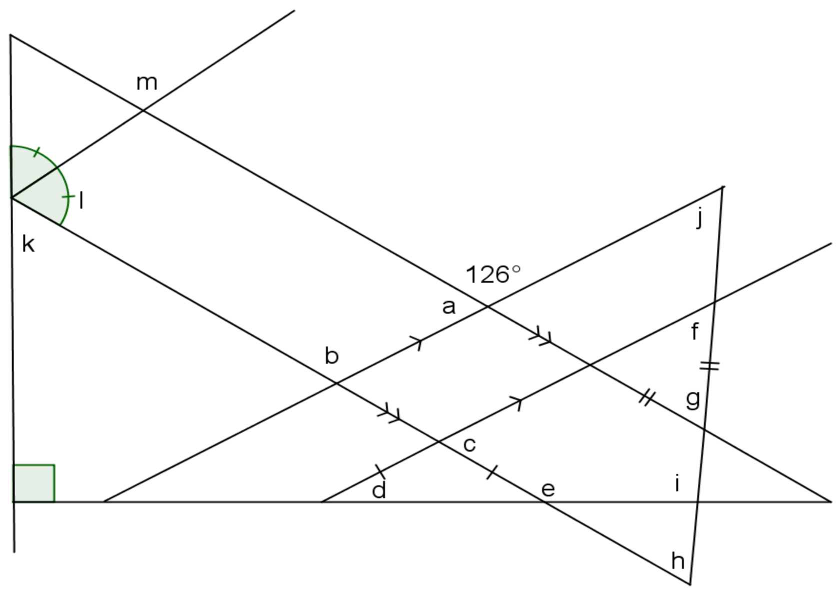
21. A triangle has sides of $2x-3,\ x+5,\ and\ 23-x$. What value(s) of x make it isosceles? Can it equilateral? Explain.

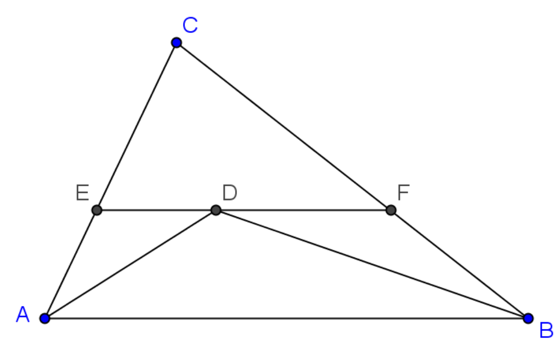
22. Crazy angles part 1. Find the measure of all lettered angles in the diagram below.



23. Given that ΔABC is isosceles, and angle A measures *x*°, where *x*<90°. Find all possible measures of angle B (in terms of *x*).

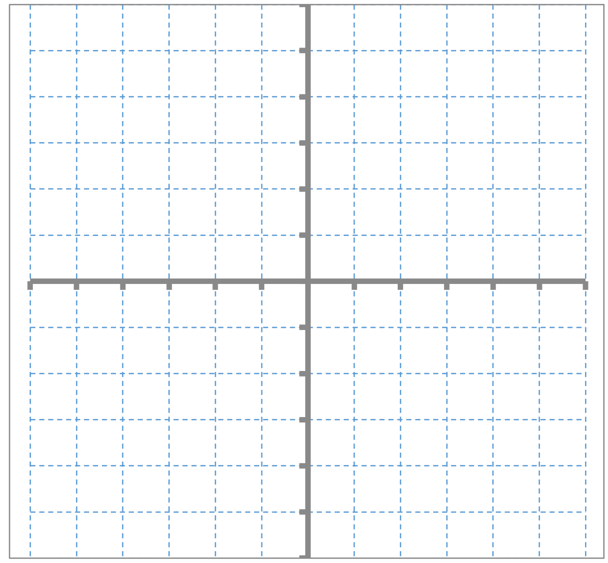
24. In scalene triangle ABC. AB=30, BC=28, and AC=20. $\overline{AD}$ *and* $\overline{BD}$ are the bisectors of angles CAB and CBA, and $\overline{EF} \parallel \overline{AB}$.

a. Are there any isosceles triangles in the diagram? Explain.



b. Find the perimeter of ΔCEF.

25. Given points A (1,1) and B(4,4) do the following:

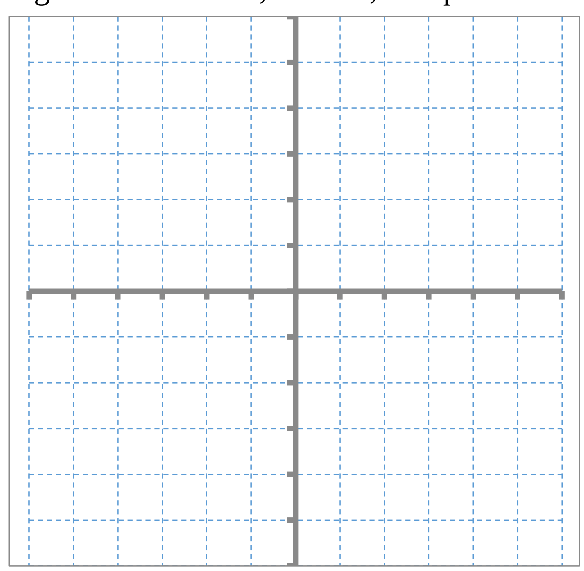


a. If ABC is an isosceles right triangle, list all possible coordinates of point C.

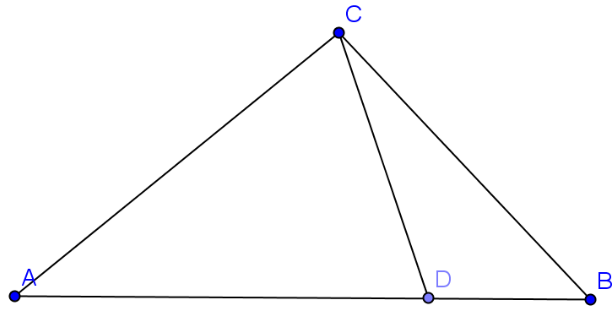
b. If ABC is a right triangle and point C is somewhere on the *x*-axis, the where could it be? There are just two possible locations.

c. Let point D be located at (0,6). If $\Delta EFG \cong \Delta ABD$ and E is (-4,2) and F is (-1,-1) then where could G be?

26. Lines $9y - 4x = 22,\ 3x + 2y = 1,\ and\ \ y = x - 2$ form a triangle. Is it isosceles, scalene, or equilateral?



27. In the triangle below, $\overline{AC} \cong \overline{AD}$ and angle ACB measures 42° more than angle ABC. Find the measure of angle BCD.



28. Helen takes an isosceles triangle and draws a line through one vertex It happens to create two smaller isosceles triangles, not necessarily identical to each other. What are the possible angles of the original triangle? Find all that work! (From Exeter, *Math 2*)

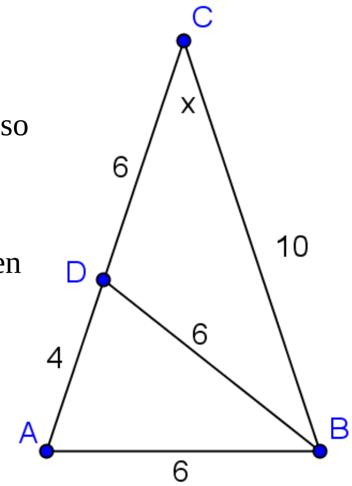
Answers

1. no: 3, 3, 4 or 3, 4, 4 2. x=70° and y=40° 3. 80° 6. 30° 7. x=36° and y=72°
8. B=ACB=76°; ACD=104° so D=36° 9. 15° 10. 70°
11. x=15 or 25 or 35 (since 3x-5 can be 40 or 70 or 100) 12. 43.5 or 33.6 13. 60° 14. 30°
15. $\Delta BED \cong \Delta ACE$ 16. 120° 17. x=3 and y=5 19. 15° 20. 20°; (540-8a) °
21. x=8 or x=9 or x=26/3; can't be equilateral→ any x that makes 2 sides = causes the 3rd to be different.
22. a=54° b=126° c=54° d=27° e=153° f=54° g=72° h=72° i=99° j=54° k=63° l=58.5° m=121.5° 23. x, 180-2x and 90-0.5x
24a. ΔAED and ΔDFB → angle bisectors and alternate interior angles b. 48
25a. (1,4), (4,1), (4,-2), (-2,4), (7,1) and (1,7) b. if A is a right angle then (2,0); if B is a right angle then (8,0); C can't be a right angle and on the x-axis… c. (1,3) or (-5,-3)
26. sides are $\sqrt{98}, \sqrt{97}, \textit{and } \sqrt{13}$ so scalene 27. 21°
28. 72-72-36 or 90-45-45

Unit 2 Handout #8: More Isosceles and Equilateral Triangles

In some more difficult problems, define one angle as a variable and write as many angles as possible in terms of that variable. Eventually another relationship should emerge that enables you to write an equation and solve it for that variable.

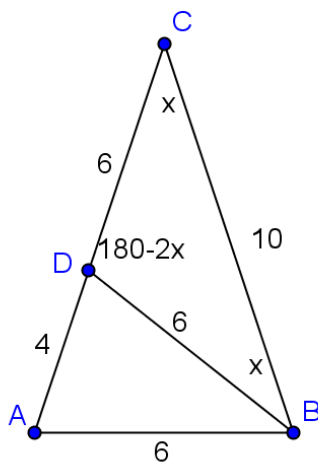
Example #1: In the diagram below, there are three isosceles triangles. Find the value of *x*.

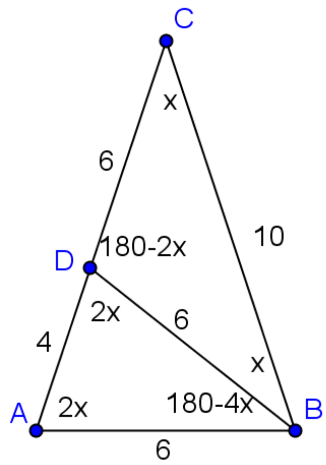


Solution:

Write as many angles as possible in terms of *x*. First work in ΔBCD. See left diagram below. Angle CBD must also be *x*, and angle CDB must be 180-2*x*, so that the angles of that triangle sum to 180°.

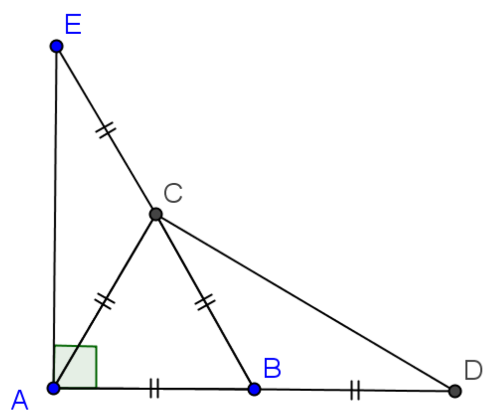
Now work in ΔABD. See right diagram below. Angles ADB and CDB are supplementary, so angle ADB must measure 2*x*. Thus angle A does also. Then angle ABD measures 180-4*x* so the sum of the angles in ΔABD is 180°.





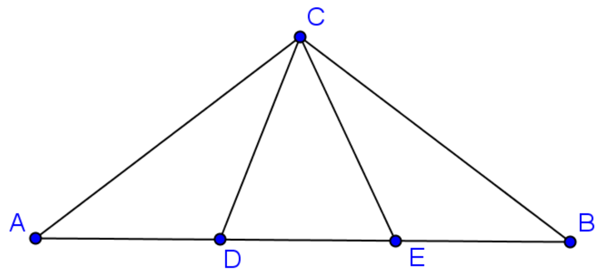
Finally, look at ΔABC. Since $\overline{AC} \cong \overline{BC}$, angles A and ABC must be congruent. So 2*x*=180-3*x* and 5*x*=180 so *x*=36°.

1a. Find the measure of $\angle ECD$ below.



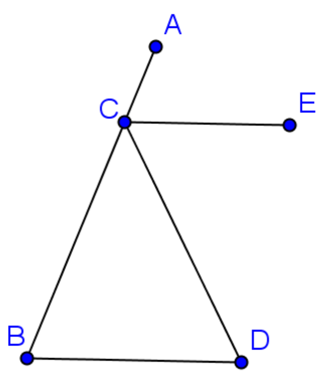
b. Explain why $\overline{AE} \cong \overline{CD}$. (no formal proof needed)

2. Given: Points D and E trisect $\overline{AB}$ and $\angle A \cong \angle B$
Prove ΔCDE is isosceles

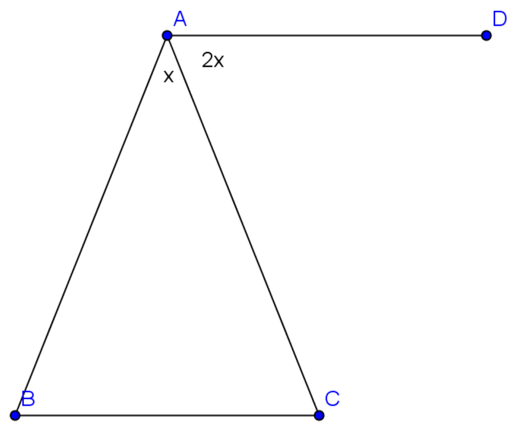


3. Show that a triangle with sides 2*x*+5, *x*+11, and 3*x*-3 cannot be equilateral. Hint: find an *x* that makes it isosceles first.

4. Given: A, B, and C are collinear. $\overline{BC} \cong \overline{CD}$; $\overline{CE} \parallel \overline{BD}$ Prove: $\overline{CE}$ bisects $\angle ACD$

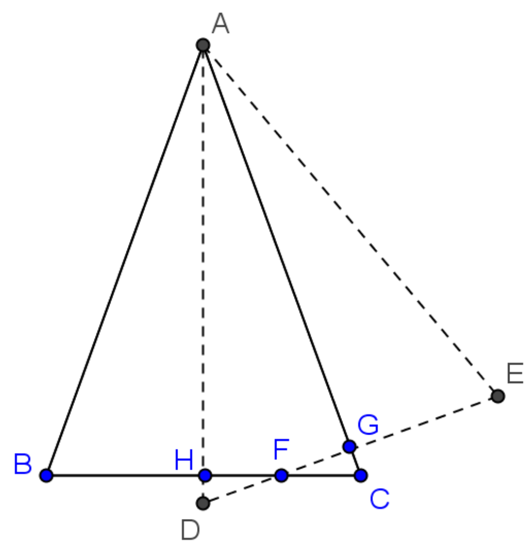


5. In the diagram below, $\overline{AB} \cong \overline{AC}$; $\overline{AD} \parallel \overline{BC}$. Find *x*.



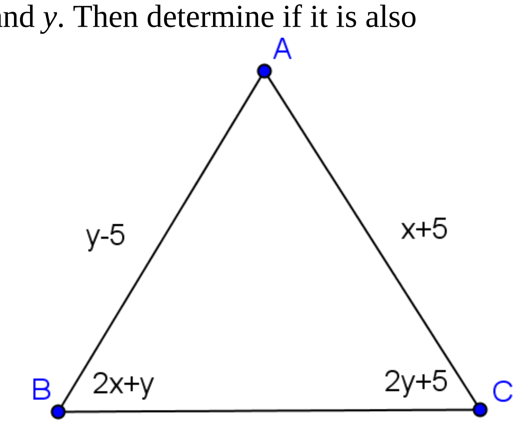
6. Isosceles ΔABC has base $\overline{BC}$ and angle B measures 70°. It is rotated 20° counterclockwise around point A, forming ΔADE.

a. What is the measure of $\angle HFD$?

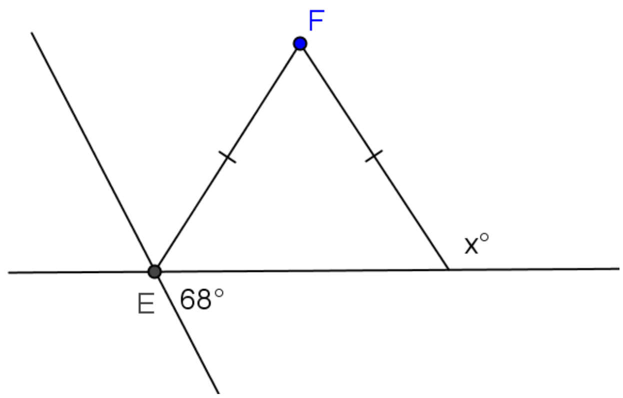


b. Must it be the case that $\Delta HFD \cong \Delta GFC$? Explain.

7. The triangle below is isosceles, with $\overline{AB} \cong \overline{AC}$. Find the values of *x* and *y*. Then determine if it is also equilateral.

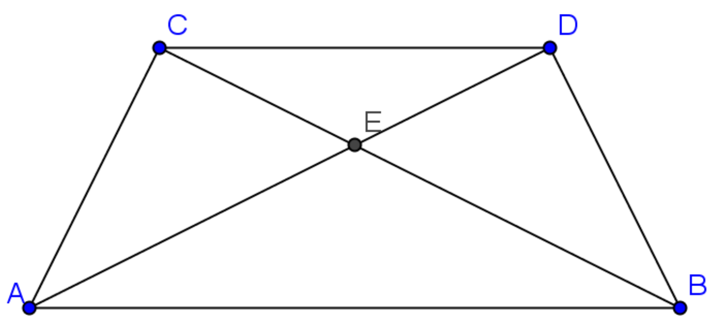


8. $\overline{EF}$ is an angle bisector; find *x*.



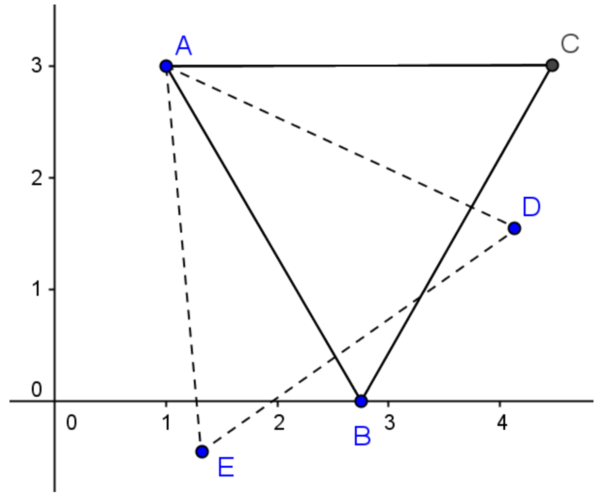
9. Given $\angle DBA \cong \angle CAB$ and $\overline{EA} \cong \overline{EB}$

Prove: $\overline{CA} \cong \overline{DB}$



10. Triangle ABC below is an equilateral triangle where $\overline{AC}$ is parallel to the *x*-axis and B is on the *x*-axis. So rays $\overrightarrow{BA}$ *and* $\overrightarrow{BC}$ trisect a straight angle. The triangle is rotated 25° clockwise about point A, creating ΔAED. Find the following:

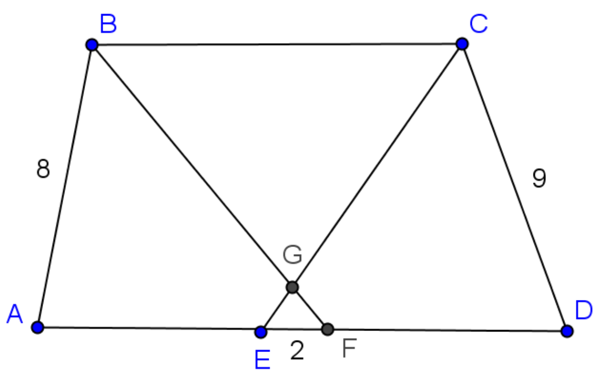
a. The acute angle at which side $\overline{DE}$ intersects the *x*-axis.



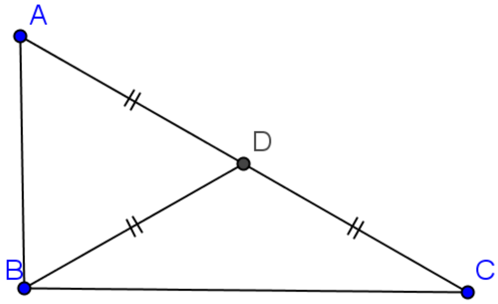
b. The acute angle at which side $\overline{AE}$ intersects the *x*-axis.

c. The acute angle at which side $\overline{DE}$ intersects side $\overline{BC}$.

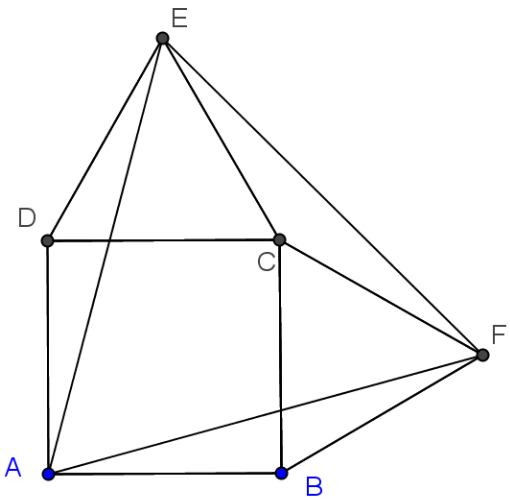
11. In the diagram below, $\overline{AD} // \overline{BC}$; $\overline{BF}$ and $\overline{CE}$ are the bisectors of angles ABC and BCD respectively. If AB=8; CD=9; and EF=2, then find AD.



12. In the triangle below, D is the midpoint of $\overline{AC}$. Show that ΔABC must be a right triangle. Hint: use the diagram below and let $\angle A$ measure *x* degrees. Then find the other angles in terms of *x*.

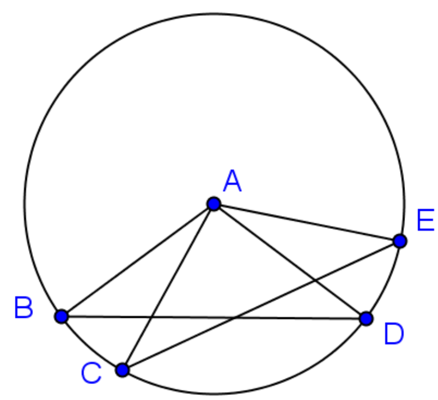


13. Starting with square ABCD, equilateral triangles CDE and BCF are drawn outside the square with sides equal to the square's sides. Must ΔAEF be an equilateral triangle? Explain. (From Exeter's *Math 2)*

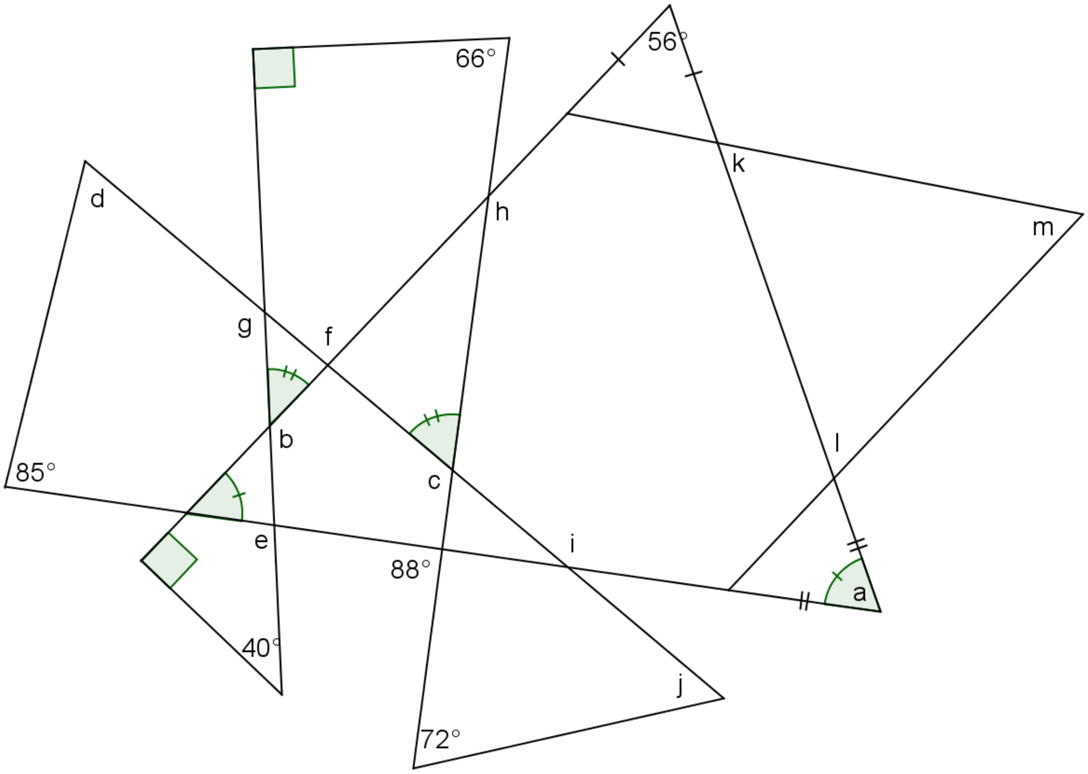


14. In the diagram below, A is the center of the circle and points B, C, D, and E are on the circle.

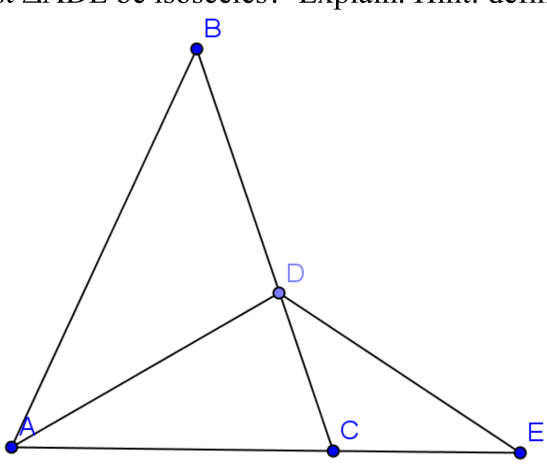
$\angle BAC \cong \angle DAE$. Prove that $\overline{BD} \cong \overline{CE}$



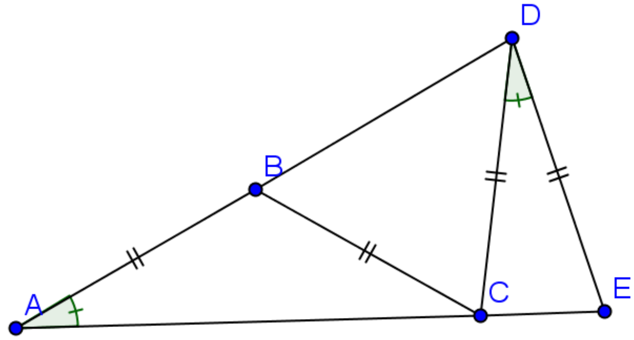
15. Crazy angles part 2! Find all of the lettered angles in the diagram below. Hint: roughly follow alphabetical order!



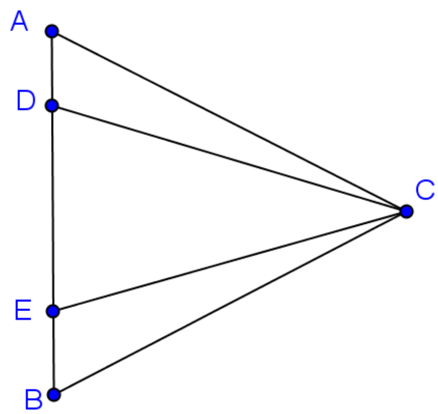
16. Below: Point C is on $\overline{AE}$; $\overline{AB} \cong \overline{BC}$; $\overline{AD}$ bisects $\angle CAB$ and $\overline{CE} \cong \overline{DC}$.
Must ΔADE be isosceles? Explain. Hint: define angle DAB as "*x*".



17. Find the measure of angle A in ΔADE below.



18. Given: $\overline{AE} \cong \overline{DB}$ and $\overline{AC} \cong \overline{BC}$
Prove ΔCDE is isosceles.



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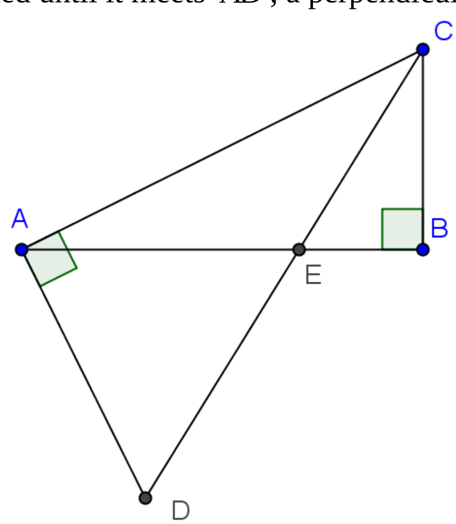
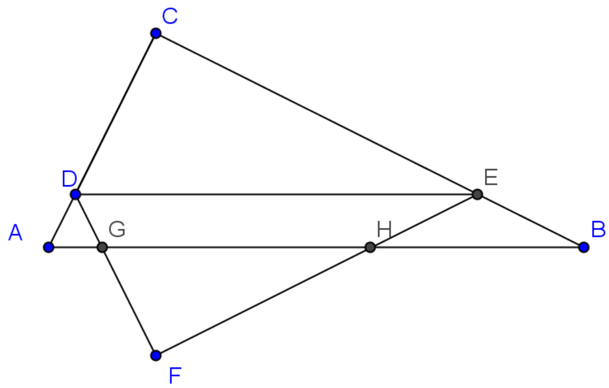
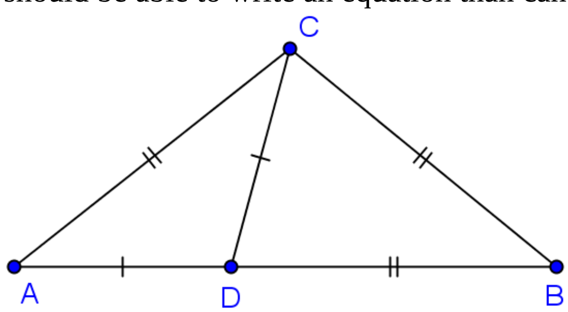
19. In ΔABC, $\overline{CE}$ is drawn to bisect angle BCA and it is extended until it meets $\overline{AD}$, a perpendicular from vertex A. Explain why ΔADE must be isosceles.



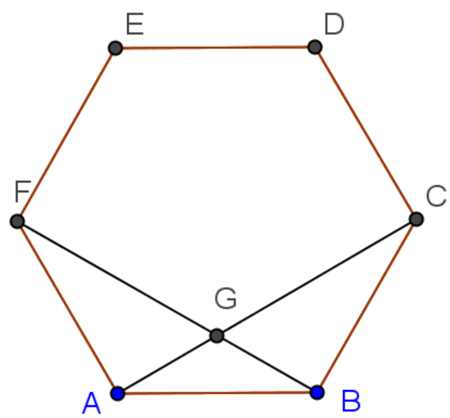
20. Given scalene ΔABC and $\overline{DE} // \overline{AB}$. Triangle CDE is reflected across $\overline{DE}$, resulting in ΔFDE. Explain why triangle HEB must be isosceles.



21. Find the measure of angle B below. Hint; call angle A "*x*" and write everything in terms of *x*. Then you should be able to write an equation than can be solved for *x*.



22. Hexagon ABCDEF is equilateral and equiangular (all sides are congruent, as are all angles). Prove: ΔAGB is isosceles



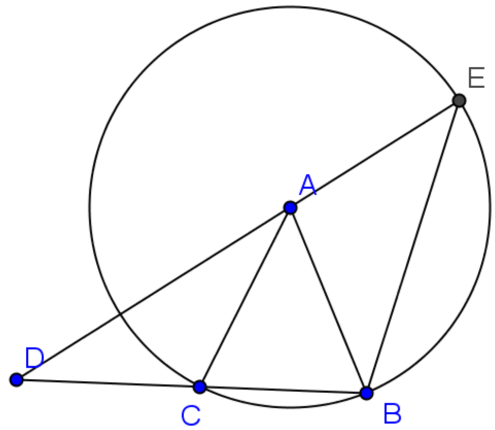
23. In the diagram, *A* is the center of the circle, $\overline{AE} \cong \overline{CD}$ and $\angle EAB \cong \angle ACD$. Points A and C are on the sides of ΔBED.

a. Explain why $\Delta ACD \cong \Delta BAE$

b. Explain why $\overline{AC} \parallel \overline{BE}$

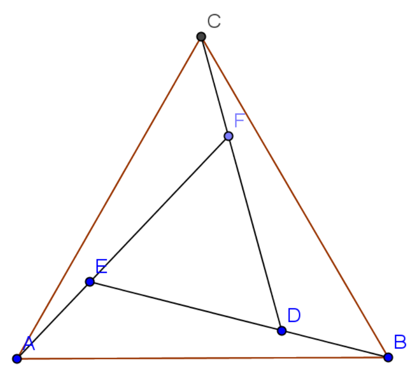
c. Show that $\angle ACB = 2\angle ABE$. (hint: call angle ABE "x")

d. Find $\angle ABE$.



24. Given: ΔABC is equilateral and $\angle BCD \cong \angle ABD \cong \angle CAF$

Prove: ΔDEF is equilateral



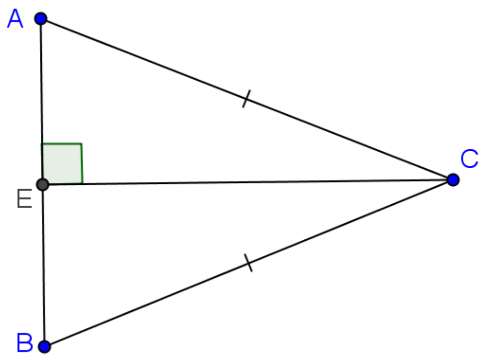
Answers

1a. 150° b. triangles ACE and DBC are congruent by SAS
3. if first two sides are equal then x=6 and then sides are 17, 17, and 18; others being = also does not work
5. 36° 6a. 20° b. yes; C=D and vertical angles at F and AD=AC and AH=AG since both are angle bisectors of the vertex angle of congruent triangles so HD=GC and AAS applies
7. x=15; y=25 so base angles are 55° and it is not equilateral 8. 124° 10a. 35° b. 85° c. 25° 11. 15
13. Yes since AD=DE=EC=CF=BF=AB and angles ABF, ADE, and ECF all measure 150°, triangles ABF, ECF, and EDA are all congruent by SAS so AE=EF=AF.
15. a=62° b=130° c=130° d= 53° e=112° f=76° g=154° h=154° i=138° j=58° k=62° l=59° m=59° 16. Yes; angle E=angle DAC 17. 180/7
19. let BCE=x then ACE=x and BEC=90-x =ADE so AED=90-x too; this AED=ADE and AD=AE so triangle AED is isosceles. 21. 36° 23d. in ΔACB sum of angles is 5x so x=36

Unit 2 Handout #9: HL ("Hypotenuse-Leg")

One other way to prove triangles congruent is ***Hypotenuse-Leg*** ("HL"). It says that if two right triangles have congruent hypotenuses and one congruent leg then they must be congruent. While it appears to be SSA, which does not determine triangle congruence, because the "A" is a right angle, it is a special case and does determine triangle congruence.

Example #1. Given $\overline{CE} \perp \overline{AB}$ **and** $\overline{AC} \cong \overline{BC}$ **Prove:** $\Delta ACE \cong \Delta BCE$.

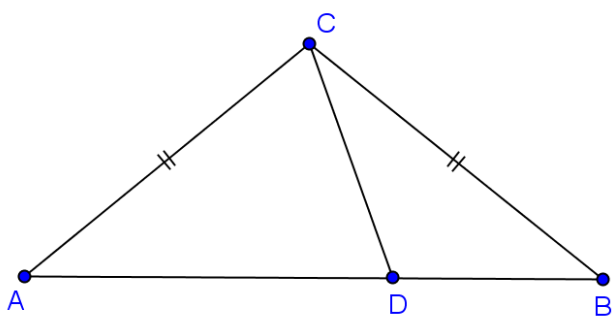


Solution

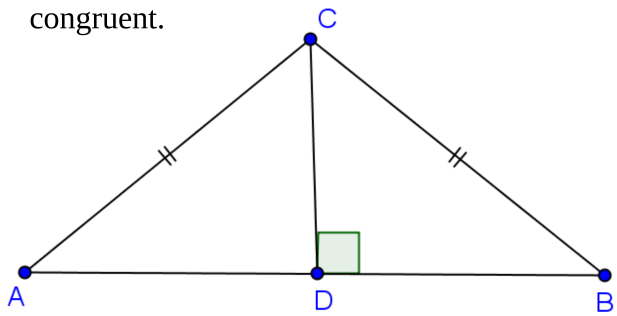
We can that we have SSA in this case. But because the angle is a right angle, we can use HL to establish congruence.

Statement	Justification
1. $\overline{AC} \cong \overline{BC}$	given
2. $\overline{CE} \perp \overline{AB}$	given
3. $\angle AEC \cong \angle BEC$	2; perp creates two congruent right angles
4. $\overline{EC} \cong \overline{EC}$	reflexive property
5. $\Delta ACE \cong \Delta BCE$.	1, 3, 4, HL

1. In the diagram below AD>DB. Show that triangles ACD and BCD have "SSA" but are not congruent.

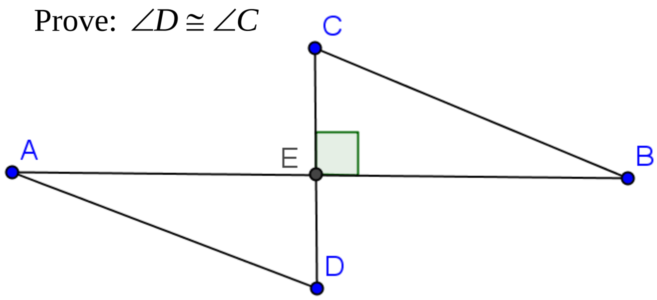


2. (continuation of #1) Instead, now let D be a right angle. Now explain why triangles ADC and BDC are congruent. Note: this shows that "HL" ("Hypotenuse-Leg") is another way to prove triangle congruence. If two right triangles have congruent hypotenuses and one set of congruent legs, then they must be congruent.



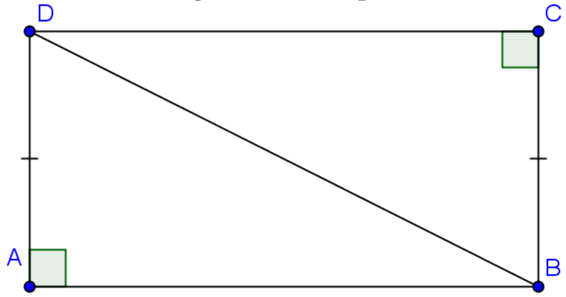
3. Given the Pythagorean Theorem, what is another way to relate HL to a triangle congruence postulate we have seen?

4. Given: $\overline{BC} \cong \overline{AD}$ and E is the midpoint of $\overline{CD}$.
Prove: $\angle D \cong \angle C$

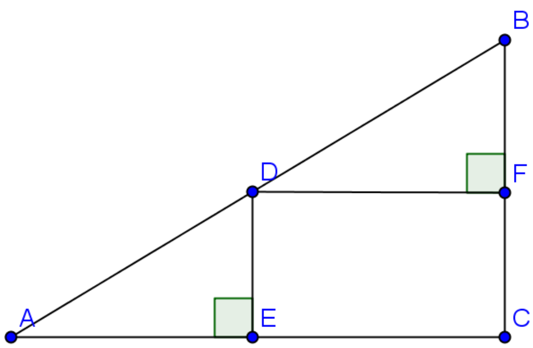


5. (continuation of #4). Explain why your result from the proof above indicates that $\overline{BC} \parallel \overline{AD}$.

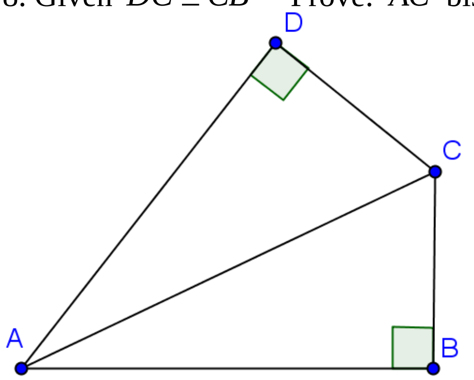
6. Given the diagram below, prove that $\overline{DC} \cong \overline{AB}$



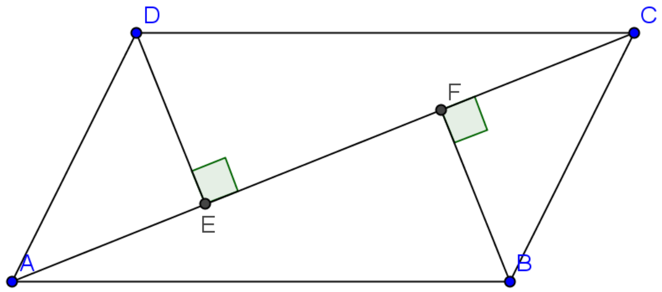
7. Given: D is the midpoint of $\overline{AB}$ and $\overline{DE} \cong \overline{FB}$ Prove $\overline{DF} \cong \overline{AE}$



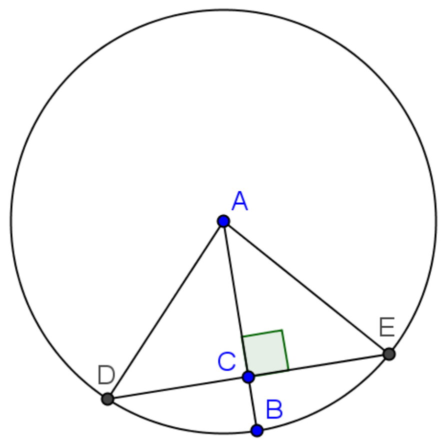
8. Given $\overline{DC} \cong \overline{CB}$ Prove: $\overline{AC}$ bisects $\angle BAD$



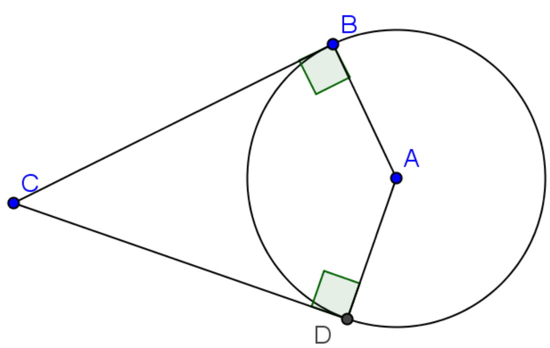
9. Given $\overline{DC} \cong \overline{AB}$ and $\overline{FC} \cong \overline{AE}$ Prove $\overline{BF} \cong \overline{ED}$



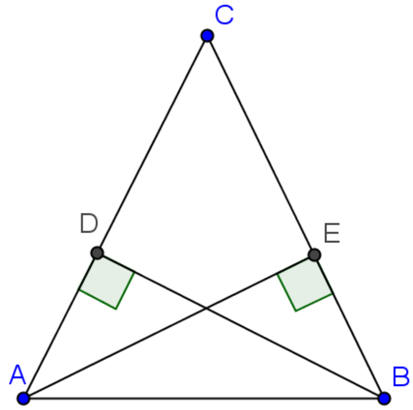
10. Use the diagram below to prove that if a radius is perpendicular to a chord then it bisects the chord. (So your given can be "circle A where $\overline{AB} \perp \overline{DE}$" and you want to prove that $\overline{AB}$ bisects $\overline{DE}$.)



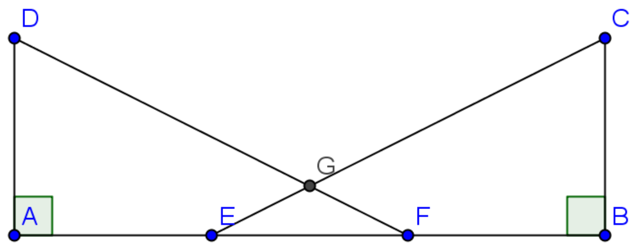
11. $\overline{CB}$ and $\overline{CD}$ are tangent segments from point C to circle A. As you'll see in geometry 2, they are perpendicular to the radii at the points of tangency. Prove that $\overline{CD} \cong \overline{CB}$. Hint: draw a line segment!



12. Given $\overline{AE} \cong \overline{BD}$ Prove: ΔABC is isosceles.

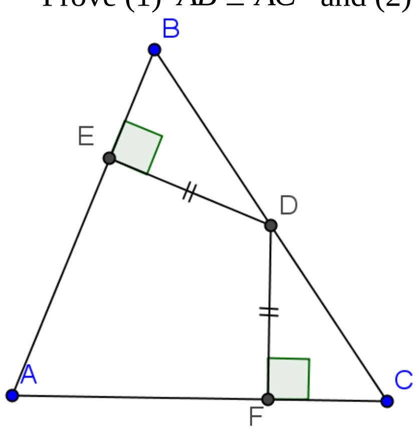


13. Given: points E and F trisect $\overline{AB}$ and $\overline{CE} \cong \overline{FD}$ Prove ΔEFG is isosceles



14. Given D is the midpoint of $\overline{BC}$ and $\overline{DE} \cong \overline{FD}$

Prove (1) $\overline{AB} \cong \overline{AC}$ and (2) and $\overline{AE} \cong \overline{AF}$ (note: do both in one proof)

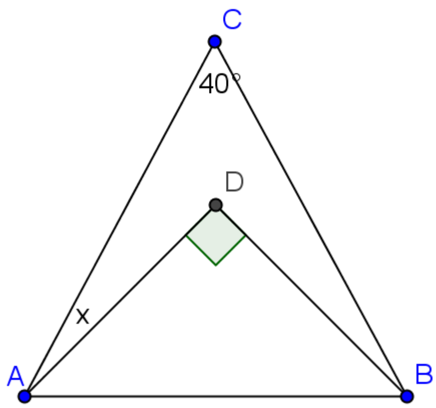


Unit 2 Handout #10: Unit 2 Review Problems

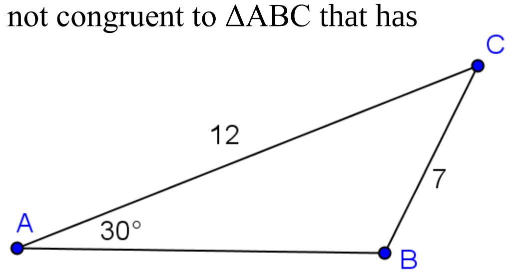
Part I: Problems

1. Given the statement, "If grapefruit then spherical" give the inverse, converse, and contrapositive. State which of this must be true.

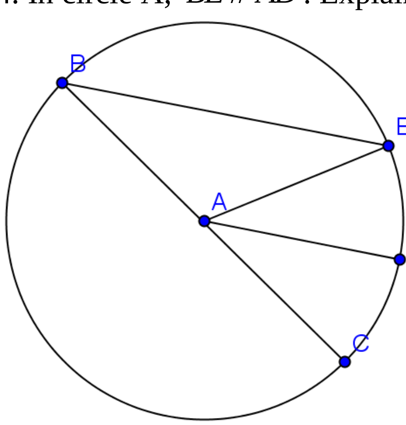
2. In the diagram below, triangles ABC and ABD are isosceles. Find the value of *x*.



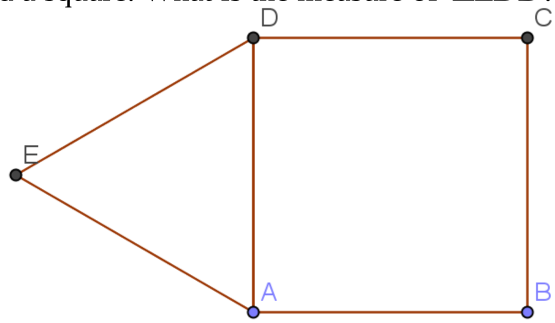
3. Once more, why SSA does not prove congruence: Draw a triangle not congruent to ΔABC that has sides of 12 and 7 with a 30° angle opposite the side of length 7.



4. In circle A, $\overline{BE} \parallel \overline{AD}$. Explain why $\overline{AD}$ must bisect $\angle CAE$

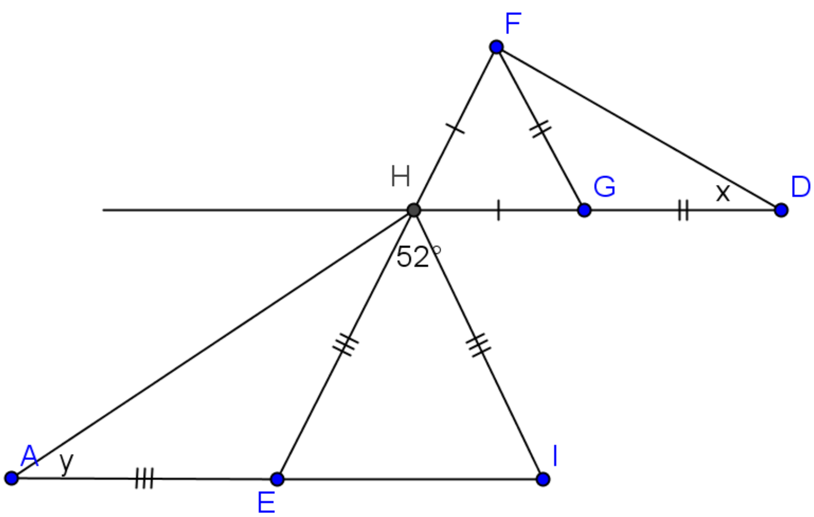


5. The diagram below consists of an equilateral triangle and a square. What is the measure of $\angle EDB$?

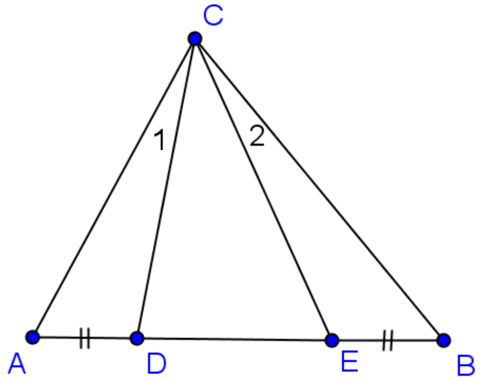


6. The sides of a triangle are 2*x*-3, *x*+5, and 3*x*-5. Find all values of *x* that make the triangle isosceles.

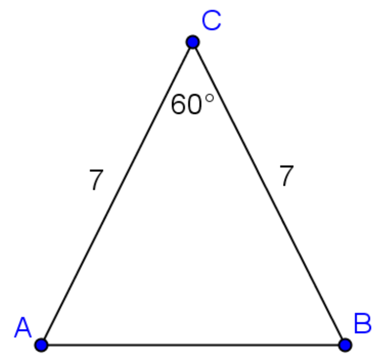
7. Given that $\overline{HD} // \overline{AI}$, find the values of *x* and *y*.



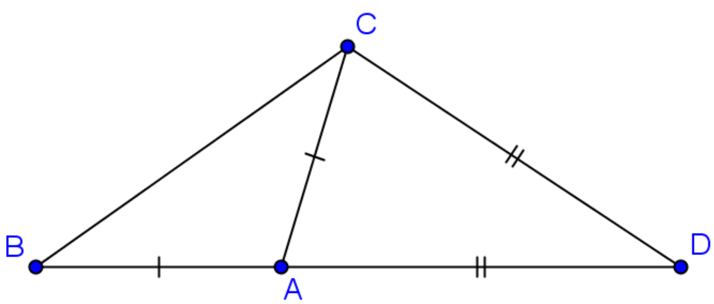
8. Alan says that $\angle 1 \cong \angle 2$ in the diagram below because angles opposite congruent sides are congruent. Is his argument convincing? Explain.



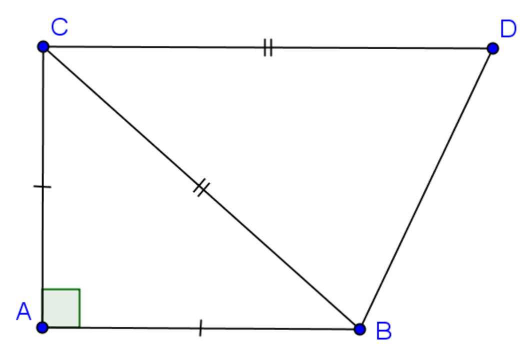
9. Find the length of $\overline{AB}$ in the triangle below.



10. Show that angle $\angle BCD$ is three times $\angle B$ diagram below. Hint: define an"x".

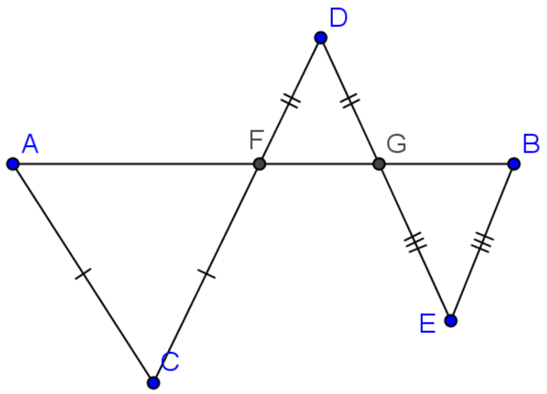


11. Give that $\overline{AB} // \overline{CD}$, find the measure of $\angle D$.

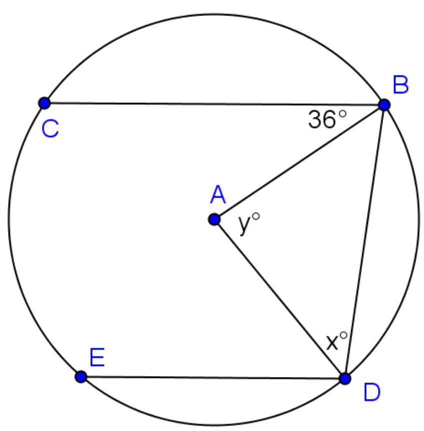


12. In triangles, we sometimes use a lower-case letter to represent the side across from the capital letter angle/vertex. So in triangle ABC, side *c* is the same as side $\overline{AB}$. Given $\Delta ABC \cong \Delta DEF$ and side *a* is three less than twice side *b*. One half of side *e* is 6 less than side *d*. How long is side *e*?

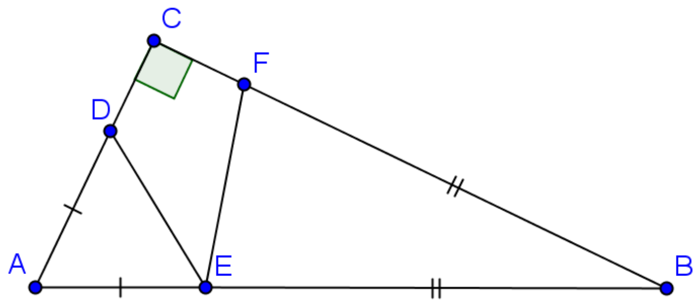
13. Explain why $\angle A \cong \angle B$ in the diagram below.



14. In circle A, $\overline{CB} \parallel \overline{ED}$ and $\overline{AD}$ bisects $\angle BDE$. Find the values of *x* and *y*.

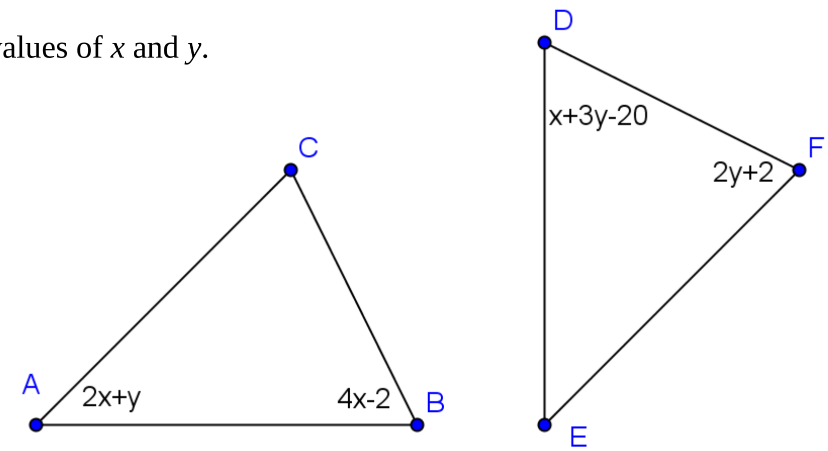


15. In ΔABC below, C is a right angle. Points D, E, and F are on the edges of the triangle to make triangles AED and BEF isosceles. Find the measure of $\angle DEF$ (hint: define angle A as "x")

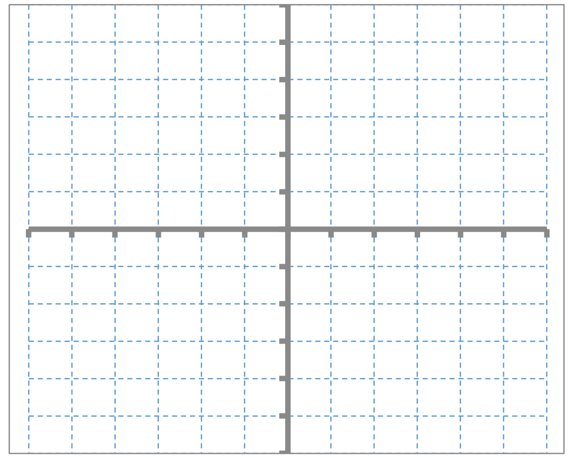


16. Two angles of an isosceles triangle measure $2x+5$ and $x+31$. Find all possible values of x

17. Below, $\Delta ABC \cong \Delta EDF$. Find the values of x and y.



18. Given the points A(-5,2), B(-1,4), C(-3,-2), D(3,0), E(5,4) and F(-1,2), draw triangles ABC and DEF.



a. Is $\angle B \cong \angle C$? Explain.

b. Is $\angle C \cong \angle F$? Explain.

c. Is $\angle A$ a right angle? Explain.

d. Point Z's coordinates are (5,1). Is it the case that $\angle FED \cong \angle EDZ$? Explain.

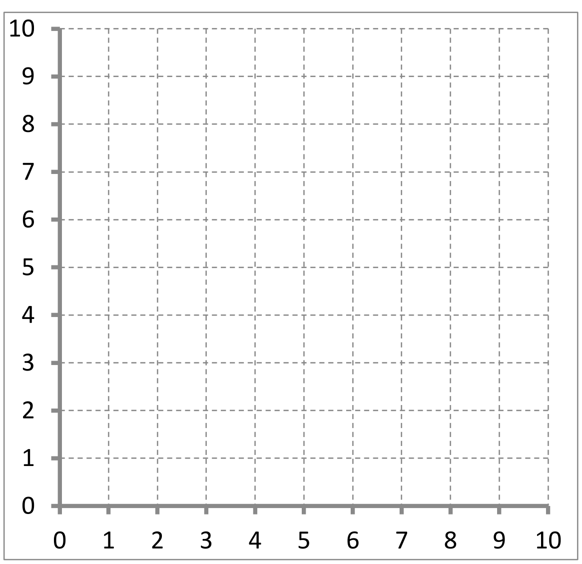
e. Where might point Z be located so that $\angle FED \cong \angle EDZ$?

19. Given the points A(2,6), B(4,8), and C(8,2) do the following:

a. Triangle ABD is isosceles where $\overline{AB} \cong \overline{BD}$. List some possible coordinates for point D.

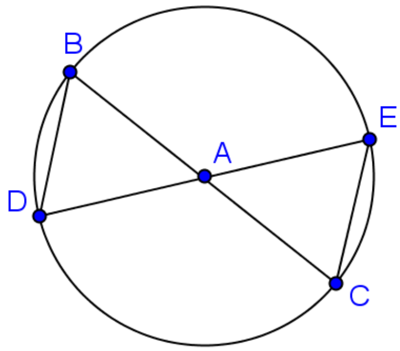
b. Triangle BCE is a right triangle where B measures 90°. List some possible coordinates for point E.

c. Triangle ACF is an isosceles right triangle. List some possible coordinates for point F.

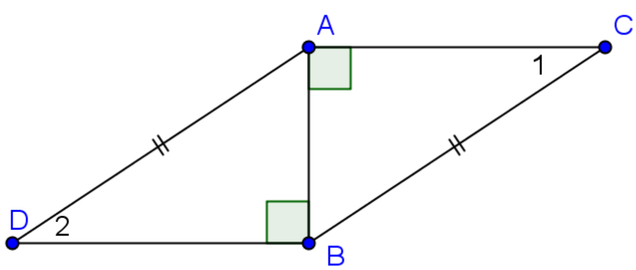


Part II: Proofs

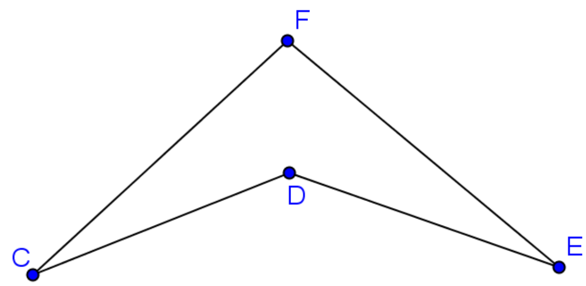
1. Given: circle A Prove $\overline{BD} \cong \overline{EC}$



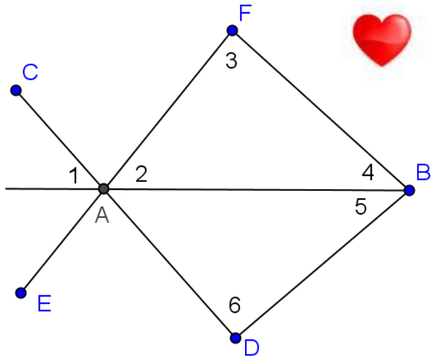
2. Prove $\angle 1 \cong \angle 2$

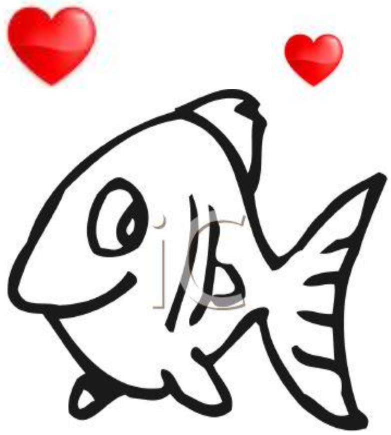


3. Given: $\overline{EF} \cong \overline{CF}$ and $\overline{ED} \cong \overline{CD}$ Prove: $\angle E \cong \angle C$

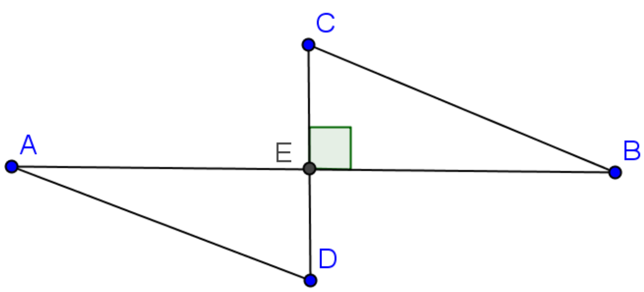


4. Given $\angle 1 \cong \angle 2$; $\angle 4 \cong \angle 5$
Prove: $\angle 3 \cong \angle 6$

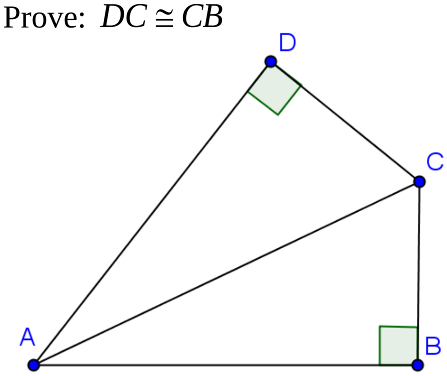




5. Given: $\overline{EC} \cong \overline{ED}$ and $\overline{BC} \parallel \overline{AD}$
Prove: E is the midpoint of $\overline{AB}$

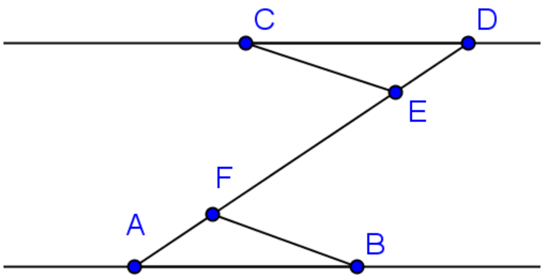


6. Given: $\overline{AC}$ bisects $\angle BAD$
Prove: $\overline{DC} \cong \overline{CB}$



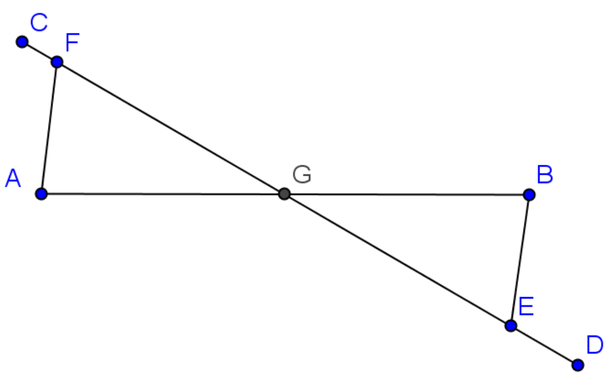
7 Given: $\overline{CD} // \overline{AB}$ and $\overline{AE} \cong \overline{DF}$ $\angle CED \cong \angle BFA$

Prove: $\overline{CD} \cong \overline{AB}$



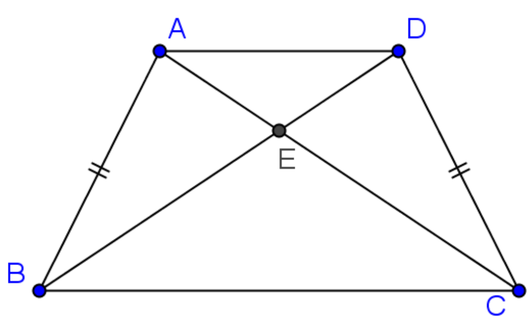
8. $\overline{BE} \cong \overline{AF}$ and $\overline{BE} // \overline{AF}$

Prove: G is the midpoint of $\overline{AB}$ *and* $\overline{EF}$

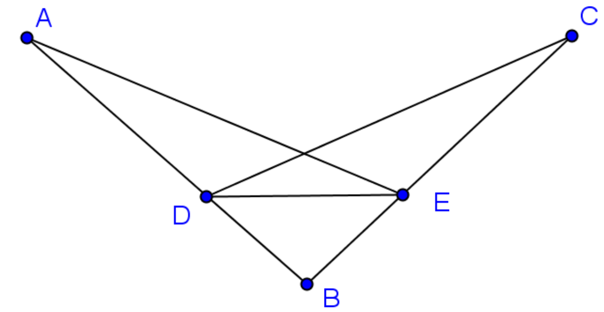


9. Given $\angle ABC \cong \angle DCB$

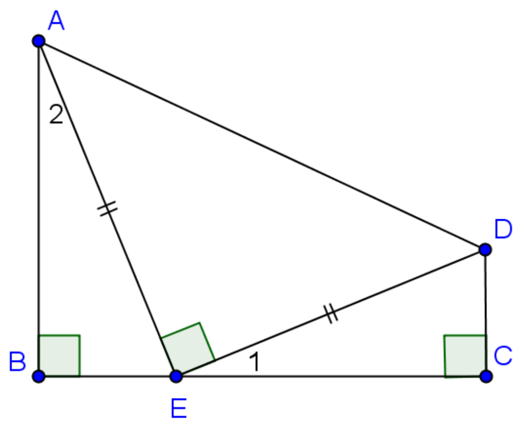
Prove: ΔBEC is isosceles



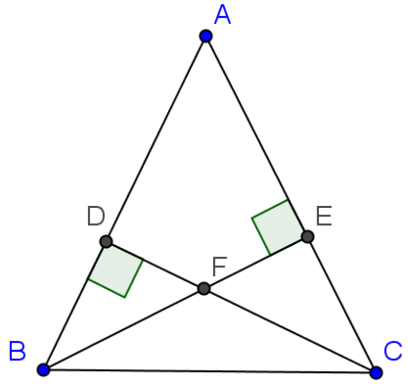
10. Given: $\overline{CE} \cong \overline{AD}$ and $\overline{BE} \cong \overline{BD}$
Prove: $\angle A \cong \angle C$



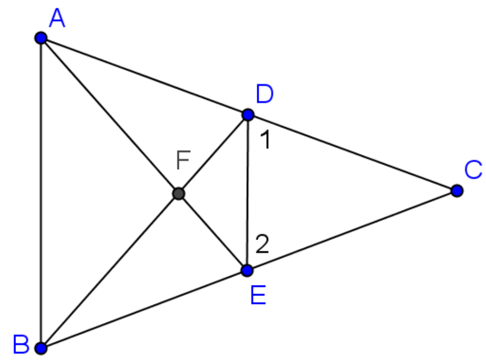
11. Prove $\angle 1 \cong \angle 2$ and $\overline{BE} \cong \overline{CD}$



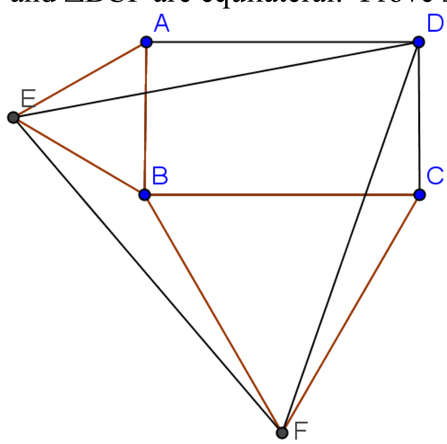
12. Given $\overline{BD} \cong \overline{CE}$
Prove $\Delta DBF \cong \Delta ECF$ then explain why ΔABC must be isosceles.



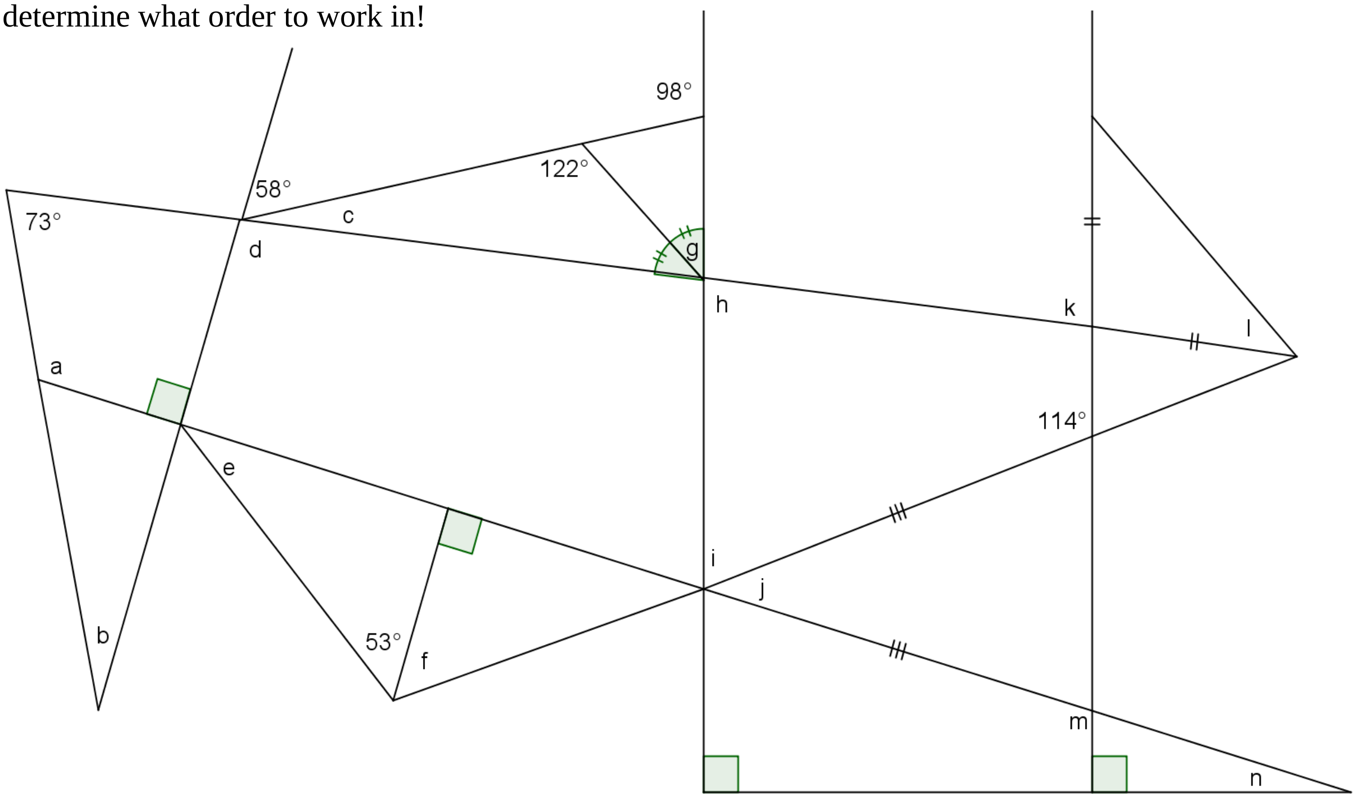
13. Given: $\angle 1 \cong \angle 2$ and $\overline{AC} \cong \overline{CB}$ Prove: ΔDEF is isosceles



14. Given: ABCD is a rectangle (four right angles and two pairs of congruent opposite sides) and ΔAEB and ΔBCF are equilateral. Prove ΔDEF is also equilateral.



Part III: Crazy Angles #3: Note that on this one, the letters of the angles don't really help you determine what order to work in!



Answers: Part I

1. inverse: "if not grapefruit then not spherical" converse: "if spherical then grapefruit"
Contrapositive: "if not spherical then not grapefruit" only contrapositive must be true
2. 25° 3. Continue AB to the right and draw the 7 from C down and right to AB…
4. angles B and E are congruent b/c ΔABE is isosceles; angles DAE and E are congruent b/c alt interior and DAC and B are congruent b/c corresponding angles…. So DAE and DAC are congruent
5. 105° 6. 5 or 8 (x=2 does not make a valid triangle) 7.x=29°; y=32°
8. No, angles opposite congruent sides ***in any given triangle*** are congruent; not in 2 different triangles!
9. 7; must be equilateral 10. If angle B is x then BAC = 180-2x and CAD=2x and ACD=2x so BCD=3x
11. 67.5° 12. 6 since a=2b-3 and 0.5e=d-6 but e=b and a=d so a=2b-3 and 0.5b=a-6
13. $\angle A \cong \angle AFC \cong \angle DFG \cong \angle DGF \cong \angle EGB \cong \angle B$ using vertical angles and isosceles triangles
14. 3x+36=180 so x=48 and y=84. 15. Angle A=x so AED=90-0.5x B=90-x so FEB=45+0.5x
So 90-0.5x + DEF + 45+0.5x = 180° and x=45°
16. those = each other → x=26; 2(2x+5)+x+31=180 → x=27.8; (2x+5)+2(x+31)=180 so x=28.25
17. 4x-2=x+3y-20 and (2x+y)+(4x-2)+(2y+2)=180 so x=18 and y=24
18a. yes, since $\overline{AB} \cong \overline{AC}$ b. Yes since $\Delta ABC \cong \Delta DEF$ by SSS c. yes since, with slopes, $\overline{AB} \perp \overline{AC}$.
d. No, since EF is not parallel to DZ (slopes are 1/3 and 1/2).

e. (6,1) works, so EF and DZ are parallel.. or any other point on the line $y = \frac{1}{3}x - 1$ where x>3

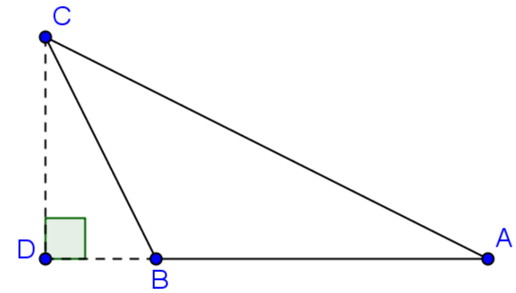
19a. two are (6,6) and (2,10); there are many others (on a circle around B)
b. anywhere on the line $y = \frac{2}{3}x + \frac{16}{3}$ including (1,6) and (4,8)
c. (4,-4), (12,8), (6,12), (-2,0) plus (harder) (7,7) and (3,1)

Part III: a=121° b=31° c=18° d=104° e=37° f=42° g=40° h=80° i=66° j=48° k=80° l=40° m=114° n=24°; Note: I started with angle *g*

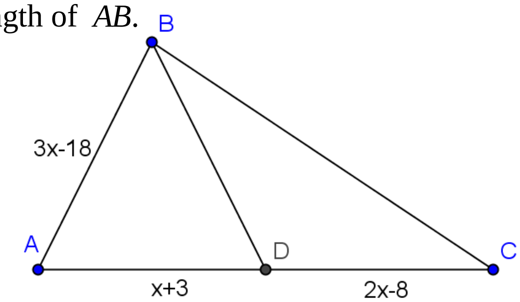
Unit 3 Handout #1: Medians and Altitudes of Triangles

Some interesting line segments associated with triangle:

1. ***Angle bisector***: As the name implies, it bisects an angle.
2. ***Median***: Connects a vertex with the midpoint of the opposite side.
3. ***Altitude***: A perpendicular segment from one vertex to the line containing the opposite side. Note: it may be outside of a triangle, as $\overline{CD}$ is on the right:



Example #1: Given that $\overline{BD}$ is a median of ΔABC, find the length of $\overline{AB}$.



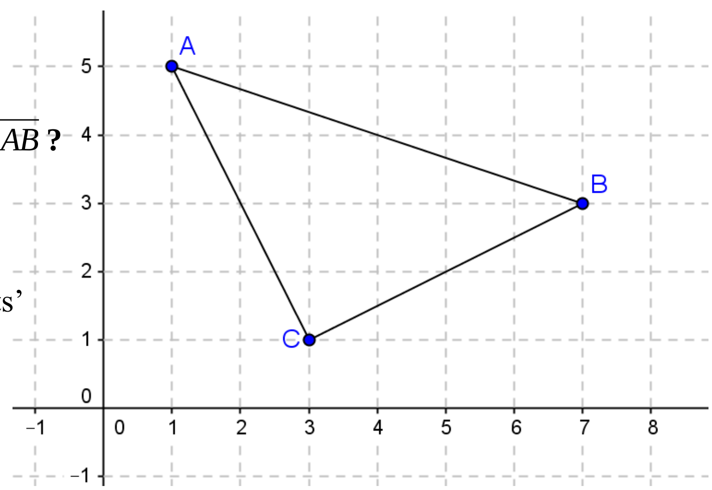
Solution

Since the median divides the opposite side into two congruent segments, we know that x+3=2x-8. Therefore x=11 and the length of $\overline{AB}$ is 3(11)-18, or 15.

Example #2: Given ΔABC below, do the following;

a. Find the length of the median from A to $\overline{BC}$.

b. Where does the altitude from C intersect side $\overline{AB}$ **?**



Solution

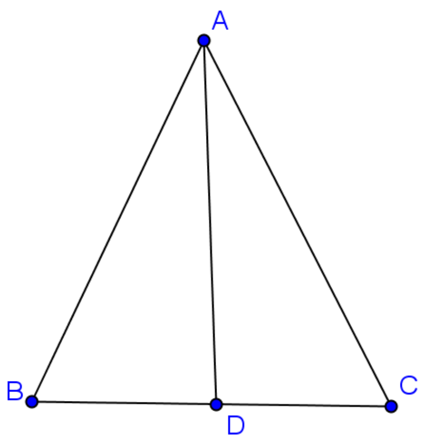
a. The median intersects $\overline{BC}$ at its midpoint. The midpoint's coordinates are the average of the endpoints' coordinates, so (5,2).

This distance is $\sqrt{(\Delta x)^2 + (\Delta y)^2}$, so $\sqrt{(5-1)^2 + (2-5)^2}$ *or* $\sqrt{25}$ *or* 5.

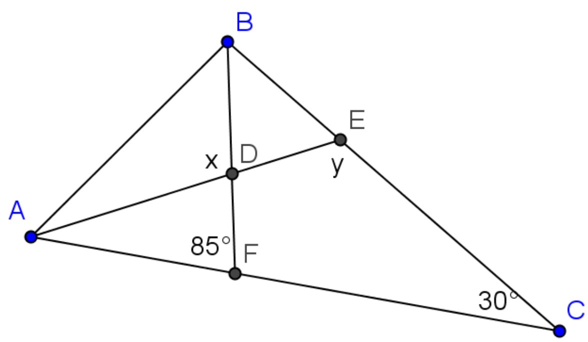
b. We first need equation of the line containing the altitude. It is perpendicular to $\overline{AB}$, whose slope is -1/3, so the altitude's slope is 3. Since it goes through, C. We can plug C's coordinates into $y = 3x + b$, giving us $1 = 3(3) + b$ *and* $b = -8$ so the altitude is on the line $y = 3x - 8$.

To find where it intersects $\overline{AB}$, we need the equation of $\overline{AB}$. It is $y = -\frac{1}{3}x + b$. Plugging A's coordinates in to solve for b, we get $y = -\frac{1}{3}x + \frac{16}{3}$. To find the intersection of these two lines, substitute for y, yielding $3x - 8 = -\frac{1}{3}x + \frac{16}{3}$ *so* $9x - 24 = -x + 16$ *and* $10x = 40$ so x=4. Plug this into either equation to see that y=4 also, so the answer is (4,4).

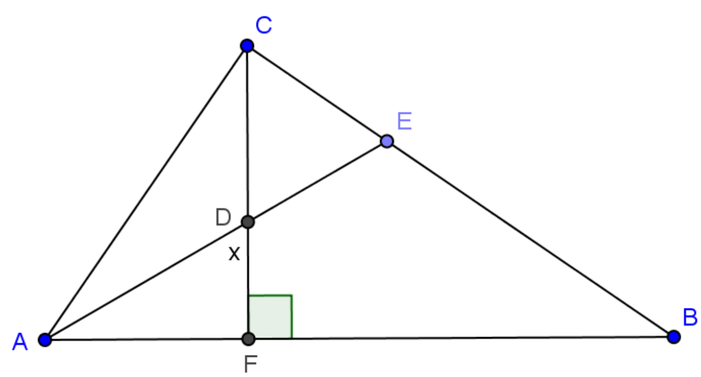
1. In triangle ABC, $\overline{AD}$ is both a median and an altitude. Explain why $\overline{AB} \cong \overline{AC}$.



2. Segments $\overline{AE}$ and $\overline{BF}$ are angle bisectors. Find the values of *x* and *y*.



3. In triangle ABC, $\overline{CF}$ is an altitude and $\overline{AE}$ is an angle bisector. For what value of *x* is triangle ADC isosceles with base $\overline{AC}$?



4. Triangle ABC has vertices A(3,2), B(10,2) and C(9,6).

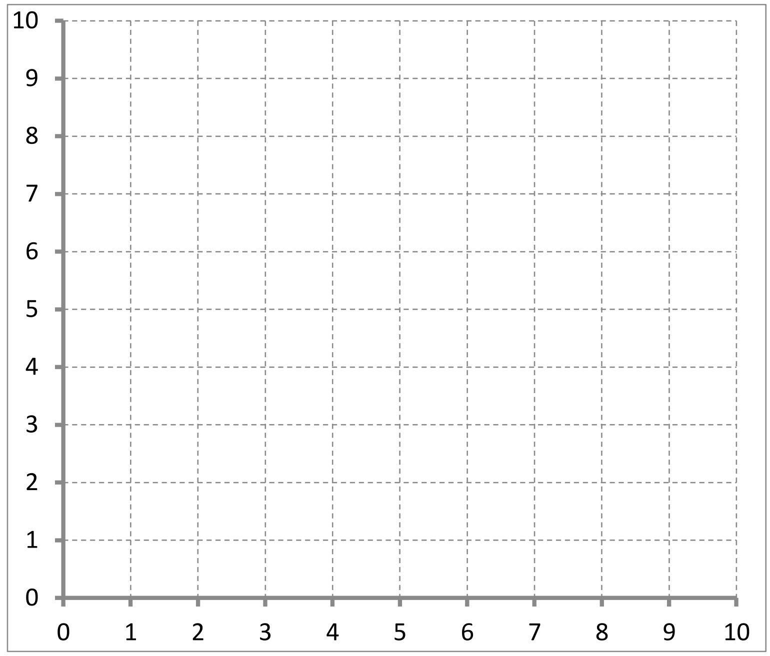
a. What are the coordinates of the point where the median from C hits side $\overline{AB}$?

b. What are the coordinates of the point where the altitude from C hits side $\overline{AB}$?

c. What are the coordinates of the point where the median from B hits side $\overline{AC}$?

d. What are the coordinates of the point where the altitude from B hits side $\overline{AC}$? (don't be afraid of fractions!)

e. $\Delta FGH \cong \Delta ABC$. Given that F's coordinates are (3,1) and G's coordinates are (3,8), give all possible coordinates of H.



5. Given the points A (-3,0), B(5,0), and C(3,6) do the following:

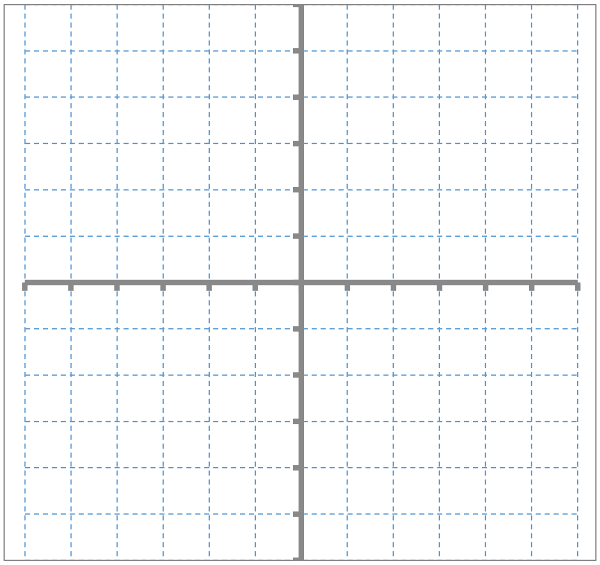
a. Find the equation of the line that contains the median drawn from C to $\overline{AB}$.

b. Find the equation of the line that contains the median drawn from B to $\overline{AC}$.

c. Find the coordinates of the point D where the medians from parts *a* and *b* intersect.

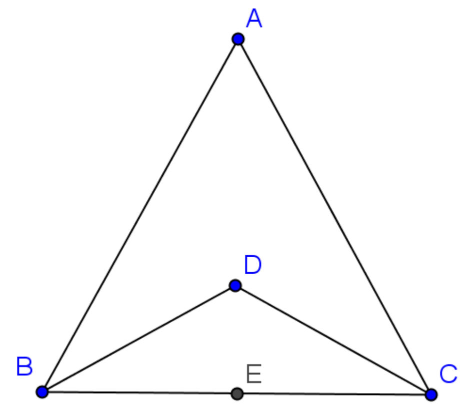
d. Find the equation of the line that contains the median drawn from A to $\overline{CB}$.

e. Does this median also go through point D? Show algebraically.



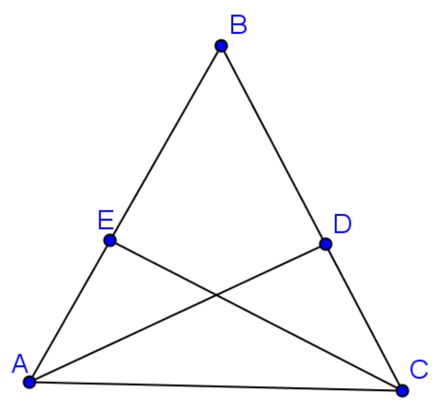
6. Given: $\overline{BD}$ *and* $\overline{CD}$ are bisectors of $\angle ABC$ *and* $\angle ACB$ respectively; also $\overline{BD} \cong \overline{CD}$

Prove $\angle ABC \cong \angle ACB$

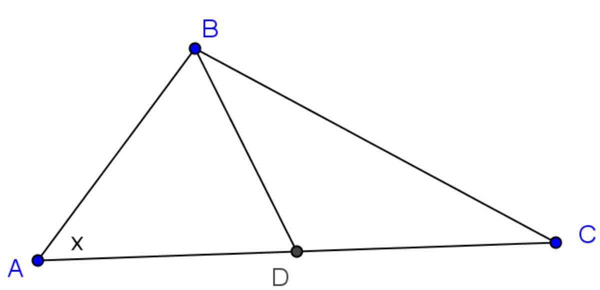


7. Given: $\overline{AD}$ *and* $\overline{CE}$ are altitudes and $\overline{AB} \cong \overline{CB}$

Prove: $\overline{CD} \cong \overline{AE}$.

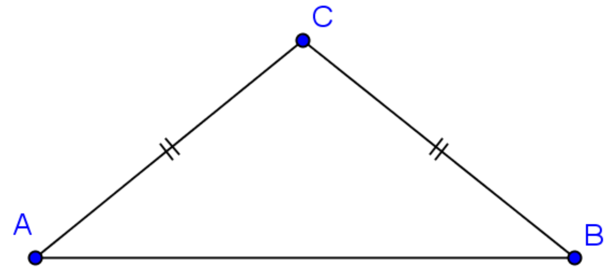


8. In ΔABC, median $\overline{BD}$ is one-half the length of side $\overline{AC}$. Write all angles in terms of *x* and then find the measure of angle ABC.



9. In the triangle below, $\overline{AC} \cong \overline{BC}$. Draw the angle bisector of angle C and then explain why each of the following must be true (note: you do not need to do a formal two-column proof!)

a. The angle bisector of C is also an altitude.



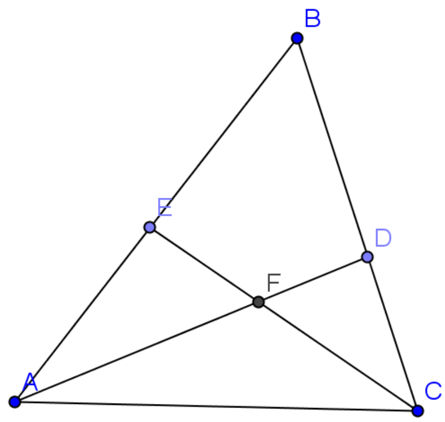
b. The angle bisector is also a median.

c. Angles A and B must be congruent.

10. Prove that the medians to the legs of an isosceles triangle are congruent.

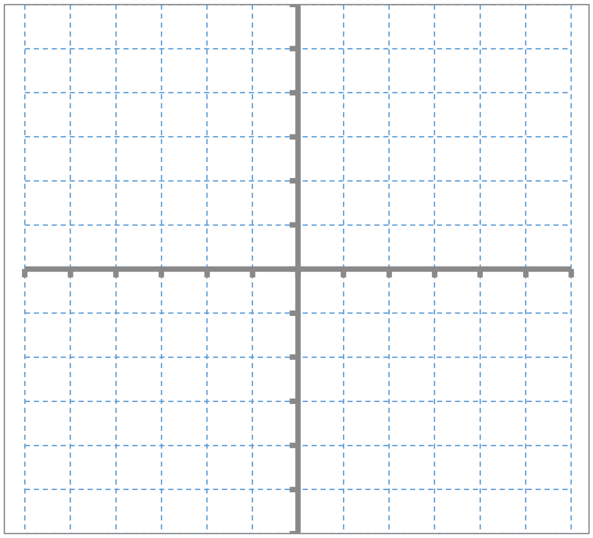
11. In ***scalene*** triangle ABC, draw the median from A to $\overline{BC}$. Now draw segments from B and C to this median that are perpendicular to it (note: you'll likely need extend the median outside of the triangle). Prove that these two perpendiculars are congruent. (Moise-Downs Geometry)

12. $\overline{AD}$ bisects angle BAC and $\overline{CE}$ is an altitude of ΔABC. If angles ECB and AFC measure 30° and 115° respectively, then find the measure of angles ACF and B.

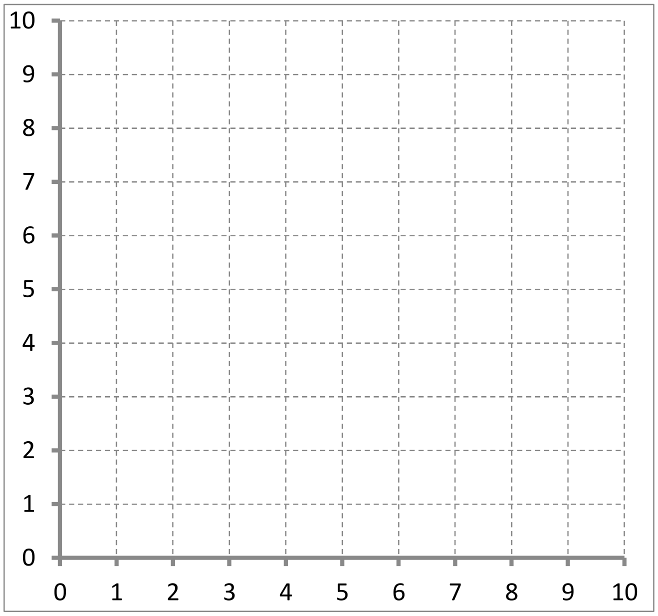


13. In ΔABC, A=100°, B=50°, $\overline{AD}$ is an altitude and $\overline{BE}$ is a median. Find the measure of angle DEC.

14. Show that the medians of the triangle whose vertices are A(0,4), B(4,0), and C(-6,0) are concurrent (meaning that they all intersect in a single point) and that point is 2/3rds of the way from each vertex to the other end of each median. Note: this point is called the ***centroid*** of the triangle.



15. In ΔABC, the medians are $\overline{AD}$, $\overline{BE}$, *and* $\overline{CF}$. The coordinates of D, E, and F are D(6,7), E(2,5), and F(5,2). Find the coordinates of A, B, and C.



Answers

2. x=105° y=130°   3. 60°   4a. (6.5,2) b. (9,2) c. (6,4) d. (102/13, 68/13) e. (-1,7) and (7,7)

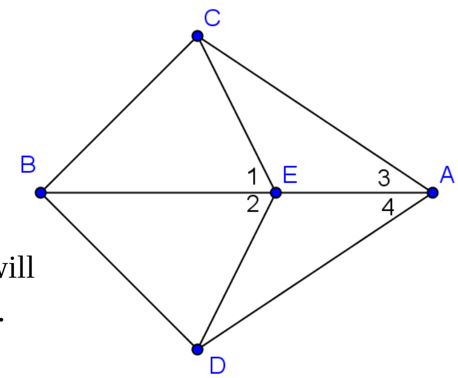
5a. $y = 3x - 3$ b. $y = -\frac{3}{5}x + 3$ c. (5/3,2)   d. $y = \frac{3}{7}x + \frac{9}{7}$ e. does (5/3,2) make eqn true? yes

8. it must be 90°   12. B=60° and ACF=40°   13. 120°   15. A(1,0), B(9,4), and C(3,10)

Unit 3 Handout #2: "Detour Proofs"

A "detour proof" is when you need to prove one pair of triangles is congruent in order to prove another pair is congruent. Typically, CPCTC is used after congruency is established in the first pair. This enables one to prove the second pair congruent.

Example: Given $\angle 1 \cong \angle 2,\ \angle 3 \cong \angle 4$ **prove:** $\overline{BC} \cong \overline{CD}$



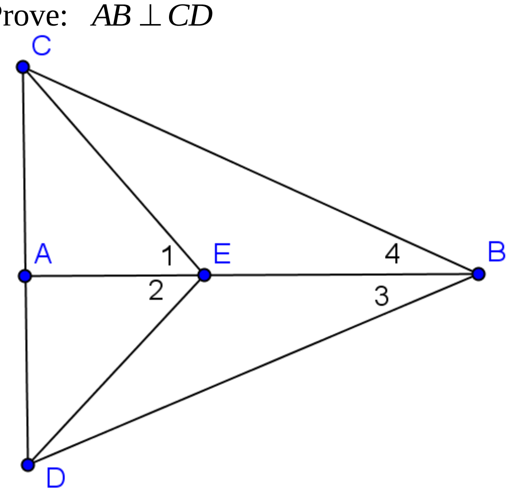
Solution

As given, we do not have enough information to prove that $\Delta BCE \cong \Delta BDE$ or that $\Delta BCA \cong \Delta BDA$. But we can prove that $\Delta ACE \cong \Delta ADE$. Then we can use CPCTC to prove parts equal that will enable us to show either that $\Delta BCE \cong \Delta BDE$ or that $\Delta BCA \cong \Delta BDA$. Then we can use CPCTC to complete the proof.

Statement	**Justification**
1. $\angle 3 \cong \angle 4$	given
2. $\angle 1 \cong \angle 2,$	given
3. $\angle CEA \cong \angle DEA$	2; supplements of congruent angles are congruent
4. $\overline{AE} \cong \overline{AE}$	reflexive property
5. $\Delta ACE \cong \Delta ADE$	1, 3, 4, ASA
6. $\overline{ED} \cong \overline{EC}$	5; CPCTC
7. $\overline{BE} \cong \overline{BE}$	reflexive property
8. $\Delta CEB \cong \Delta DEB$	2, 6, 7, SAS
9. $\overline{BC} \cong \overline{CD}$	8; CPCTC

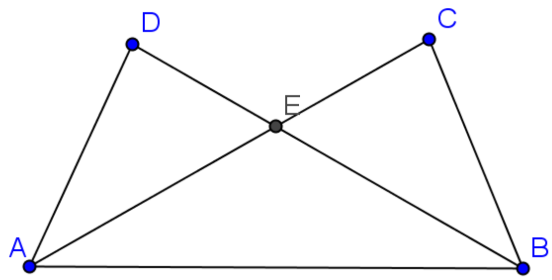
1. Given $\angle 1 \cong \angle 2,\ \angle 3 \cong \angle 4$

Prove: $\overline{AB} \perp \overline{CD}$



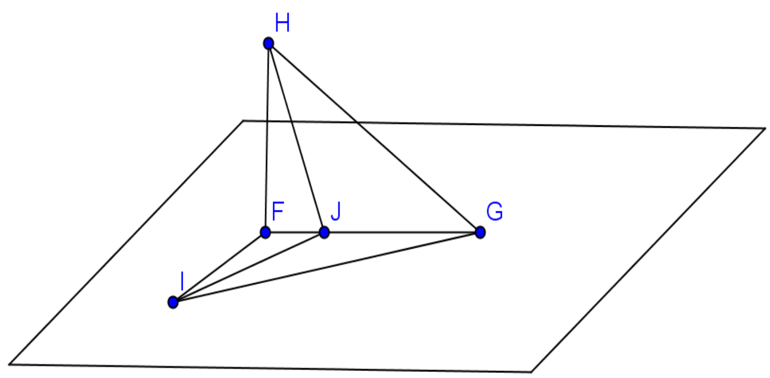
2. Given $\overline{ED} \cong \overline{CE}$, $\angle D \cong \angle C$

Prove: $\angle EBA \cong \angle EAB$



3. In the diagram on the left below, ΔFIG is in the plane while point H is not.

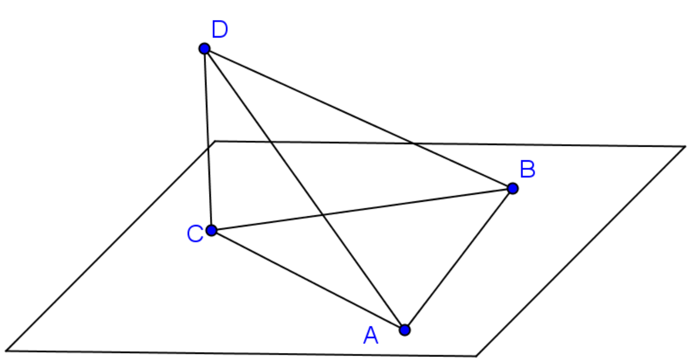
Given $\overline{HJ} \cong \overline{IJ}$ and $\overline{HG} \cong \overline{IG}$ Prove: $\overline{HF} \cong \overline{IF}$



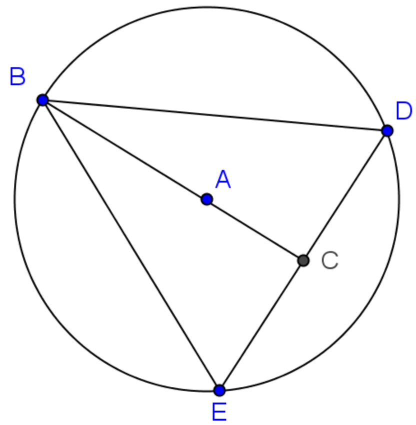
4. In the diagram below, point D is not coplanar with ΔABC. $\overline{DC}$ is perpendicular to the plane containing ΔABC, which means that it is perpendicular to all lines in the plane that contain point C.

Given: $\overline{AD} \cong \overline{BD}$

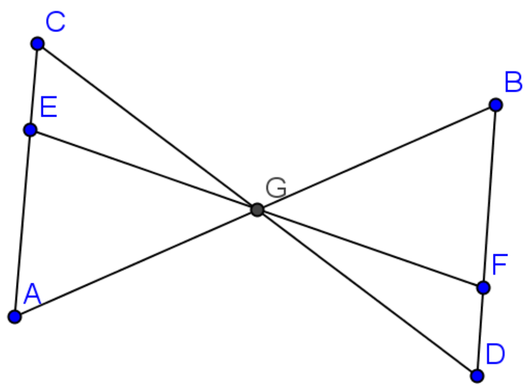
Prove: ΔABC is isosceles



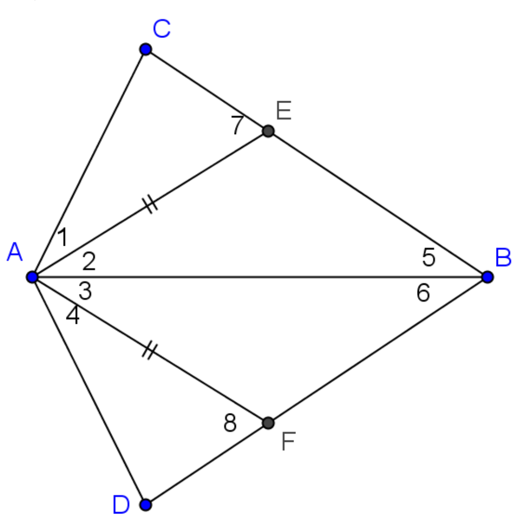
5. Given: Circle A where $\overline{BD} \cong \overline{BE}$
Prove: C is the midpoint of $\overline{DE}$.
Hint: draw some radii!



6. Given: G is the midpoint of $\overline{AB}$ and $\overline{CD}$. Points E, G, and F are collinear. Prove $\overline{AE} \cong \overline{BF}$

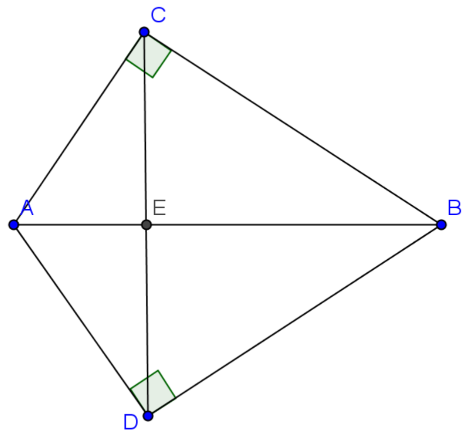


7. Given $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$ Prove $\overline{CE} \cong \overline{DF}$

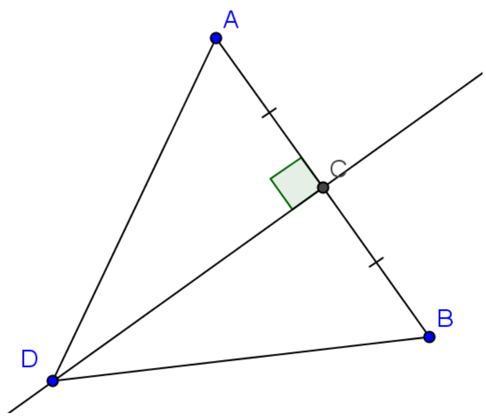


8. Given: $\angle ACB$ *and* $\angle ADB$ are right angles, $\overline{BC} \cong \overline{BD}$

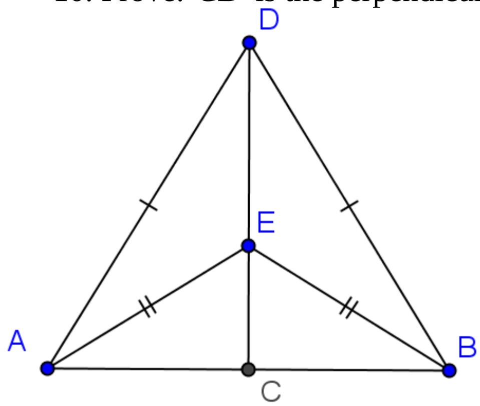
Prove: $\overline{DC} \perp \overline{BA}$



9. Prove that any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment. Using the diagram on the left below, write givens and prove statements and complete the proof.

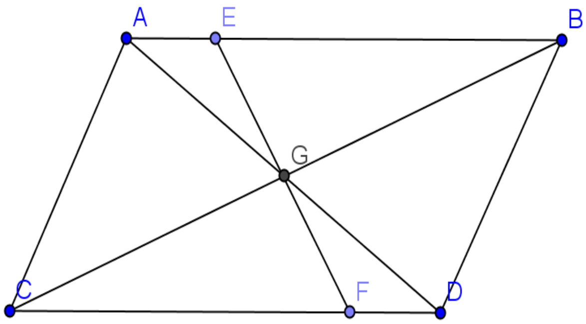


10. Prove: $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$



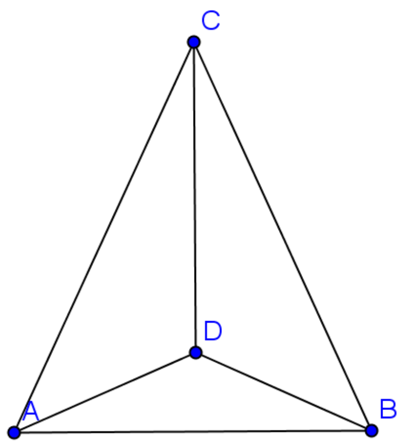
11. Given: $\overline{AB} \parallel \overline{CD}$ *and* $\overline{AB} \cong \overline{CD}$

Prove: $\overline{EG} \cong \overline{FG}$



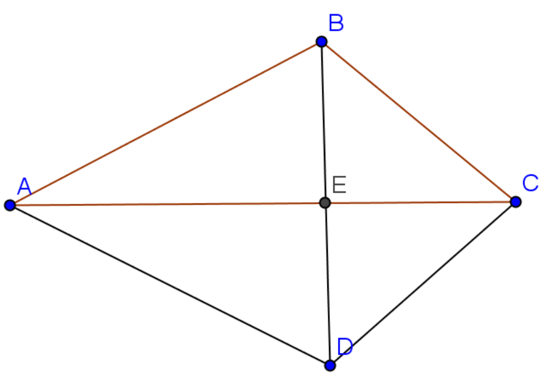
12. Given: $\angle DAB \cong \angle DBA$ and $\angle CAD \cong \angle CBD$

Prove: $\overline{CD}$ bisects ACB $\angle ACB$



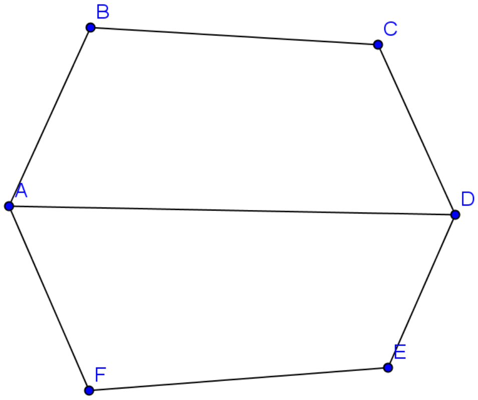
13. Given: $\overline{AC}$ bisects both $\angle BAD$ *and* $\angle BCD$

Prove: $\overline{AC}$ is the perpendicular bisector of $\overline{BD}$.



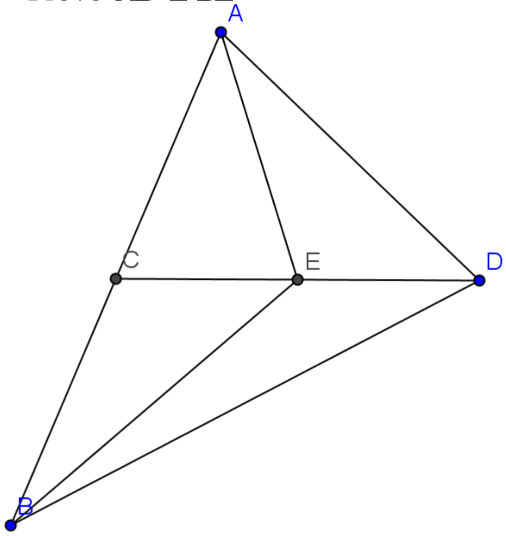
14. Given: $\overline{AB} \cong \overline{AF}$, $\overline{BC} \cong \overline{EF}$, $\overline{CD} \cong \overline{DE}$ and $\overline{AD}$ bisects $\angle BAF$

Prove: $\angle B \cong \angle F$

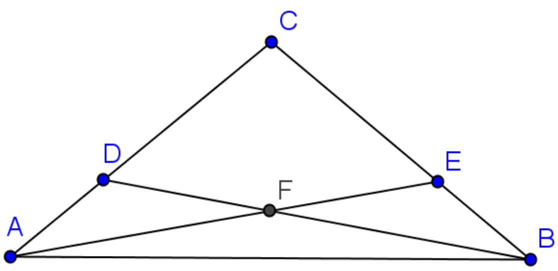


15. Below, C and E are midpoints of $\overline{AB}$ and $\overline{CD}$; $\overline{AC} \cong \overline{AE}$

Prove $\overline{AD} \cong \overline{BE}$

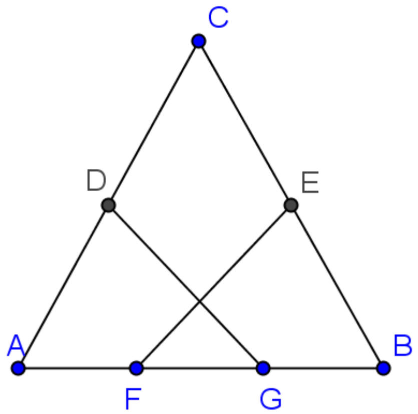


16. Given: $\angle ADB \cong \angle BEA$ and $\angle EAB \cong \angle DBA$ Prove: $\overline{CD} \cong \overline{CE}$



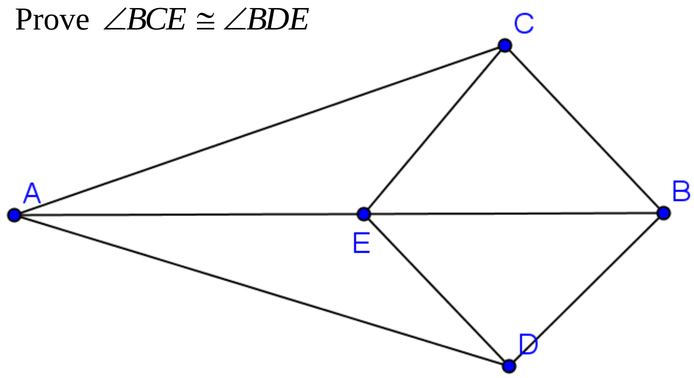
17. Given: D and E are midpoints of $\overline{AC}$ *and* $\overline{BC}$; F and G trisect $\overline{AB}$, $\overline{CD} \cong \overline{CE}$

Prove: $\angle ADG \cong \angle BEF$

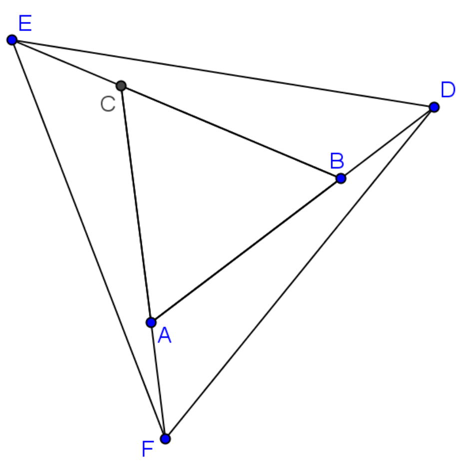


18. Given: $\overline{AC} \cong \overline{AD}$ and $\angle CAE \cong \angle DAE$

Prove $\angle BCE \cong \angle BDE$

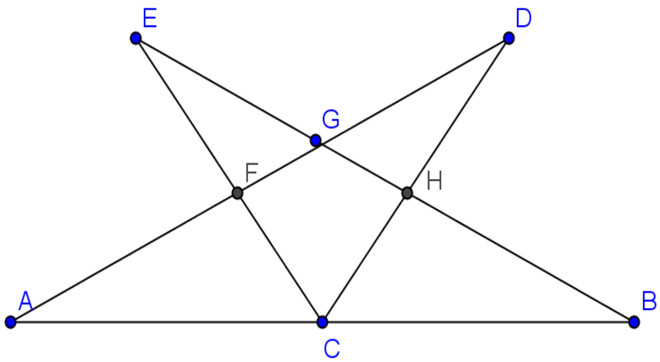


19. ABC is an equilateral triangle and its sides are extended to points D,E and F so that the extensions $\overline{BD}$, $\overline{CE}$, *and* $\overline{AF}$ are equal in length. Prove that ΔDEF is also equilateral



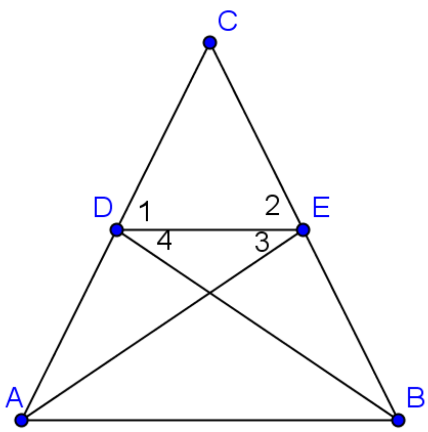
20. Given $\angle ACF \cong \angle BCH$; C is the midpoint of $\overline{AB}$; $\angle A \cong \angle B$

Prove: $\overline{EG} \cong \overline{DG}$



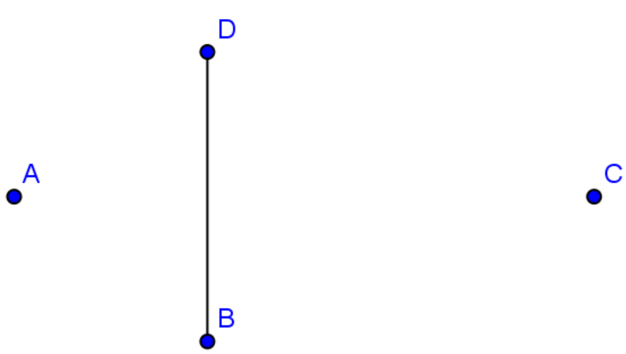
21. Given: $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$

Prove: ΔABC is isosceles.



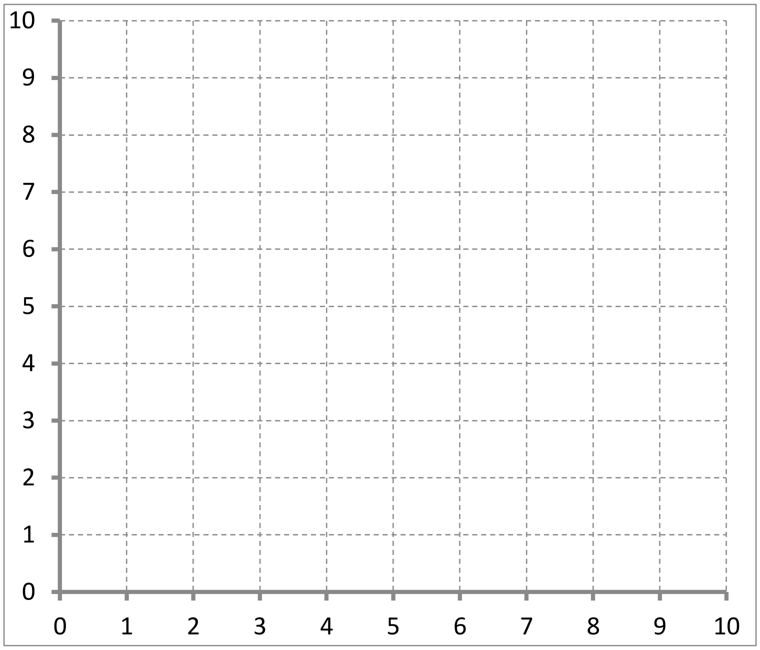
22. Given A and C are both equidistant from the endpoints of $\overline{BD}$

Prove: $\overleftrightarrow{AC}$ is the perpendicular bisector of $\overline{BD}$



23. Given triangle ABC with vertices A (1,2), B(9,6), and C(7,0), do the following:

a. Find the perimeter.



b. Find the equation of the line containing the median from vertex A to side $\overline{BC}$.

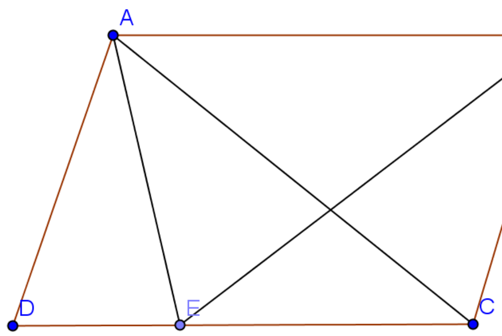
c. Find the equation of the line containing the altitude from vertex C to side $\overline{AB}$.

d. Find the coordinates of the point where the altitude from C to AB intersects side $\overline{AB}$.

e. Is C a right angle? Justify your answer.

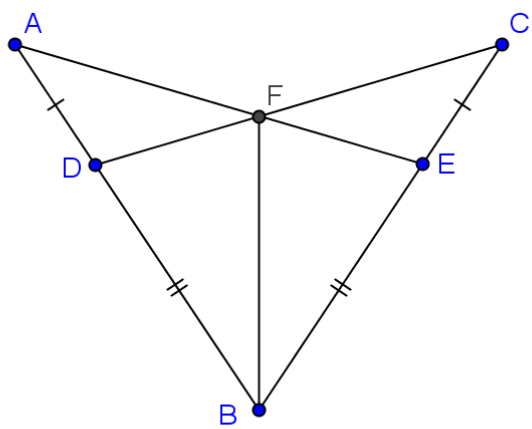
24. Given: $\overline{AD} \cong \overline{AE}$ and $\angle DAE \cong \angle CAB$ and $\angle D \cong \angle AEB$

Prove: $\overline{AB} \cong \overline{AC}$



25. Given: Point F is the intersection of $\overline{AE}$ *and* $\overline{CD}$.

Prove $\overline{BF}$ bisects $\angle ABC$



Answers

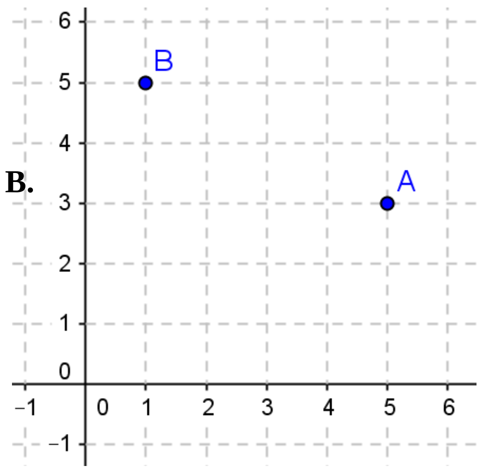
23a. $4\sqrt{10}+4\sqrt{5}$ b. $y=\frac{1}{7}x+\frac{13}{7}$ c. $y=-2x+14$ d. (5,4)

e. yes; the slope of AC is -1/3 & the slope of BC=3; since they are negative reciprocals, C is a right angle

24. look at triangles AEB and ADC   25. look at triangles ABE and CBD…

Unit 3 Handout #3: Equidistance

The ***perpendicular bisector*** of a segment contains of all of the points in a plane that are ***equidistant*** from the endpoints of the segment. This can be helpful for constructing isosceles triangles given a base and for finding the center of a circle given three points.



Example #1: Given A(5,3) and B(1,5), find the following:

a. The equation of the perpendicular bisector of $\overline{AB}$.

b. The point on the *x*-axis that is equidistant from point A and B.

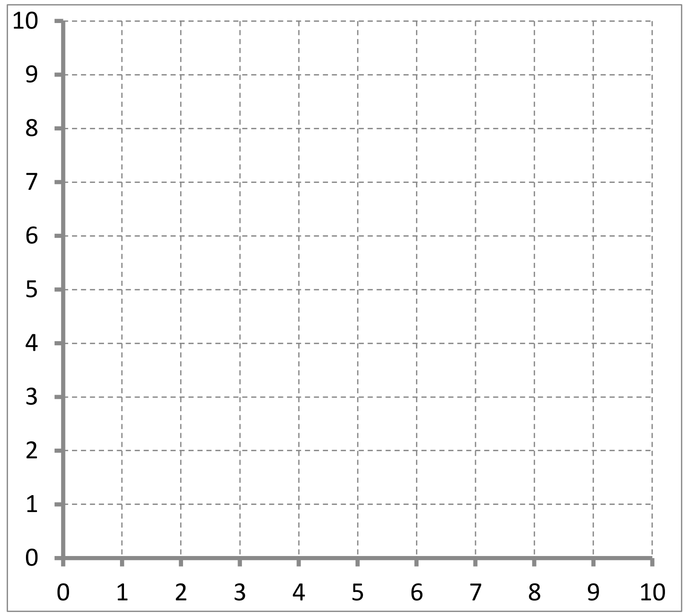
Solution

a. The perpendicular bisector is a line, so we need a point and the slope. A point is the midpoint of $\overline{AB}$, which is (3,4). The slope of the perpendicular bisector is the negative reciprocal of the slope of $\overline{AB}$. Since $\overline{AB}$'s slope is -½, the slope of the perpendicular bisector is 2. Thus the perpendicular bisectors's equation is $y = 2x + b$, and putting (3,4) in we get *b*=-2 so $y = 2x - 2$.

b. The perpendicular bisector is all points equidistant from A and B. So it is the point on the perpendicular bisector on the *x*-axis. Plugging zero for *y* to the equation $y = 2x - 2$ yields *x*=1. So the point is (1,0). A quick check shows that (1,0) is 5 units from both A and B.

1. Prove that, for any point P on the perpendicular bisector of segment $\overline{AB}$, $\overline{PA} = \overline{PB}$.

2. Given the points A(2,1), B(2,5), and C(6,3), write the equation of the perpendicular bisector of each segment below:



a. $\overline{AB}$

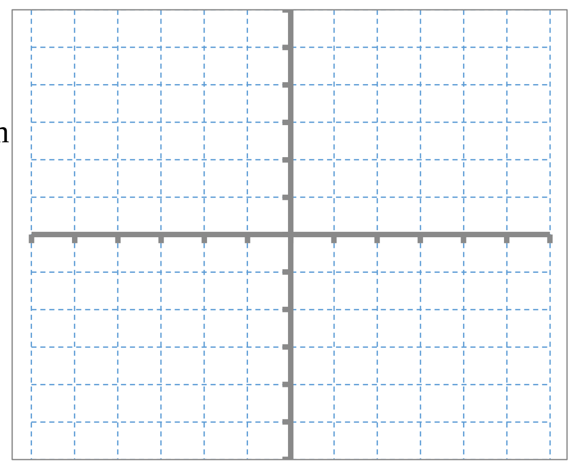
b. $\overline{AC}$

c. $\overline{BC}$

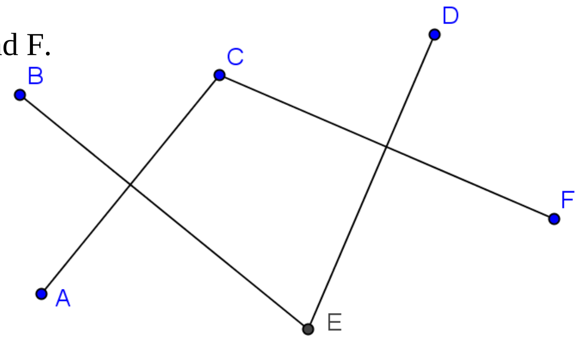
3a. Write the equation of the set of points that are equidistant from the points A(-4,2) and B(2,-6).

b. Find the exact coordinates of the point on the *x*-axis equidistant from points A and B.

c. Are there any points on the line $y = 5 - x$ that are equidistant from points A and B? If so, find their coordinates algebraically.

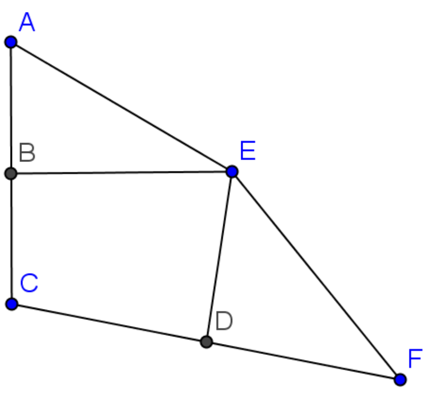


4. In the diagram below, $\overline{BE}$ is a perpendicular bisector of $\overline{AC}$, and $\overline{DE}$ is a perpendicular bisector of $\overline{CF}$. Prove that point E is equidistant from points A, C, and F.

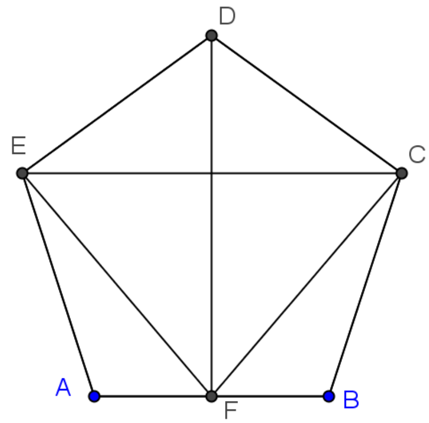


5. How can you apply what you did in question 4 above to find the center of a circle that goes through any three given points? By the way, are there ever three given points that do not determine a circle?

6. In the diagram below, $\overline{BE}$ is a perpendicular bisector of $\overline{AC}$, and $\overline{DE}$ is a perpendicular bisector of $\overline{CF}$. Prove that $\overline{AE} \cong \overline{EF}$. Hint: draw $\overline{CE}$.



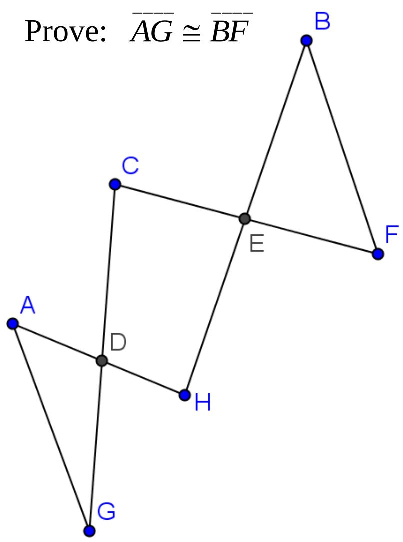
7. Given that ABCDE is equilateral and equiangular and F is the midpoint of $\overline{AB}$. Prove that $\overline{DF}$ is the perpendicular bisector of $\overline{EC}$.



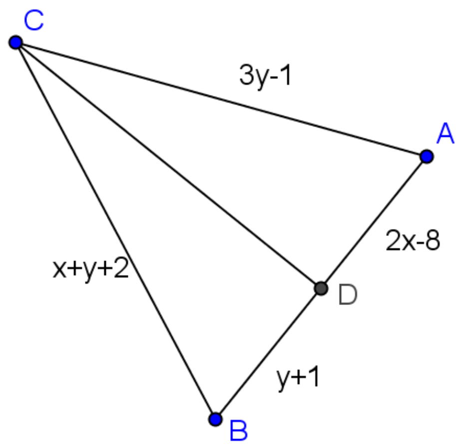
8. Circles A and B are congruent. They intersect at points C and D. Prove that $\overline{AB}$ is the perpendicular bisector of $\overline{CD}$.

9. Given : $\overline{AH}$ and $\overline{GC}$ bisect each other, and $\overline{BH}$ and $\overline{FC}$ bisect each other

Prove: $\overline{AG} \cong \overline{BF}$

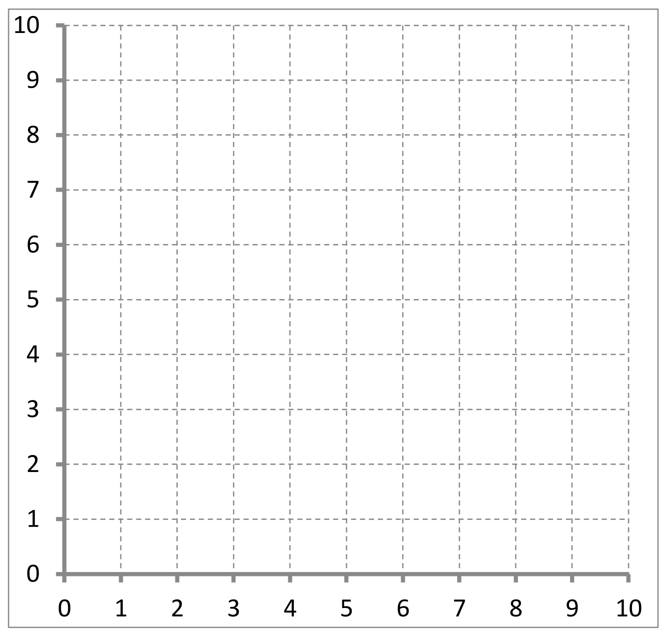


10. Given that $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$, find the perimeter of ΔABC.

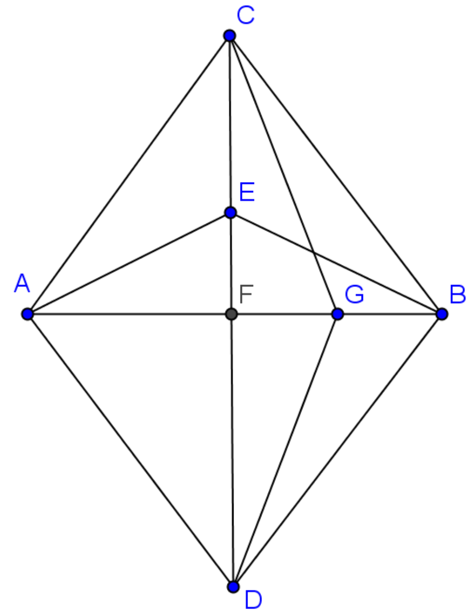


11a. Given points A(2,6) and B(6,4), draw all possible points C where triangle ABC is isosceles with $\overline{AC} \cong \overline{BC}$.

b. What, if anything, would be different if part *a* did not specify which two sides of ABC must be congruent? In other words, draw all points C where ΔABC is isosceles.



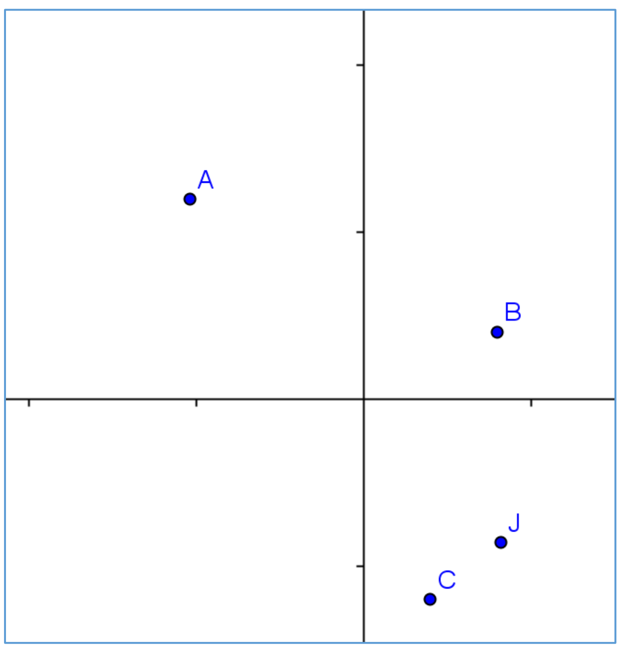
12. Segments $\overline{AB}$ *and* $\overline{CD}$ are the perpendicular bisectors of each other.



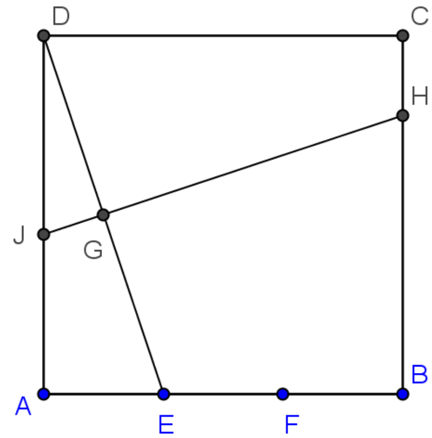
a. Name all pairs of congruent segments in the diagram.

b. Must it be the case that $\overline{AC} \cong \overline{BD}$? Explain.

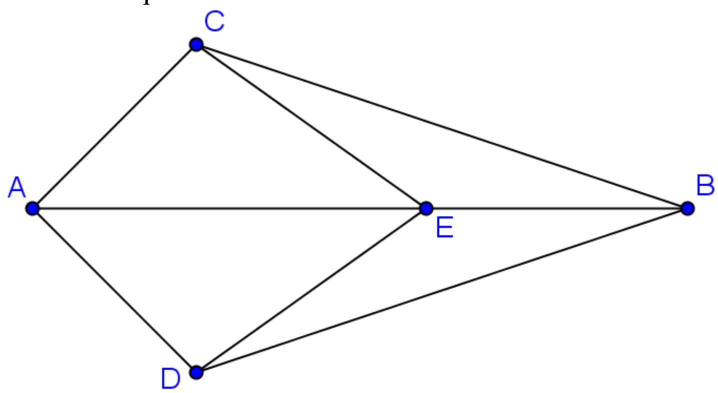
13. A rectangular town has three schools: Augustus (A), Brutus (B), and Cleopatra (C) shown below. Each person goes to the school that he or she lives closest to; so Julius (J) goes to Cleopatra. Shade the area showing the part of town that people live in if they go to the Augustus school. An approximate answer is fine; ***no calculations are necessary!*** Briefly explain your approach.



14. Square ABCD has side length of 6. Points E and F trisect side $\overline{AB}$ and $\overline{JH}$ is the perpendicular bisector of $\overline{DE}$. What are the length of segments $\overline{AJ}$ and $\overline{CH}$? Hint: put it on the coordinate plane!

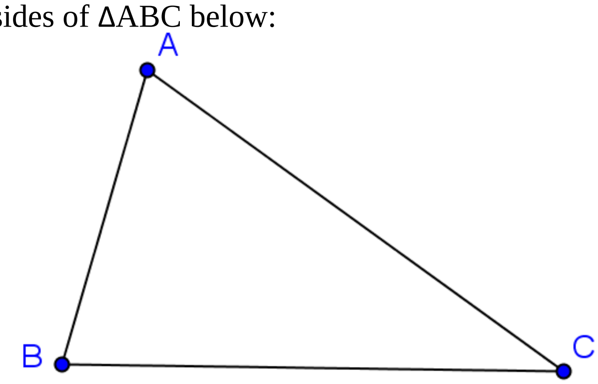


15. Given: A is equidistant from C and D; B is equidistant from C and D.
Prove: E is equidistant from C and D.



16. The set of points in a plane equidistant from two points is a line: the perpendicular bisector. What is the set of points equidistant from two points in space (three dimensions)?

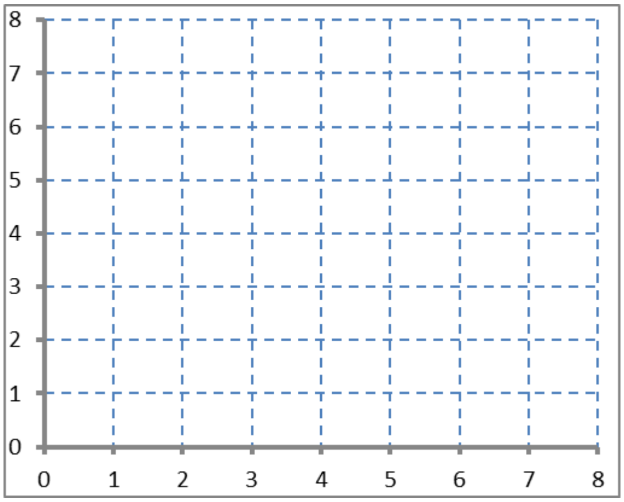
17a. Draw (approximate) perpendicular bisectors for the three sides of ΔABC below:



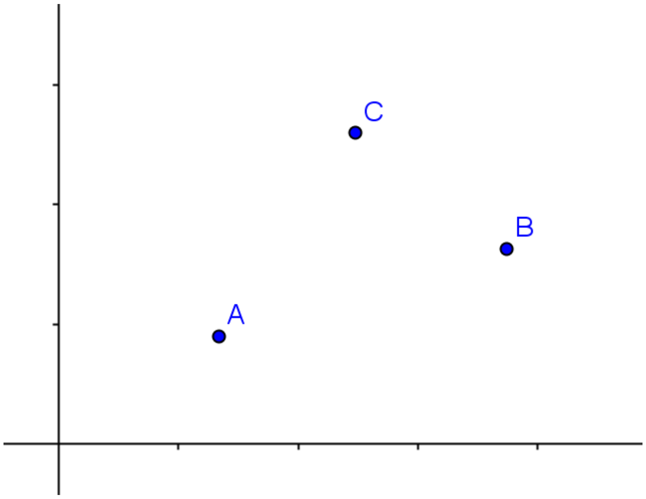
b. Intuitively, why must they all meet in one point? In other words, given that two perp bisectors meet at that point, why can you be sure that the third one must also go through it? Note: the point is called the circumcenter.

c. What is the significance of this point?

18. Given points A (0,4), B(4,6), C(6,2), and D(8,0). Triangles ABE and CDE are both isosceles, with bases $\overline{AB}$ and $\overline{CD}$. What are E's coordinates?



19. Given points A, B, and C below, show where point D can be located such that $\angle ABC \cong \angle BCD$. Draw an approximate answer and describe how you could get the exact answer. Note: your answer should be two rays meeting at one point.



20. Given A(0,0), B(-3,0), and C(4,3), draw the bisector of angle BAC. Then find its exact slope. See the answers for a hint.

Answers

2a. y=3 b. y=-2x+10 c. y=2x-4 3a. y=(3/4)x-5/4 b. (5/3,0) c. (25/7, 10/7)

5. draw perp bisectors of two segments connecting the points and find where they meet

10 y=5 and x=7 so the perimeter is 40

11a. anywhere on the perp bisector of AB → y=2x-3 (except for (4,5) since then there's no triangle)

b. another 2 sides could be congruent… so C could be anywhere on a circle centered at either A or B with a radius of $\sqrt{20}$

12a. AC & AD, BC & BD, CF & DF, CG & DG, AC&BC, AE&EB, AF&BF; AD&DB

b. yes, by transitivity: AC=BC and BC=BD so $\overline{AC} \cong \overline{BD}$?

13. draw perp bisectors connecting AB, BC, and AC 14. AJ=8/3 and CH=4/3 16. a plane

17b. the point where two meet is equidistant from the 3 vertices; the 3rd perp bisect is points equidistant from two vertices so it must include the intersection of the first two. C. If a circle is circumscribed around the triangle (going through the three vertices) then this would be its center.

18. (5,-1)→ it must be on perp bisectors of both AB and CD

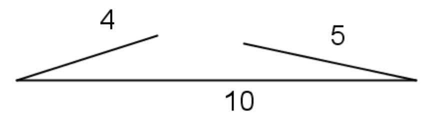
19. D can be to the right of C where CD // AB (alt interior angles); also, draw the perp bisector of BC; let it intersect AB at point E. D can be anywhere on ray CE since BCE is isosceles. Note: D cannot be exactly at point C, since we need two rays to form an angle.

20. Hint: it is hard to write the equation of an angle bisector, but easier to write one of the median or altitude. So create a triangle that contains angle BAC where the angle bisector is also a median or altitude! You'll end up with a slope of -3.

Unit 3 Handout #4: Inequalities

There are two types of inequalities involving triangles in this unit:

1. ***Side lengths***. The longest side of a triangle must be shorter than the sum of the other two sides. Otherwise the three lengths do not form a triangle: see below:



2. ***Sides and Angles***: In any triangle, the largest angle is opposite the longest side and the smallest angle is opposite the shortest side. Note: this only applies within a given triangle! A side opposite an angle of 10° in one triangle may be longer than a side opposite an angle of 120° in a different triangle!

Example #1: If two sides of a triangle are 3 and 9, what are possible lengths for the third side?

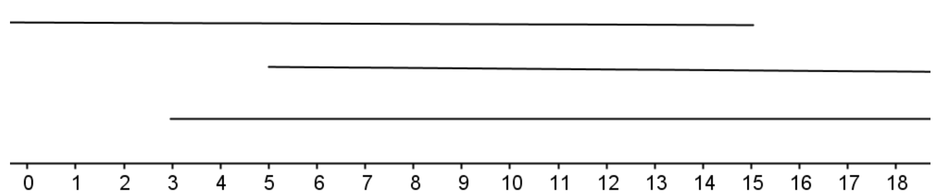
Solution:

Call the length of the third side x. We know that $x+3>9$ *and* $3+9>x$. We could also write that $x+9>3$ but that is not necessary, since three cannot be the longest side.
Now solve each inequality: $x>6$ *and* $x<12$. So our answer is $6<x<12$.

Example #2: For what values of *x* can 5, *x*+3, and 2*x*-7 be a valid triangle?

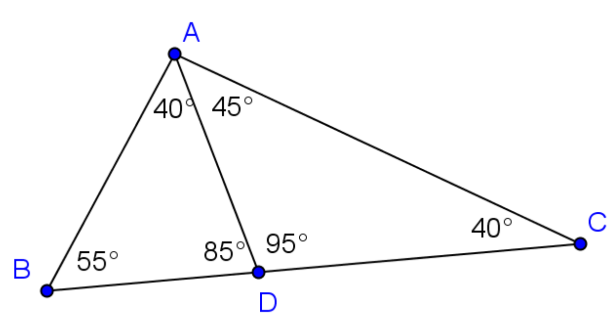
Solution

Each side must be shorter than the sum of the other two. Thus $5<(x+3)+(2x-7)$ and $x+3<5+2x-7$ and $2x-7<5+x+3$. Solving these we get $x>3$ *and* $x>5$ *and* $x<15$. The "and" inequality is true when all three parts are true. The diagram below shows that all are true when $5<x<15$.



Example #3: In the diagram below, determine which of each pair of segments is longer or indicate that it cannot be determined.

a. $\overline{AD}$ *or* $\overline{CD}$ **b.** $\overline{AB}$ *or* $\overline{AC}$ **c.** $\overline{BD}$ *or* $\overline{AC}$



Solution

a. In ΔACD, $m\angle DAC > m\angle C$, so CD>AD.

b. In ΔABC, $m\angle B > m\angle C$, so AC>AB.

c. In ΔACD, $m\angle ADC > m\angle C$, so AC>AD. And in ΔABD, $m\angle B > m\angle BAD$, so AD>BD. Since AC>AD and BD<AD, it must be the case that AC>BD.

Inequalities with side lengths

1. Warm-up. Given each system of inequalities, write the answer in the simplest form. You may want to graph them on a number line.

a. $x < 3$ *and* $x > -2$ *and* $x \geq 0$

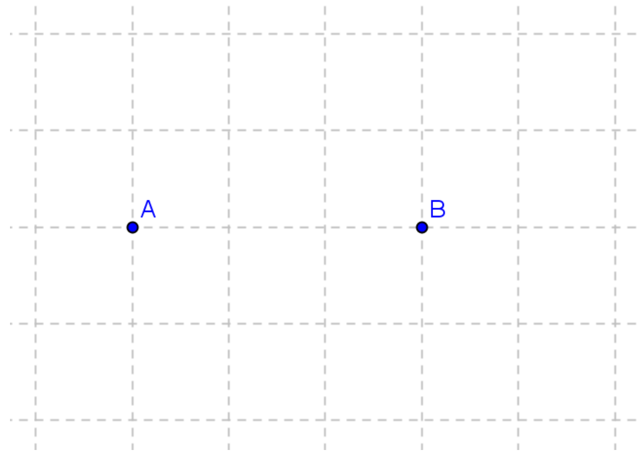
b. $-2 < x < 5$ *and* $x > 3$

2. Given that two sides of a triangle measure 6 and 9, find all possible lengths of the third side. Express your answer as an inequality.

3a. An isosceles triangle has sides of 3 and 4. What perimeters are possible?

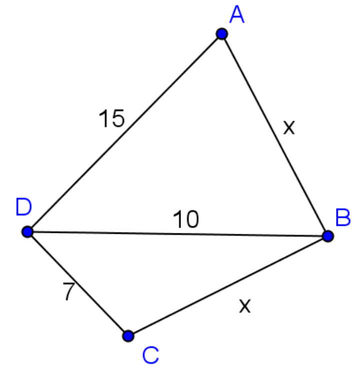
b. An isosceles triangle has sides of 3 and 7. What perimeters are possible?

4. The distance from A to B is 9 units and from B to C is 6 units. What are the possible distances from A to C? Also, given points A and B below, draw where C might be located.



5. The perimeter of a scalene triangle is 60. Write an inequality expressing the possible length of the longest side.

6. Write an inequality describing all possible values of x in the diagram below (not to scale—angles that appear acute may actually be obtuse, and vice versa).

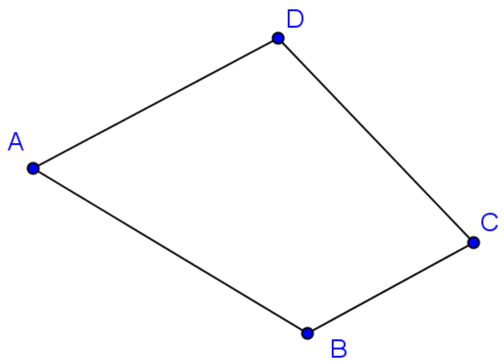


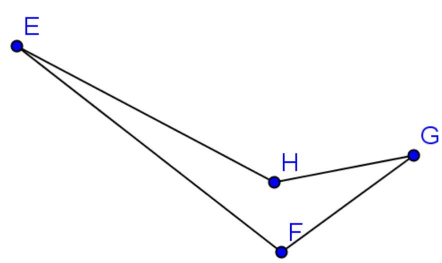
7. The sides of a triangle are 12, x, and $3x-18$. For what values of x is $3x$-18 the longest side? Make sure it is a valid triangle!

8. Find all values of x such that 12, x, and $3x-18$ can be sides of a triangle. Hint: write three inequalities and solve them, since we do not know which is the longest.

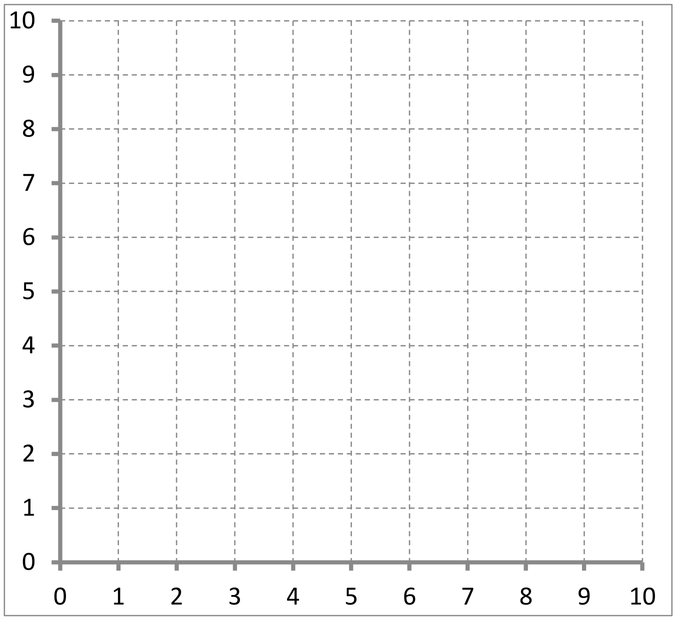
9. How many different isosceles triangles have sides that are all integers and perimeters of 19?

10. Explain why the perimeter of quadrilateral ABCD must be greater than twice the length of the longer diagonal. Does this also mean that the perimeter of ABCD must be greater than the sum of the lengths of the two diagonals? Explain. (note: ABCD is a "convex" quadrilateral in that no vertex angle exceeds 180°-- quadrilateral EFGH is NOT convex; it is concave because of angle H)



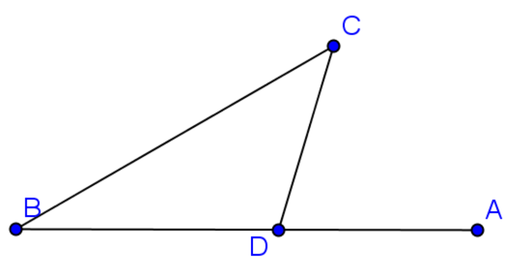


11. The sides of a triangle are 4, x, and y. Graph the set of points (x,y) that make this a valid triangle.



Inequalities with angles

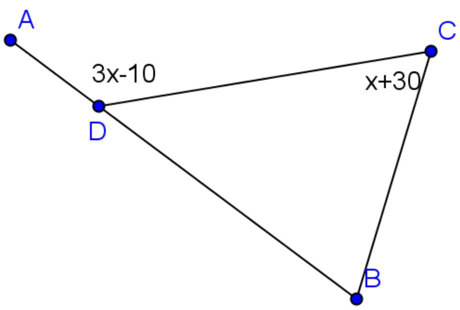
12. Explain why an exterior angle of a triangle is equal in measure to the sum of the two "remote interior angles." (these are the two angles of the triangle not adjacent to the given exterior angle—like angles B and C for exterior angle ADC below).



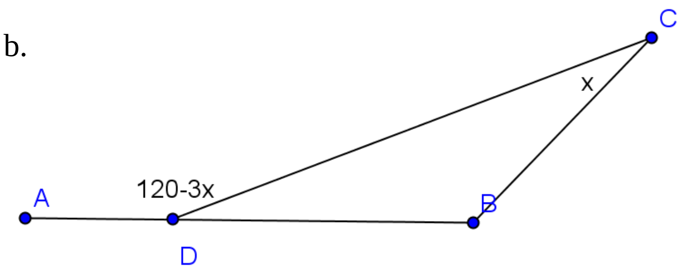
13. From the previous problem, can you conclude that each exterior angle of a triangle must be larger than either remote interior angle? Explain.

14. Find all possible values of *x* below. The diagrams may not be to scale, so make no assumptions about what angles are obtuse or acute. Be sure to look for the largest and smallest values as well (can angles have a negative measure?)

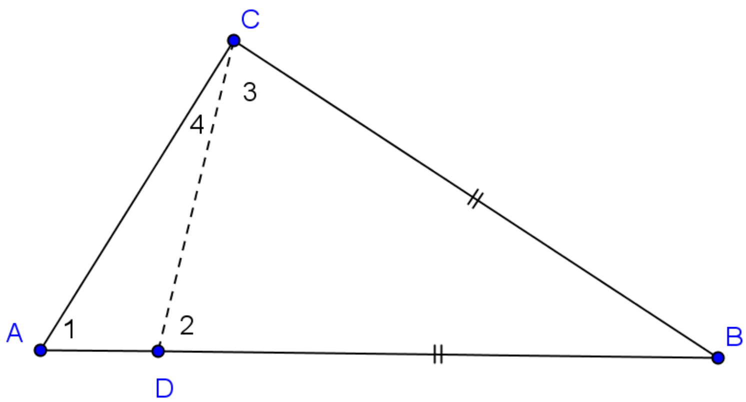
a.



b.

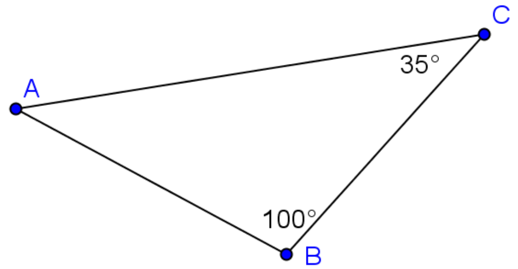


15. It turns out that, in any triangle, longer sides are opposite larger angles. The diagram below should help explain why. In triangle ABC, $\overline{AB}$ is longer than $\overline{BC}$.. Point D is placed on $\overline{AB}$ such that BC=BD. Show that angle ACB must be greater than angle CAB.



16. In the triangles below, rank the lengths of the sides from longest to shortest. Explain your answers. Note: we typically use lower-case letters for sides, where the side has the same letter as the opposite angle. So side AC can be referred to as side *b*.

a.



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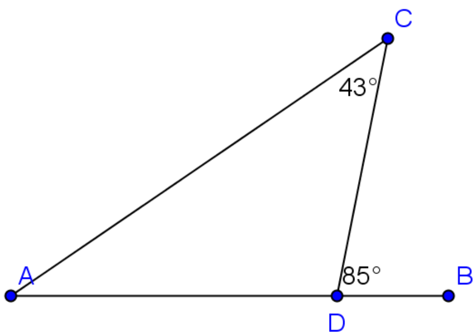
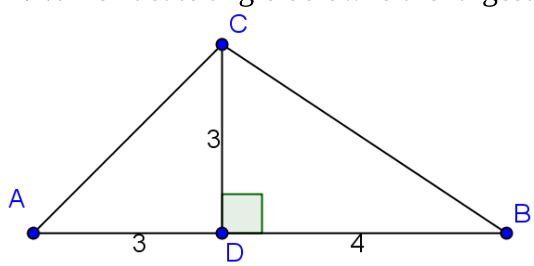
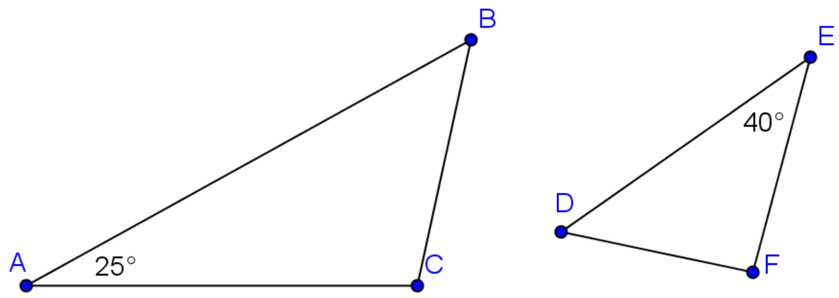


17. Which acute angle below is the largest? Explain.

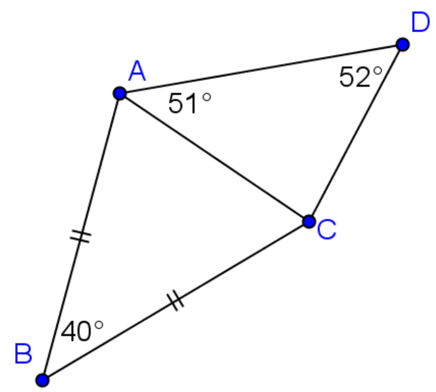


18. $\overline{AB}$ is the longest side of ΔABC and angle B measures 75°. What are possible measures for angle A?

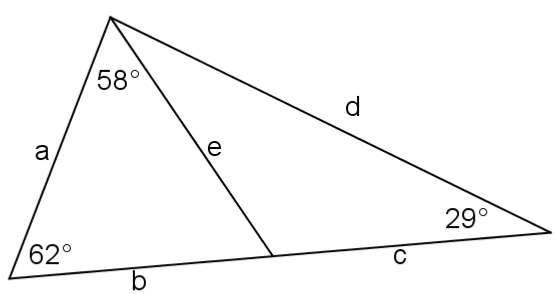
19. Despite appearances to the contrary, Aaron argues that $\overline{DF}$ must be longer than $\overline{BC}$ because it is opposite a larger angle. What do you think?



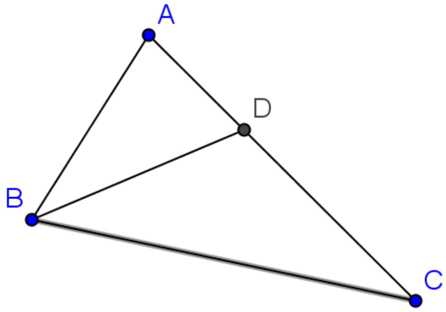
20. Why must $\overline{CD}$ be the shortest of the five sides below?



21. Rank the five sides (*a*, *b*, *c*, *d*, *e*) in length from long to short.

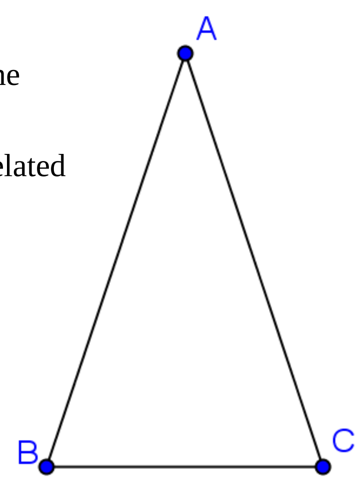


22. Given that $\overline{BD}$ bisects $\angle ABC$ below, explain why $\overline{BC} > \overline{CD}$.



23. In scalene triangle ABC, *a*>*b*>*c*. One angle measures 60°, which one?

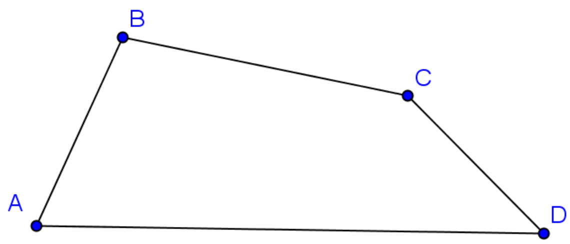
24. In triangle ABC, $\overline{AB} \cong \overline{AC}$.



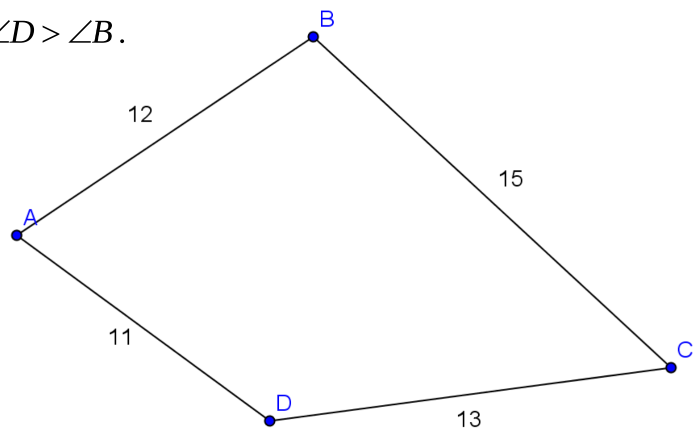
a. Point D is somewhere inside the triangle and $\overline{BD} < \overline{CD}$. Shade the part of the triangle where point D can be located. Hint: you may want to start by showing where $\overline{BD} = \overline{CD}$ (often in dealing with inequalities, it helps to start with the related equation!)

b. Explain why $\angle DCA > \angle DBA$.

25. It seems obvious that, in the diagram below, AB+BC+CD > AD. Use what you know about triangles to justify it.



26. In the quadrilateral below, explain why $\angle D > \angle B$.



27. You should know that the area of a triangle is one half of the product of the base and the altitude to that base. Two altitudes of a triangle are 5 and 8

a. Can the third altitude be 12? Explain.

b. Can the third altitude be 14? Explain.

c. What must the third altitude be between?

Answers

1a. $0 \le x < 3$ b. $3 < x < 5$ 2. $3<x<15$ 3a. 10 or 11 b. only 17

4. anywhere from 3 to 15; in a circle around B with a radius of 6 5. $20<x<30$ 6. $5<x<17$

7. 3x-18 > 12 and 3x-18>x so x>10 but $3x-18 < x+12$ so $x<15$ so $10<x<15$

8. $12 < 4x-18$ and $x < 3x-6$ and $3x-18 < x+12$ so x>7.5 and x>3 and $x<15$ → $7.5<x<15$

9. There are five: 5-5-9; 6-6-7; 7-7-5; 8-8-3; 9-9-1

10. the longer diagonal divides it into two triangles; in each the sum of the other two sides must exceed the third side (diagonal)… I think it works for EFGH as well… 11. x+y>4 and $y<x+4$ and y>x-4

12. ADC+BDC=180 and B + C + BDC=180 so by transitivity ADC = B+C

13. yes; all angles measures must be positive numbers

14a. 3x-10 > x+30 and $0<3x-10<180$ and $0<x+30<180$ so $20<x<190/3$

b. $0<120-3x<180$ and $0<x<180$ and 120-3x>x so $x<30$ and x>0 → $0<x<30$

15. 2=3 and 2>1 so 3>1 thus 3+4>1 so larger angle is opposite longer side

16a. b>a>c b. d>c>a 17. Angle DCB since it is greater than CBD and thus must be greater than 45°

18. $0<A<30°$ since C must be greater than 75°

19. the theorem states that, ***in a given triangle***, longer sides are opposite larger angles. It says nothing about sides in different triangles. Sorry Aaron!

20. AC is the shortest side of ABC. But in ACD, CD is shorter than AC 21. d>c>e>a>b

22. BDC is an external angle to triangle ABD so it is larger than ABD. But ABD is the same as DBC so BDC>DBC 23. Angle B; the middle one

24a. in the left 1/2 of the triangle b. since $DB<DC$, $BCD<DBC$; ACB=ABC so DCA>DBA

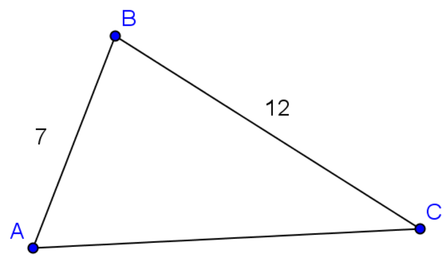
25. draw AC; AB+BC>AC and AC+CD>AD . add the two inequalities and subtract AC….

26. Draw BD. $\angle BDC > \angle DBC$ and $\angle BDA > \angle DBA$; add and we get $\angle D > \angle B$

27a. yes; since the area is the same whichever bases you use, then the sides would be A/5, A/8, and A/12. And $A/8 + A/12 < A/5$ so triangle inequality holds b. No, A/5 > A/12 + A/14 c. $40/13<x<40/3$

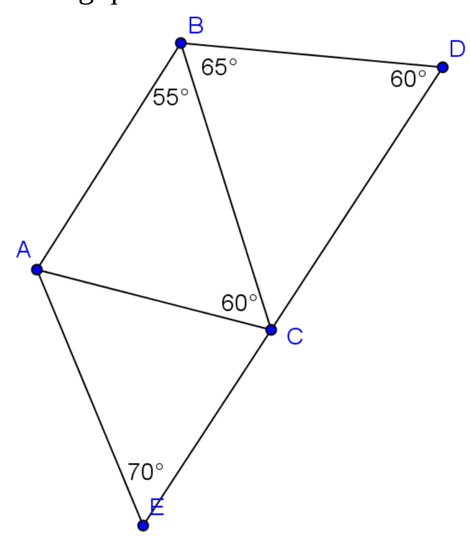
Unit 3 Handout #5: More Inequalities and the Hinge Theorem

1a. In the triangle below, $\angle C < \angle B < \angle A$. What range of perimeters are possible for ΔABC?



1b. Instead, if $\angle C < \angle A < \angle B$ then what range of perimeter are possible?

2. In the diagram below, points E, C, and D are collinear. Answer the following questions.



a. Is $\overline{AB} // \overline{DE}$? Explain.

b. Is $\Delta CAB \cong \Delta DBC$? Explain.

c. Which side of which triangle is the longest? Why?

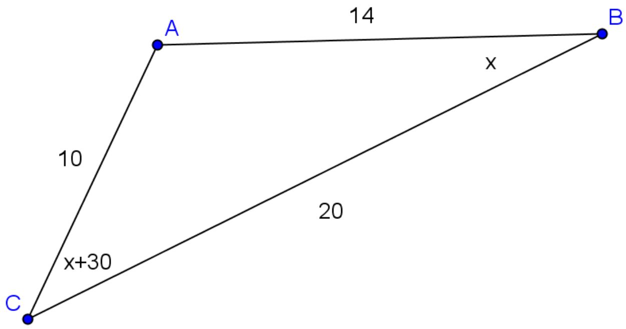
d. Of the seven triangle sides in the diagram, which are the three smallest? (in order). While you cannot (yet!) easily prove that AC<BD, you may take it as a given.

e. Can we know which of the seven sides is the 2nd longest? Explain.

f. Can we know which of the seven sides is the 3rd longest? Explain.

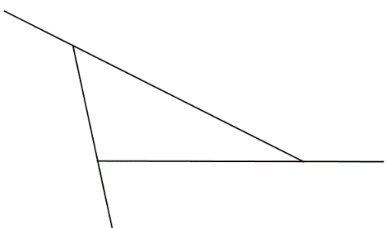
3. The sides of a triangle are 9, *x*, and 30-2*x*. What values of *x* are possible?

4. What values of *x* make this a valid triangle? First find the measure of angle A in terms of *x*.



5. The sides of a triangle are 2*x*+3, 3*x*+8, and 6*x*+7. Find the possible values of *x*.

6a. What is the sum of the external angles of a triangle (one at each vertex, as shown below)?

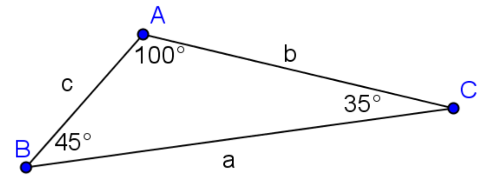


b. The external angles of a triangle are in the ratio 3:4:5. What is the triangle's smallest angle?

7. Can the external angles of a triangle be in the ratio 6:2:1? Explain.

8. If the ratios of the angles of a triangle is 4:5:6, then what is ratio of the triangle's external angles?

9. In ΔABC below, note that $m\angle A > m\angle B + m\angle C$. Which one of the following statements is true? Explain.



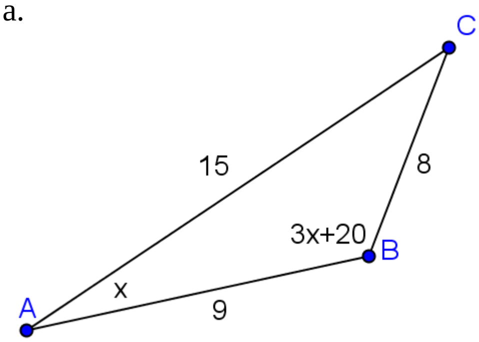
a. The length of side *a* **must** exceed the sum of the lengths of sides *b* and *c*.

b. The length of side *a* **may** exceed the sum of the lengths of sides *b* and *c*.

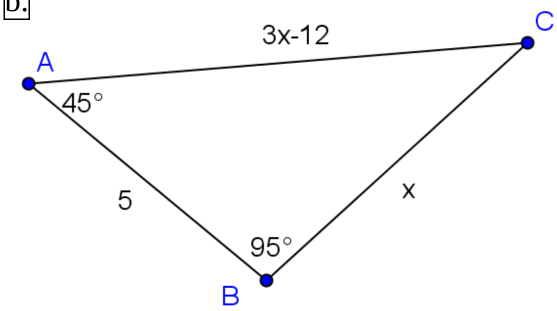
c. The length of side *a* **cannot** exceed the sum of the lengths of sides *b* and *c*.

10. What values of *x* are possible in the diagrams below?

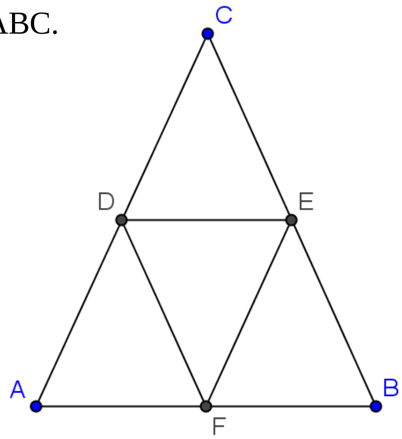
a.



b.



11. Prove that the triangle formed by joining the midpoints of the 3 sides of an isosceles triangle is also isosceles. Use the diagram below, where $\overline{AC} \cong \overline{BC}$ and D, E, and F are the midpoints of the sides of ΔABC.



12. True or false: in a right triangle, the altitude to the hypotenuse is the shortest of the three altitudes. Explain.

13. Given points A(1,1), B(9,3), and C(3,7) do the following. Be careful with fractions!

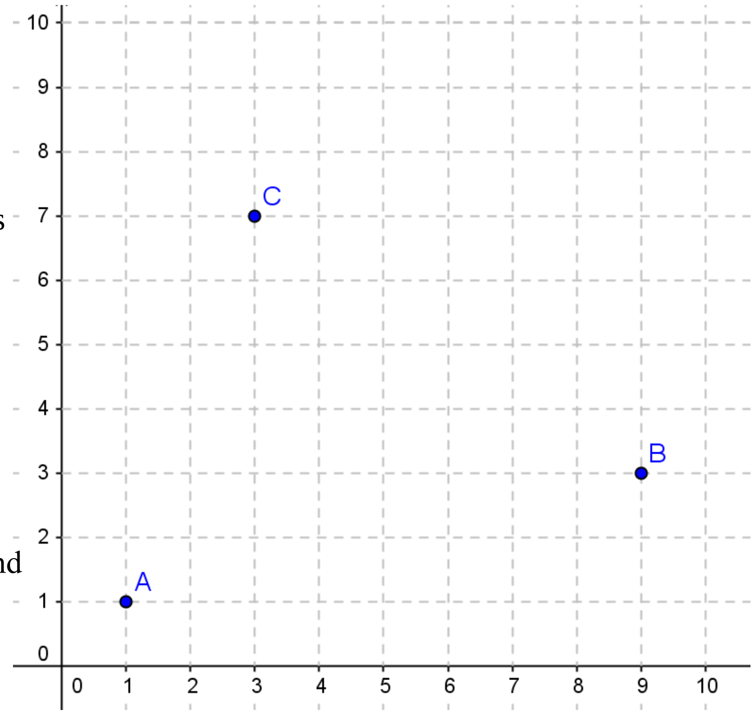
a. ΔABD is isosceles with $\overline{AD} \cong \overline{BD}$. If the *x*-coordinate of D is -2, then find the *y*-coordinate.

b. Given that $\angle ABC \cong \angle BCE$ and E's *x*-coordinate is 8, find its *y*-coordinate. Hint: alternate interior angles are congruent!

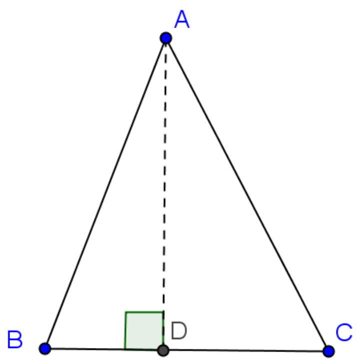
c. Where does altitude from A intersects side $\overline{BC}$?

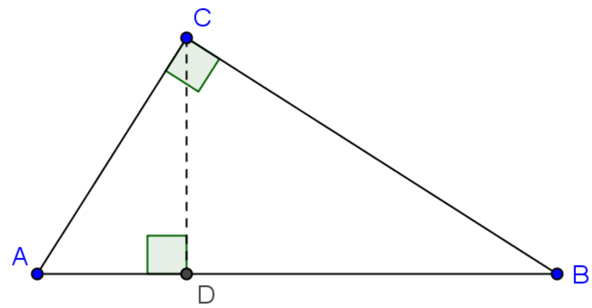
d. The three medians of the ΔABC intersect in one point. What are the coordinates of that point?

e. There is one circle containing points A, B, and C. One could say that the circle is "circumscribed" around the triangle (or that the triangle is "inscribed" in the circle). Find the center and radius of the circle.

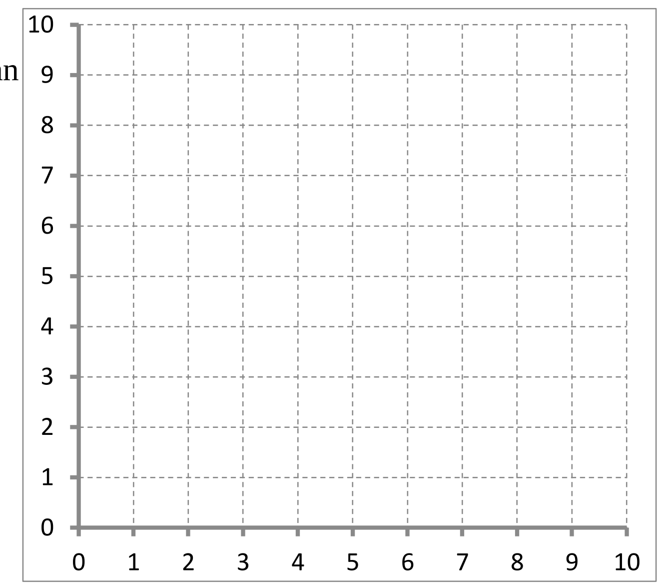


14. In some triangles, an altitude to a side is longer than the side (see the triangle at the left below). Prove that the altitude to the hypotenuse of a right triangle cannot be longer than the hypotenuse (see answers for a hint).

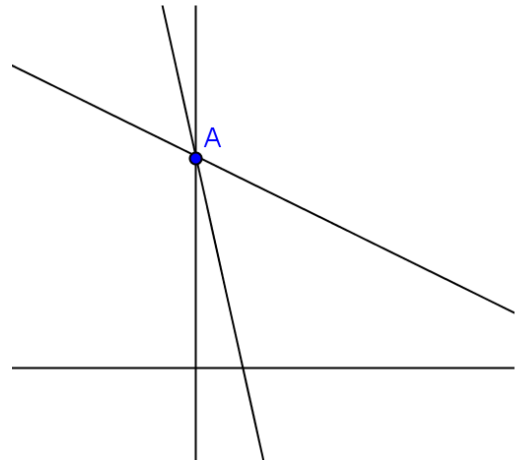




15. Given the points A(1,0) B(4,9), and C(10,3), determine whether angle A is greater than 60°, equal to 60°, or less than 60°. Justify your answer.



16. Three mathematicians are looking at two lines that intersect at point A.



Larry says, "I randomly put point B on one line and point C on the other line—both on the same side of A (meaning both with positive *x*'s or negative *x*'s). It turned out that $\overline{BC}$ was longer than both $\overline{AB}$ and $\overline{AC}$. I bet that will always happen, wherever I put B and C as long as they are on the same side of A."

Curly replies, "I'm not sure. I'd say that no matter where you put B and C (one on each line, both on the same side of A), $\overline{BC}$ will be longer than ***at least*** one of the distances $\overline{AB}$ and $\overline{AC}$."

Moe says, "You guys are reading way too much into one random set of points that Larry chose. I don't think you can conclude anything."

Who do you think is right, and why?

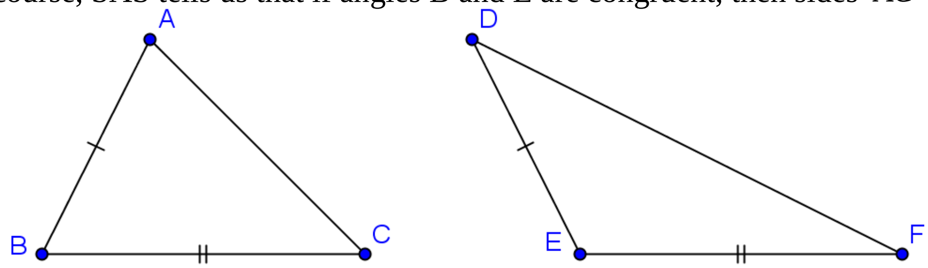
17. Follow up:

Larry says, "I drew a different pair of two lines (also intersecting at A). I picked a point on each line (on the same side of A) and found that $\overline{BC}$ was shorter than both $\overline{AC}$ and $\overline{AB}$. Can we conclude anything?"

Moe says, "If life were fair, $\overline{BC}$ should be longer than the other two next time…"

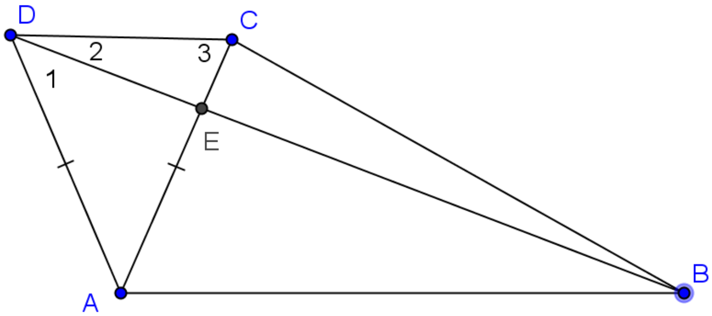
What do you think?

The Hinge Theorem: In triangles ABC and DEF, let $\overline{AB} \cong \overline{DE}$ *and* $\overline{BC} \cong \overline{EF}$. If angle B exceeds angle E, then $\overline{AC}$ will be longer than $\overline{DF}$. If angle E exceeds angle B then $\overline{DF}$ will be longer than $\overline{AC}$. [And, of course, SAS tells us that if angles B and E are congruent, then sides $\overline{AC}$ and $\overline{DF}$ will be congruent.]

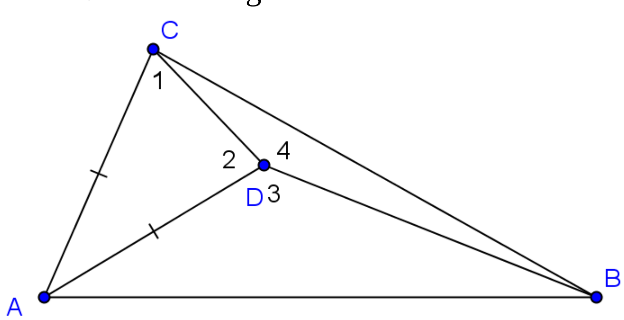


18. To show why the Hinge Theorem works, we need to look at two scenarios. In both cases AC=AD.

a. In the scenario below, triangles ABD and ABC both share two sides; the angle between them is larger for triangle ABD, so we want to show that $\overline{BD}$ is longer than $\overline{BC}$. How can you explain it?

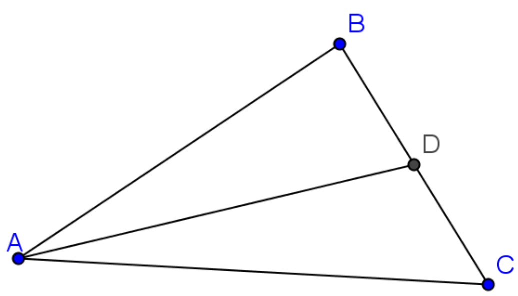


b. In the second scenario (below), the side of the triangle with the smaller angle falls within the other triangle. But still. Triangles BAD and BAC share two sides and the included angle is larger for triangle BAC. Use the diagram to show that $\overline{BC}$ must be longer than $\overline{BD}$.

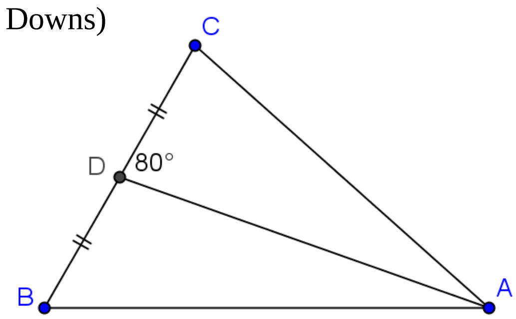


19. If a triangle is a right triangle, then the square of the longest side is equal to the sum of the squares of the other sides. Imagine a triangle where the square of the longest side is greater than the sum of the other two sides. Must it be obtuse? Explain.

20. D in the midpoint of $\overline{BC}$, and AC>AB, it appears that angle ADB is acute. Must it be? Explain.

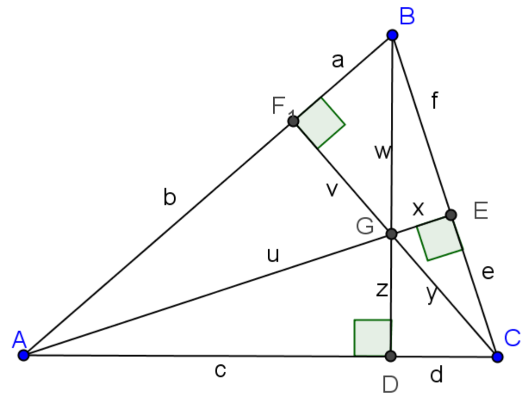


21. In ΔABC, $\overline{AD}$ is a median. Explain why $\angle C > \angle B$ (from Moise Downs)



22. Let D be a point on the interior of ΔABC. Show that $AD + BD + CD > \frac{1}{2}(AB + BC + AC)$

23. Show that the perimeter of a triangle be greater than the sum of the altitudes and less than twice the sum of the altitudes. Use the diagram below (this assume that the altitudes meet inside the triangle..)



Answers

1a. 26 < perim < 31 b. 31 < perim < 38 2a. yes b. no (angle's same but not the corresponding side) c. CD d. EC < AE < AC e. BC f. no, either BD or AB

3. 9<30-x and x<39-2x and 30-2x<x+9 so 7<x<13

4. A=150-2x so 150-2x > x+30 and x+30>x which means x<40 so 0<x<40 5. -0.4<x<4

6a. 360° b. 30° 7. No: need an external angle of 240°, which is impossible 8. 9:10:11 (cool!)

9. C 10a. 20<x<32 b. 6<x<8.5 (longest side can't be more than the sum of the other two!)

12. The altitude divides the original triangle into two smaller right triangles. In these triangles, the hypotenuses are the legs(altitudes) of the original triangle, so they must be larger than the other sides of these two smaller triangles, one of which is the altitude to the hypotenuse of the original triangle.

13a. 30 b. 33/4 c. (57/13, 79/13) d. (13/3, 11/3) e. (52/11, 34/11) sqrt(2210)/11

14. Hint: consider 3 cases: A=45°; A>45°; A<45°. If A=45° then AD=AC and BD=CD so the hypotenuse, AB is twice the altitude CD. If A>45° then B<45° and DCB>45° so DB>CD and thus AB>CD. If A<45° then ACD>45° so AD>CD and thus AB>CD. Another less formal approach: it seems difficult for any line segment on the inside of a right triangle can be larger than the hypotenuse.

15. smallest side is opposite A so under 60°

16. Curly. Given what Larry found, the lines intersecting at A must form an angle greater than 60°. So the side opposite, BC, does not have to be the longest side (imagine a 95-61-24 triangle) but can't be shortest.

17. the lines meet at an angle less than 60° so BC cannot be the longest side (sorry Moe!), but does not have to be the shortest.

18a. Angle (1+2) = angle 3 so $\angle 3 > \angle 2$ and angle DCB > angle 3. So DCB>angle 2 and thus DB>BC

b. angle 2 < 90 and angle 3 < 180 so angle 4>90 so BC is the longest side in BCD, longer than BD

19. Yes, by Hinge Theorem, if the longest side is longer then the opposite angle is larger, given that the other two sides don't change.

20. Yes, by hinge theorem, $\angle ADC > \angle ADB$ and they are supplementary.

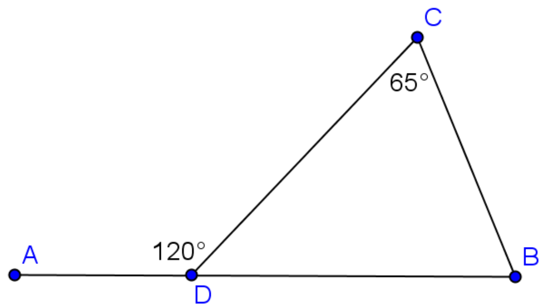
21. by Hinge Theorem, AB>AC, so in ΔABC, $\angle C > \angle B$

23. Look at the six right triangles formed where one vertex the intersection of the altitude and one is a vertex of ABC (like AGF and BGE). We get u+v>b, v+w>a, w+x>f, x+y>e, y+z>d and u+z > c. Adding these up we get 2(u+v+w+z+y+z) > a+b+c+d+e+f so the perim is less than twice the sum of the altitudes.

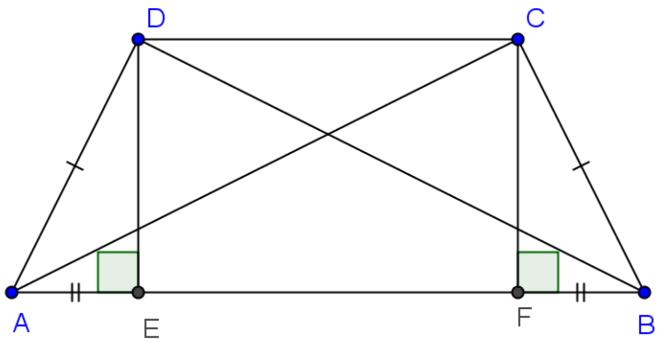
Now look at ABE, E is a right angle so a+b > u + x. In AEC, c + d > u + x for the same reason. In BDC and BDA we get e+f > w + z and a+b > w+z . In AFC and BFC we can c+d>v+y and f+e>v+y. Add up these six inequalities and we get 2(a+b+c+d+e+f)>2(u+x+w+z+v+y); divide by 2 and the perimeter is greater than the sum of the lengths of the altitudes. Phew!

Unit 3 Handout #6: Unit 3 Practice Problems

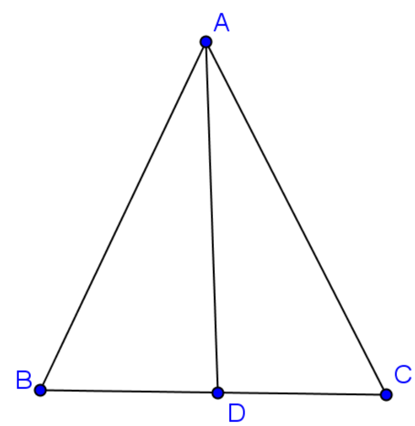
1. Rank the sides of triangle BCD from longest to shortest.



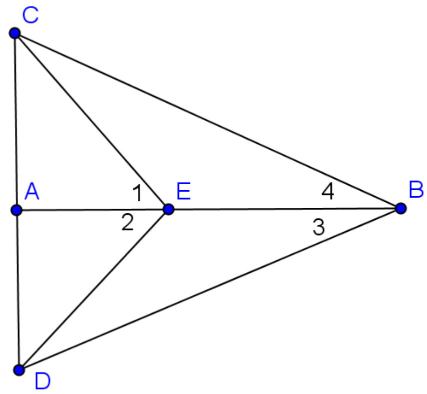
2. In the diagram below, $\overline{AB} \parallel \overline{CD}$. Prove that $\overline{AC} \cong \overline{BD}$.



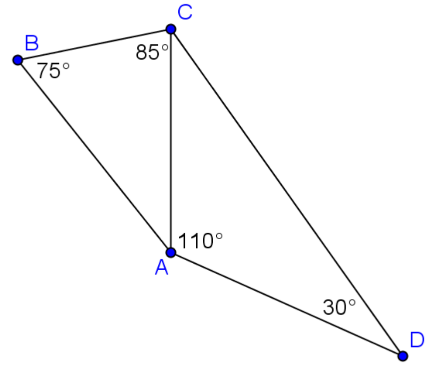
3. In triangle ABC, prove that if $\overline{AD}$ is both an angle bisector and an altitude then it must also be a median.



4. Given $\angle 1 \cong \angle 2,\ \angle 3 \cong \angle 4$ Prove: $\overline{AB} \perp \overline{CD}$

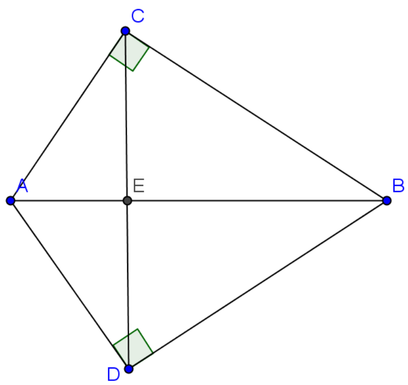


5. The diagram below is not to scale. Determine the longest and shortest of all sides or explain why it cannot be determined.



6. Given: $m\angle ACB = m\angle ADB = 90^\circ$ and $\overline{BC} \cong \overline{BD}$

Prove: $\overline{CD} \perp \overline{AB}$



7. Points A, B, and C are in a plane, not necessarily collinear. If $\overline{AB} = 10$ *and* $\overline{BC} = 15$, then what are the largest and smallest possible lengths of $\overline{AC}$?

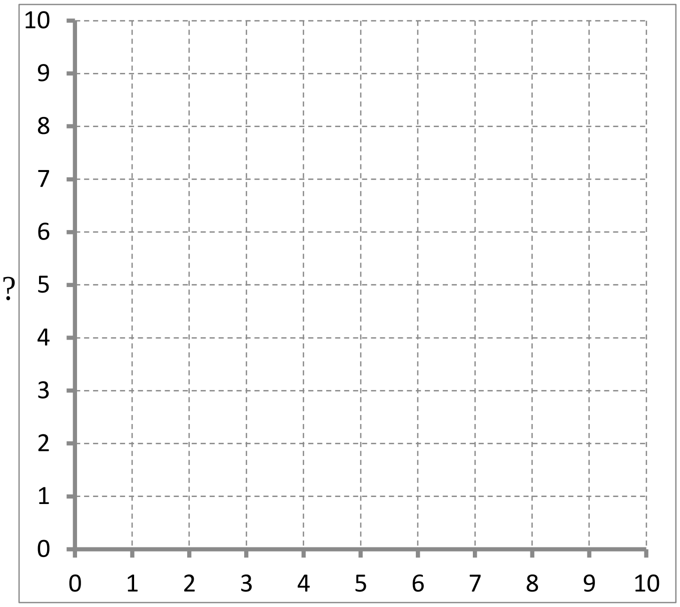
8. Given points A (2,3), B(8,1), and C (4,9), do the following:

a. What is the equation of the perpendicular bisector of $\overline{AB}$?

b. Point D is somewhere on the *x*-axis; its coordinates are (*d*,0). If D is closer to point A than to point B, what values of *d* are possible?

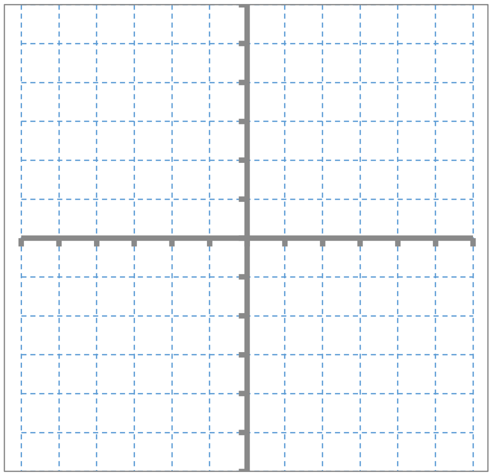
c. Is the point (-3,-1) on the perpendicular bisector of $\overline{BC}$? Justify your answer. Note: you do not need to find the equation of the perpendicular bisector to answer this question! Think a bit!

d. Which is the largest angle in triangle ABC? Explain.



9. Three sides of a triangle are 7, 2*x*+1, and 3*x*-5. What are possible values of *x*?

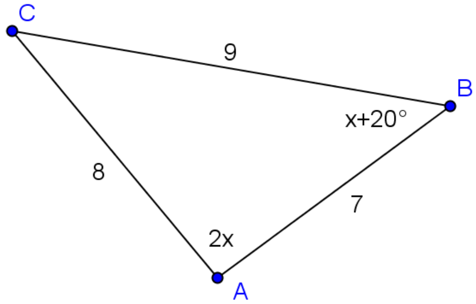
10. Find the center and radius of the circle containing the points (-3,-3), (1,5), and (-3,5).



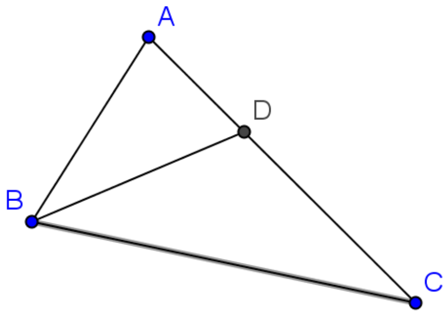
11. Explain why any point on the perpendicular bisector of segment $\overline{AB}$ is equidistant from points A and B.

12. Given any chord of a circle. Must the perpendicular bisector of the chord pass through the center of the circle? Explain your reasoning, but no proof is required.

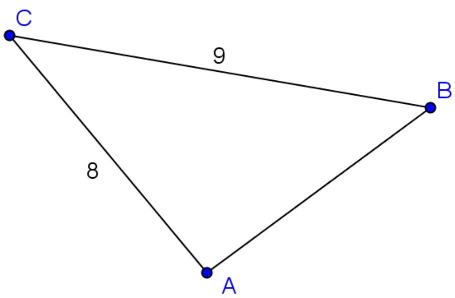
13. Find all possible values of *x* in the triangle below.



14. Given that $\overline{BD}$ bisects $\angle ABC$ below, explain why $\overline{AB} > \overline{AD}$.

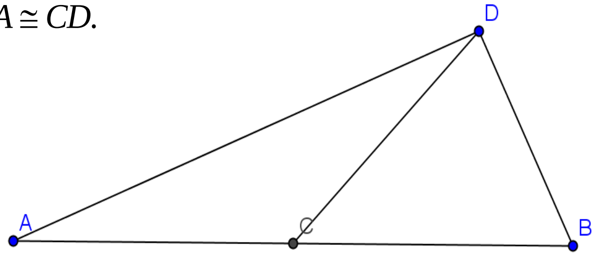


15. In ΔABC below, $\angle C < \angle B < \angle A$. Write an inequality describing the possible perimeters for ΔABC.



16. The ratios of angles of triangle is 2:3:4. What is the largest external angle?

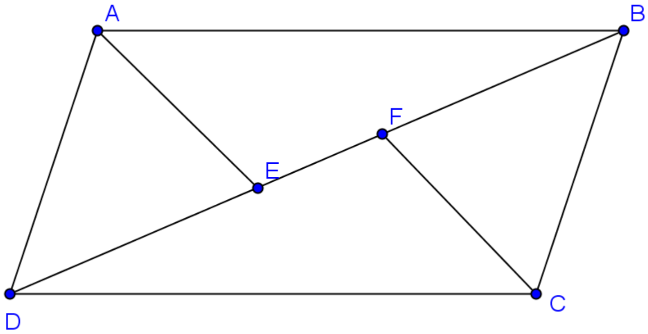
17. In the diagram below, $\overline{DC}$ is a median of ΔADB. Also, $\overline{CA} \cong \overline{CD}$.



a. If $\angle CAD$ measures 35° then find the measure of angle B.

b. Instead, if $\angle CAD$ measures $x°$ (where $x<90$) then find the measure of angle B in terms of x.

18. Given $\overline{AB} \parallel \overline{CD}$, $\overline{AD} \parallel \overline{CB}$ and $\overline{DE} \cong \overline{BF}$. Prove $\overline{AE} \cong \overline{CF}$



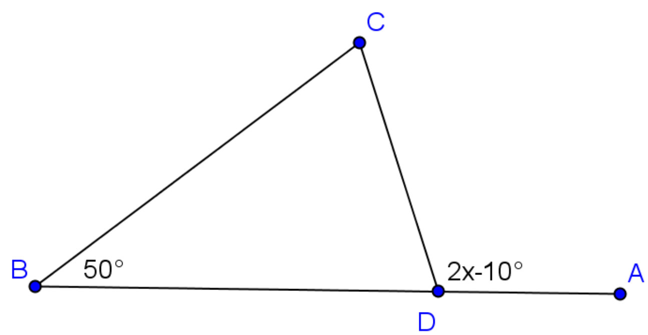
19. In the diagram below, do the following:

a. Find all possible values of x.

b. Now assume that $\overline{BD} \cong \overline{BC}$. What are the possible values of x now?

c. Instead, assume that $\overline{BC}$ is longer than $\overline{BD}$. Now what values of x are possible?

d. Finally, assume that $\overline{BC} > \overline{BD} > \overline{CD}$. Now what x's are possible?



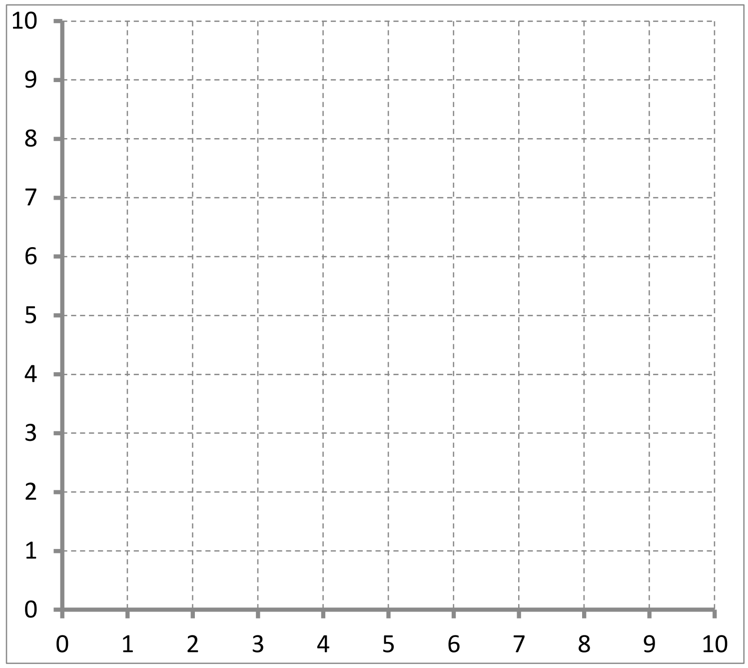
20. Given the points A(2,6), B(4,8), and C(8,2) do the following:

a. What are the coordinates where the median from vertex B intersects side $\overline{AC}$?

b. Triangle ABD is isosceles where $\overline{AB} \cong \overline{BD}$. List two possible sets of coordinates for point D and then describe all possible sets.

c. What are the coordinates where the altitude from vertex B intersects side $\overline{AC}$?

d. What are the coordinates where the median from vertex B intersects the median from vertex A?



Answers

1. BD>BC>CD

2.

2.	1. $\overline{AE} \cong \overline{BF}$	given
	2. $\overline{AD} \cong \overline{BC}$	given
	3. $\angle AED = \angle BFC = \angle FED = \angle EFC = 90^\circ$	all right angles are congruent
	4. $\Delta AED \cong \Delta BFC$	1, 2,3, HL
	5. $\overline{ED} \cong \overline{FC}$	4, CPCTC
	6. $\overline{AF} \cong \overline{BE}$	1, addition property (add EF to both)
	7. $\Delta BED \cong \Delta AFC$	3, 5, 6, SAS
	8. $\overline{BD} \cong \overline{AC}$	7, CPCTC

4.	1. $\angle 1 \cong \angle 2,\ \angle 3 \cong \angle 4$	given
	2. $\angle CEB \cong \angle DEB$	1, supplements of cong angles are congr
	3. $\overline{EB} \cong \overline{EB}$	reflexive
	4. $\Delta CEB \cong \Delta DEB$	1, 2, 3, ASA
	5. $\overline{CB} \cong \overline{DB}$	4, CPCTC
	6. $\overline{AB} \cong \overline{AB}$	reflexive
	7. $\Delta CAB \cong \Delta DAB$	1, 5, 6, SAS
	8. $\angle CAB \cong \angle DAB$	7, CPCTC
	9. $m\angle CAB = 90^\circ$	8, congruent supplements are right angles
	10. $\overline{AB} \perp \overline{CD}$	9, def of perpendicular

5. BC must be the shortest of all sides since AC is the shortest side of ACD and the middle side of ABC. While it looks like CD is the longest of all sides, it is possible that AB is longer.

6.	1. $\angle ABC = \angle ADB = 90^\circ$	given
	2. $\overline{BC} \cong \overline{BD}$	given
	3. $\overline{AB} \cong \overline{AB}$	reflexive
	4. $\Delta CAB \cong \Delta DAB$	1, 2, 3, HL
	5. $\angle CBE \cong \angle DBE$	4, CPCTC
	6. $\overline{EB} \cong \overline{EB}$	reflexive
	7. $\Delta CEB \cong \Delta DEB$	2, 5, 6, SAS
	8. $\angle CEB \cong \angle DEB$	7, CPCTC
	9. $m\angle CAB = 90^\circ$	8, congruent supplements are right angles
	10. $\overline{AB} \perp \overline{CD}$	9, def of perpendicular

7. $5 \le \overline{AC} \le 25$ (it can be equal to 5 or 25 because it may not be a triangle, as the points could be collinear)

8a. $y = 3x - 13$ b. perp bisector hits x-axis at x=13/3; so anywhere below that .. d < 13/3

c. easiest way is to see if it is equidistant from B & C… distance from B is $\sqrt{125}$; from C is $\sqrt{149}$ so no.

d. angle A since BC is the longest side

9. 7 < 5x-4 and 2x+1 < 3x+2 and 3x-5 < 2x + 8 so x>2.2 and x>-1 and x<13 so 2.2 < x< 13

10. center is where perp bisectors meet: they are x=-1 and y=1 so (-1,1) and radius is $\sqrt{20} = 2\sqrt{5}$

11. use SAS

12. yes since the center of the circle is equidistant from all points on the circle, it must be equidistant from the ends of the chord. And the perp bisector is all points equidistant from the endpoints, so it must include the center.

13. angle C is 160-3x so.. 2x > x+20 and x+20 > 160-3x so x>35 and all angles must be positive so 160-3x>0 and x < 53.333 so 35 < x < 53.3333

14. angle ADB is larger than angle DBC because it is an exterior angle. Since DBC = DBA (bisect) we know that ADB > DBA so $\overline{AB} > \overline{AD}$.

15. AB is shortest side less than 8 and more than 1 (or not a valid triangle) so 18<perim<25

16. 140° 17. 55° b. 90°-x

18.

1. $\overline{AB} \parallel \overline{CD},\ \overline{AD} \parallel \overline{CB}$	given
2. $\angle ABD \cong \angle CDB$	1, alternate interior angles
3. $\angle DBC \cong \angle BDA$	1, alternate interior angles
4. $\overline{DB} \cong \overline{DB}$	reflexive
5. $\Delta BDC \cong \Delta DBA$	2, 3, 4, ASA
6. $\overline{DC} \cong \overline{AB}$	5, CPCTC
7. $\overline{DE} \cong \overline{BF}$	given
8. $\overline{DF} \cong \overline{BE}$	7, additive prop (add EF to both)
9. $\Delta CDF \cong \Delta ABE$	2, 6, 8, SAS
10. $\overline{AE} \cong \overline{CF}$	9, CPCTC

19a. 2x-10 > 50 so x>30 and also 2x-10<180 so x<95 … 30<x<95

19b. angle C and CDB are equal so 65° and 2x-10=115 so x=62.5° is the only possibility

19c. tough… BDC=190-2x so C=2x-60 so 190-2x > 2x-60 x<62.5 so 30<x<62.5

19d. 190-2x > 2x-60 and 2x-60 > 50 so 55 < x < 62.5

20a. (5,4) b. (6,6) and (2,10); on a circle of radius $2\sqrt{2}$ around B except at (2,6) or (6,10)

c. (32/13, 74/13) d. (14/3, 16/3)

Unit 4 Handout #1: Introduction to Polygons

Polygons are two-dimensional shapes consisting of vertices connected by edges. The shape must be closed, in that each vertex has two edges; edges must also be line segments and may not cross each other. If adjacent sides are collinear then they are considered a single side. In a ***convex polygon***, each interior angle measures less than 180°.

An ***equilateral*** polygon has sides that are all congruent; an ***equiangular*** polygon has angles that are all congruent. If a polygon is both equilateral and equiangular, then it is called a ***regular polygon***. Examples include an equilateral triangle and a square.

Some formulas for polygons:

-The sum of the angles in a polygon with *n* sides is $180\,(n-2)$. It can be divided into (*n*-2) triangles, each of whose angles sums to 180°.

-A convex polygon with *n* sides (where *n* is at least 3) has $\frac{n(n-3)}{2}$ diagonals. From each vertex, there are (*n*-3) diagonals since there are no diagonals to the vertex itself or its neighbors. Since there are *n* vertices, each with (*n*-3) diagonals, multiply *n* by *n*-3 to (apparently) get the number of diagonals. But this counts each diagonal twice, because it counts it one time at each endpoint, so divide by two. Hence the formula above!

<u>Quadrilaterals have four sides. Some important types of quadrilaterals are:</u>

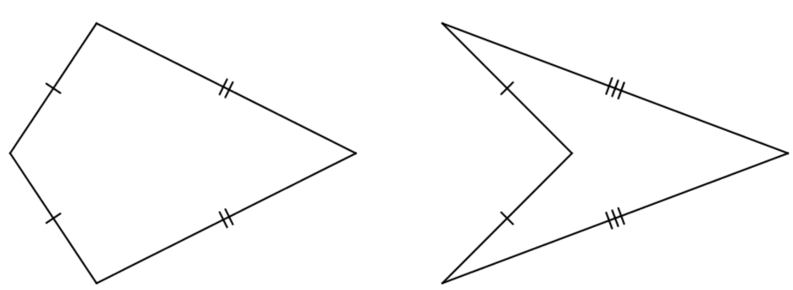
-***Parallelogram***: has two pairs of parallel sides, opposite from each other.

-***Rectangle***: a parallelogram with a right angle.

-***Rhombus***: a quadrilateral where all four sides are congruent. Note: this definition does not say "*parallelogram"* (although it can be shown that a rhombus must be a parallelogram)!

-***Trapezoid***: a quadrilateral with exactly one pair of parallel sides (called bases). If the other two sides happen to be congruent, then it is an ***isosceles trapezoid***.

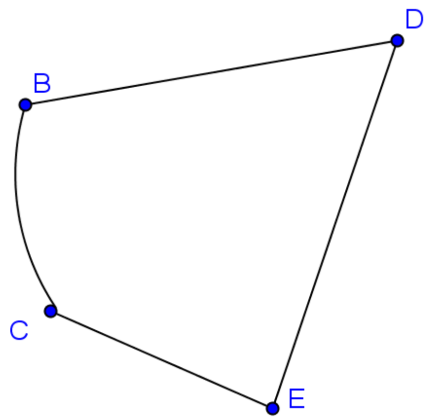
-***Kite***: a quadrilateral with exactly two pairs of congruent adjacent sides and no parallel sides. By this definition, a rhombus is not a kite. Something that meets this definition but is not convex is often called an "arrowhead". (See below)



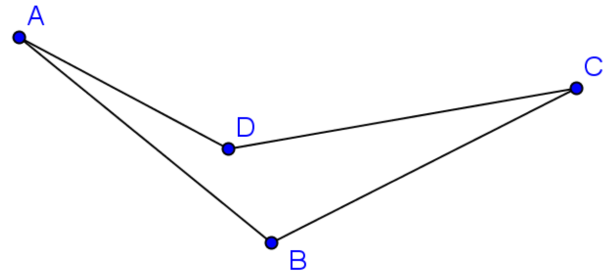
Kite Arrowhead

1. Which of the following are polygons? For those that are not, indicate why. For those that are, name them.

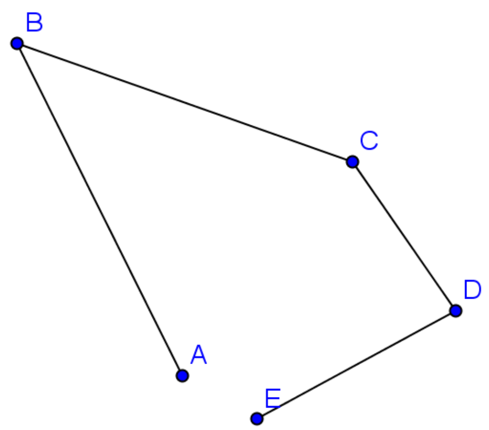
a.



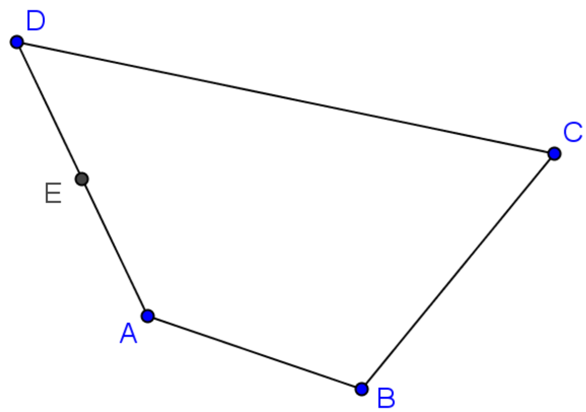
b.



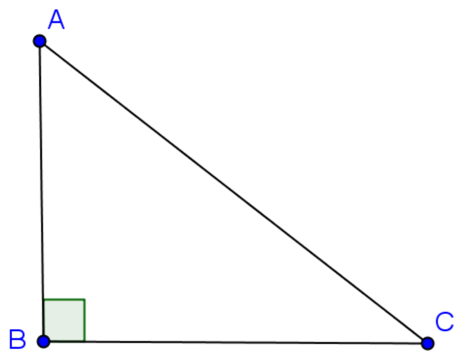
c.



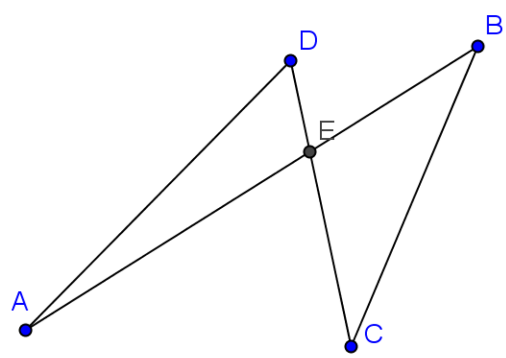
d.



e.

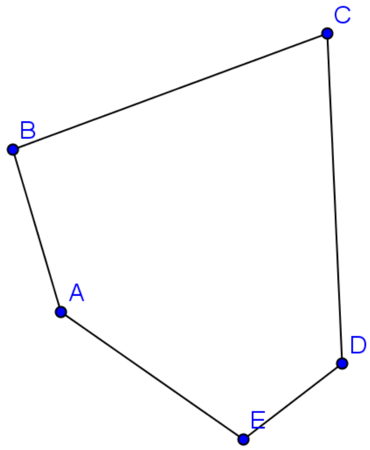


f.

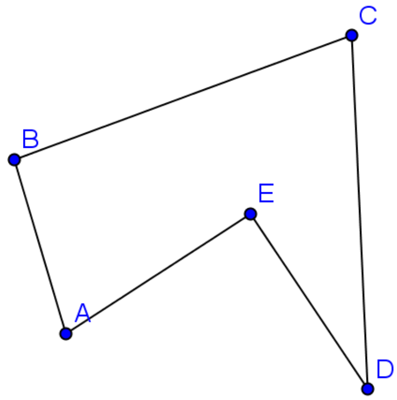


2. Find the sum of the angles in each of the following polygons.

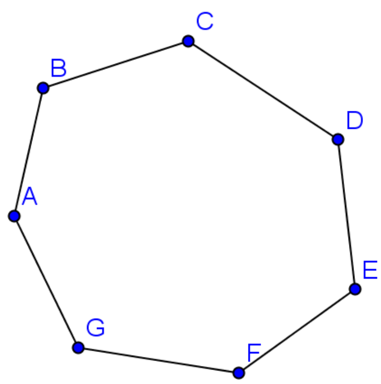
a.



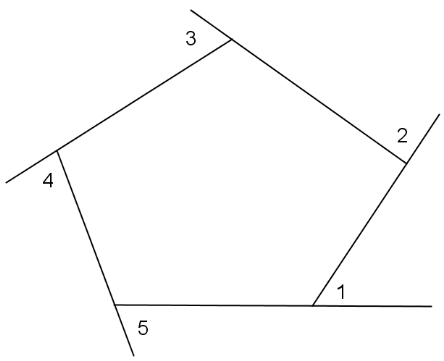
b.



c.



3. Find the sum of the external angles of the pentagon below. Do not assume all angles are the same. Are you surprised by the answer?



4a. For a convex polygon with *n* sides, find the sum of the external angles.

b. If the convex *n*-gon is equiangular, then what is the measure of each external angle?

5. The ratio of the angles of a quadrilateral is 3:4:5:6. What is the largest angle?

6. Ben claims to have a pentagon whose angles are in the ratio 1:2:3:4:5. Adrian says no such pentagon exists. Who is right and why?

7. A hexagon is equiangular, meaning that all of its angles are equal.

a. What is the measure of each angle?

b. Must it also be equilateral (in other words, must all sides be the same length)? Explain.

c. Find the measure of each interior angle of an equiangular *n*-gon.

8. How many diagonals does a convex polygon with 5 sides have? How about 6 sides? How about 100 sides? Or simply with *n* sides (assume *n* is an integer at least 3).

9. To prove triangles congruent, we can use SSS, SAS, ASA, or AAS (in addition to HL). Which of the following can be used to prove that two quadrilaterals are congruent? Hint: not all quadrilaterals are convex! Come up with counterexamples when you can.

a. SSSS

b. SASAS

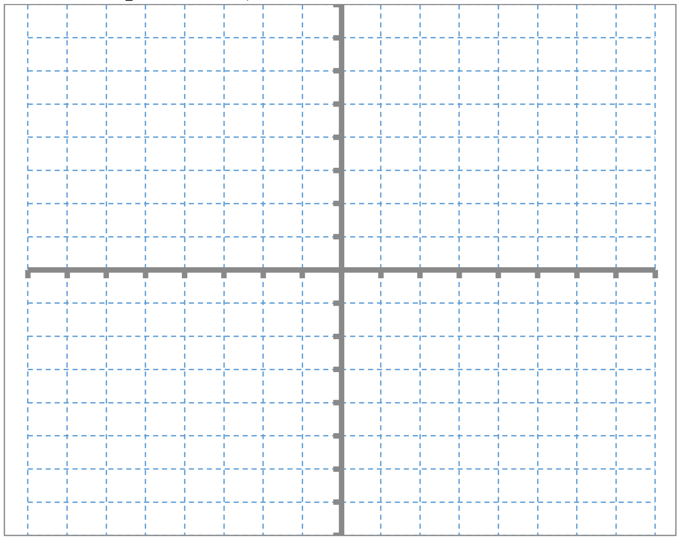
c. SASA

d. ASASA

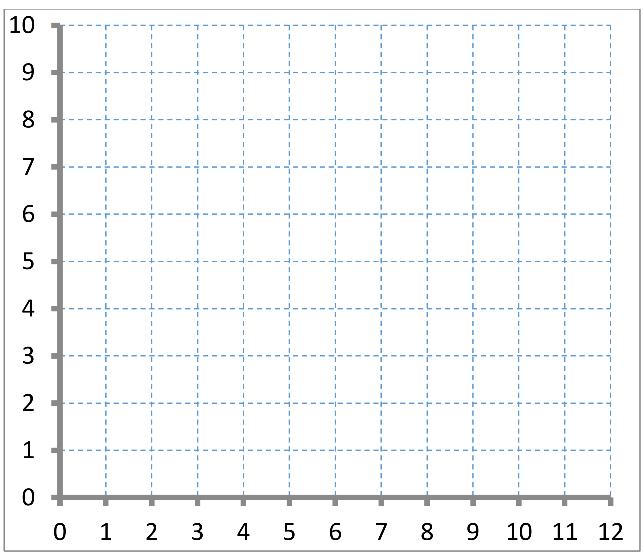
e. AASA

f. SASAA

10a. Three vertices of parallelogram ABCD are A(-4,-4), B(3,1), and C(-1,2). Find the coordinates of D.
b. Three vertices of a parallelogram are A(-4,-4), B(3,1), and C(-1,2). Find the coordinates of D. (is it the same problem?)

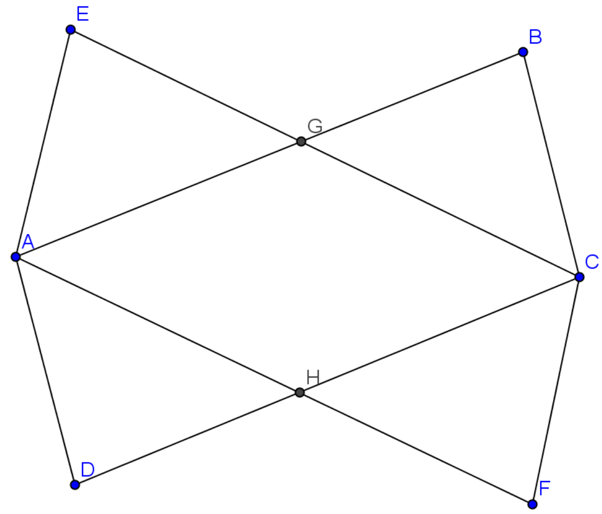


11. Given that two vertices of a square are (3,2) and (7,2), find all possible coordinates for the other two vertices. Note: these two do not have to be adjacent!



12. In the diagram below ABCD and AECF are parallelograms.

a. Must AGCH also be a parallelogram? Explain.



b. Find all angles congruent to ∠BGC.

c. Give the vertices of a trapezoid.

13. Determine whether each statement below is sometimes correct, always correct, or never correct:

a. A square is a rhombus

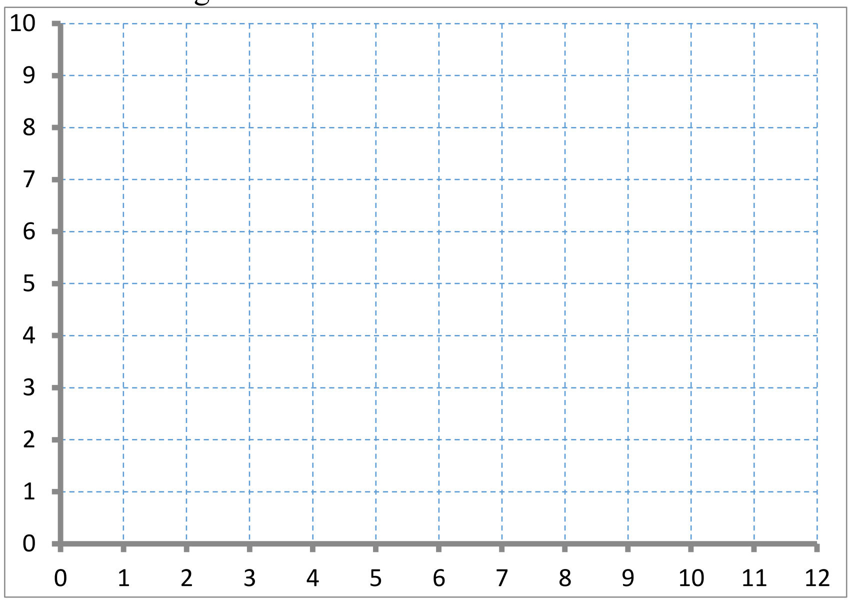
b. A kite is a parallelogram

c. A polygon has as many vertices as it has sides.

d. A polygon has more diagonals than sides.

e. A parallelogram is a rectangle

14. Given points A(6,1), B(2,0), C(1,5)…. do the following:



a. If ABCD is a parallelogram, then where is point D?

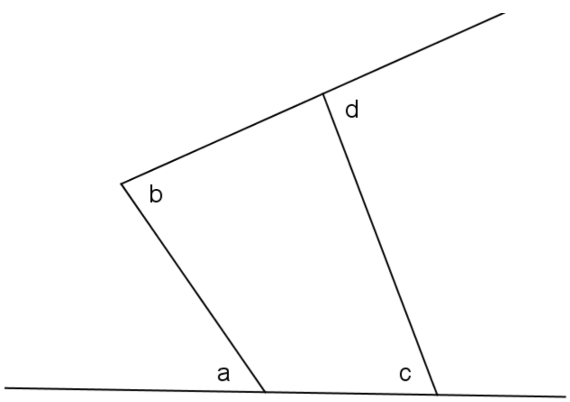
b. Is ABCD a rectangle? Justify your answer.

c. Given points E(8,3) and F(12,10). Is BCFE a trapezoid? Explain.

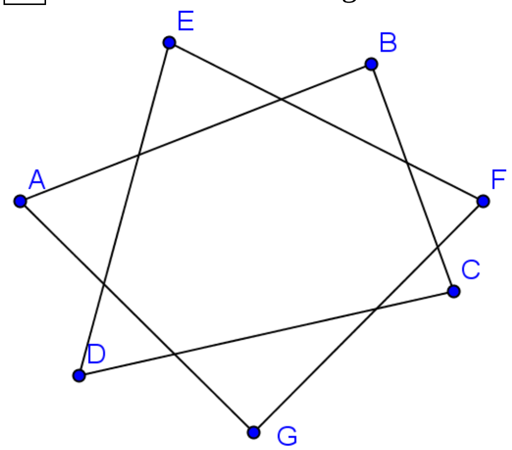
d. Point G has an *x*-coordinate of 12. If BCGE is a trapezoid with bases $\overline{BE}$ and $\overline{CG}$ then find G's *y*-coordinate.

e. Is BCGE an isosceles trapezoid? Justify your answer.

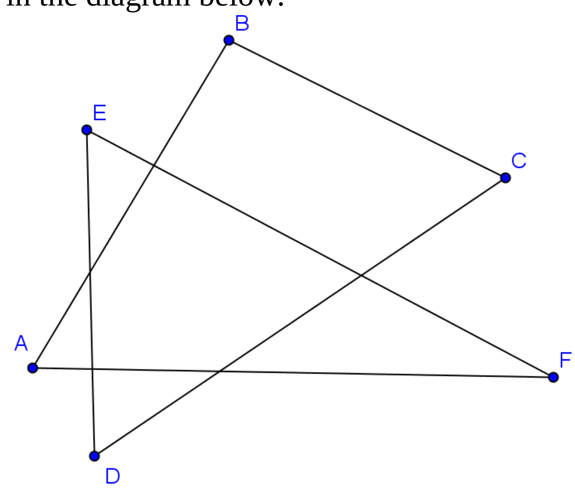
15. Explain why $a+d = b+c$



16. Find the sum of the angles at vertices A, B, C, D, E, F, and G.



17. Find the sum of the measures of angles A, B,C, D, E and F in the diagram below.

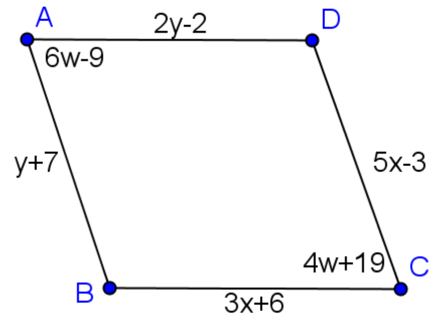


Answers

1. b, d, and e are polygons (though d is a quadrilateral and not a pentagon) 2a. 540° b. 540° c. 900°
3. 360°; no-if you were walking around the perimeter and turned at those angles, you'd make a circle
4a. 360° b. 360/n 5. 120° 6. Adrian: it isn't a pentagon b/c one angle=180° 7a.120° b. no c. 180-360/n
8. 5; 9; 4850; n(n-3)/2 → from each vertex n-3 diags; times n verts; divide by 2 since counting both ends
9. b, d, f 10a.(-8,-3) b.(-8,-3) or (6,7) or (0,-5) 11. (3,6) and (7,6) or (3,-2) and (7,-2) or (5,0) and (5,4)
12a. yes; 2 pairs of // opposite sides b. EGA; GCH; GAH; CHF; AHD c. AGCD is one…
13a. always b. never c. always d. sometimes e. sometimes
14a. (5,6) b. no; slopes of AB & BC are not negative reciprocals c. no; no two sides are parallel
d. 10.5 e. no. BC is not congruent to GE
15. because (180-a) + (180-d) + b + c =360…. Or (fancy!) draw a diagonal splitting angles b and c. d and a are exterior angles to the two triangles created so they are each equal to the sum of parts of b & c
16. 540° 17. 360°

Unit 4 Handout #2: Proving and Applying Quadrilateral Properties

Example #1: Given parallelogram ABCD below, find the values of *x*, *y*, and *z* and then determine if the diagonals $\overline{AC}$ *and* $\overline{BD}$ are perpendicular to each other.



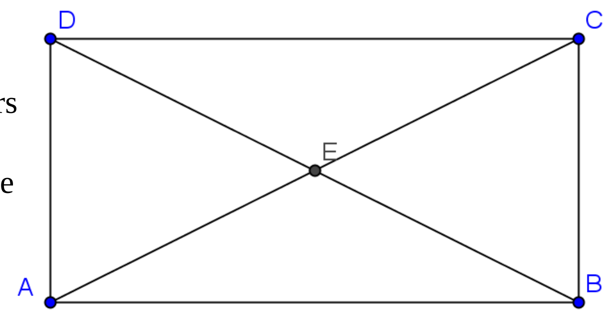
Solution

The opposite sides of parallelograms are congruent. So $2y-2=3x+6$ *and* $y+7=5x-3$. Solving this system yields *x*=4 and *y*=10.

Opposite angles of parallelograms are congruent, because they are both supplements of the other vertex angles. Therefore $6w-9=4w+19$ *and* $2w=28$ *and* $w=14$.

For the diagonals of the parallelogram to be perpendicular to each other, it must be a rhombus. Plugging *x* and *y* in to find the lengths of the sides, we see that two are 18 units long and two are 17. Therefore it is not a rhombus and thus its diagonals are not perpendicular to each other.

Example #2: Given rectangle ABCD Prove: ΔCEB is isosceles. Start with definitions; no using properties of quadrilaterals.



Solution

The definition of a rectangle is a quadrilateral with two pairs of parallel opposite sides (a parallelogram) and one right angle. If we can prove that $\Delta ABC \cong \Delta DCB$ then we can use CPCTC to show that $\angle ACB \cong \angle DBC$ and we are just about done. *Note: if we could use properties then we could take line 6 as a given and proceed from there.*

Statement	Justification
1. $\overline{CD} // \overline{BA}$; $\overline{AD} // \overline{BC}$	given; definition of parallelogram
2. $\angle BAC \cong \angle DCA$	1; alternate interior angles
3. $\angle BCA \cong \angle DAC$	1; alternate interior angles
4. $\overline{CA} \cong \overline{CA}$	reflexive property
5. $\Delta ABC \cong \Delta CDA$	2, 3, 4, ASA
6. $\overline{AB} \cong \overline{CD}$	5; CPCTC
7. $\overline{CB} \cong \overline{CB}$	reflexive property
8. $\angle ABC = \angle DCB = 90$	definition of rectangle
9. $\Delta ABC \cong \Delta DCB$	6, 7, 8, SAS
10. $\angle ACB \cong \angle DBC$	9; CPCTC
11. $\overline{CE} \cong \overline{EB}$	10; sides in a Δ opp congr angles are congr
12. ΔCEB is isosceles	11; definition of isosceles

1. List the definitions of each. This does NOT mean all of the properties, merely the definition!

a. Parallelogram

b. Rectangle

c. Kite

d. Rhombus

e. Trapezoid

f. Isosceles trapezoid

2. Remember the notation $\cup$ *and* $\cap$? (union and intersection). If A is the set of rectangles and B is the set of rhombi, then what is $A \cap B$?

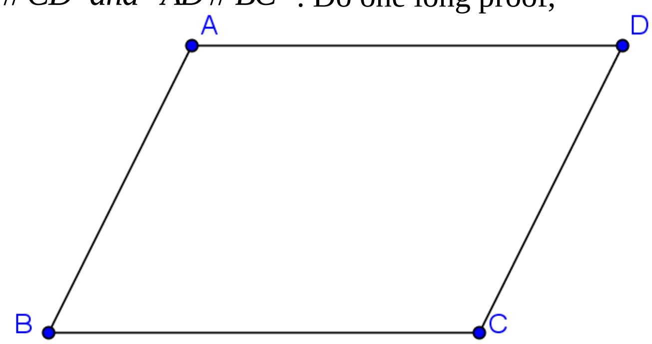
3. Create a Venn diagram that includes the following sets: polygons, quadrilaterals, squares, rectangles, rhombi, parallelograms, kites, trapezoids, isosceles trapezoids.

4. Given: ABCD is a parallelogram, so by definition $\overline{AB} \parallel \overline{CD}$ *and* $\overline{AD} \parallel \overline{BC}$. Do one long proof, showing all of the following:

a. $\overline{AB} \cong \overline{CD}$ *and* $\overline{AD} \cong \overline{BC}$

b. $\angle A \cong \angle C$ *and* $\angle B \cong \angle D$.

c. The diagonals bisect each other.



5. Prove that the diagonals of a rectangle are congruent. Provide your own diagram. You may use properties of quadrilaterals that you proved in the previous question.

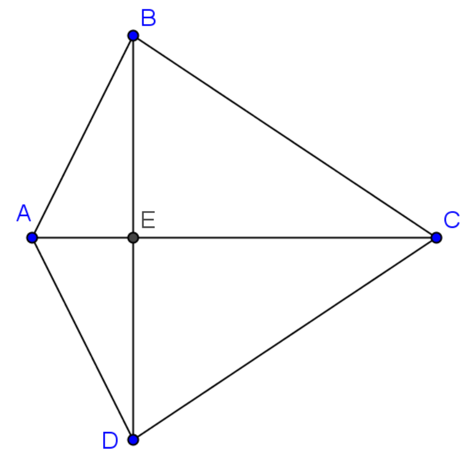
6. Given kite ABCD, prove the following (in one long proof):

a. $\angle ABC \cong \angle ADC$

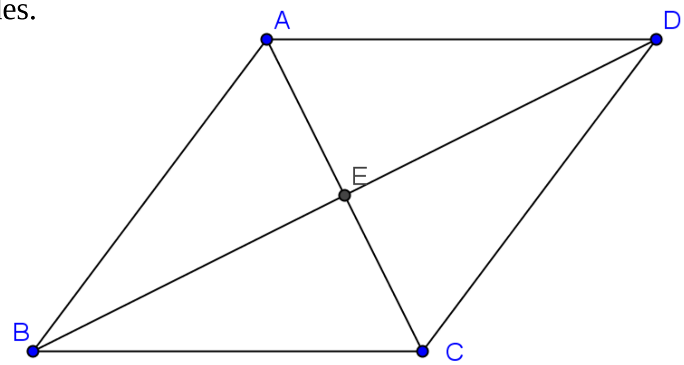
b. The diagonals are perpendicular.

c. E is the midpoint of $\overline{BD}$

d. $\overline{AC}$ bisects both $\angle BAD$ *and* $\angle BCD$.



7. Given rhombus ABCD below, prove the following (in one long proof). Start with the definition of rhombus as a quadrilateral with four congruent sides.



a. $\overline{AD} \parallel \overline{BC}$ *and* $\overline{AB} \parallel \overline{CD}$

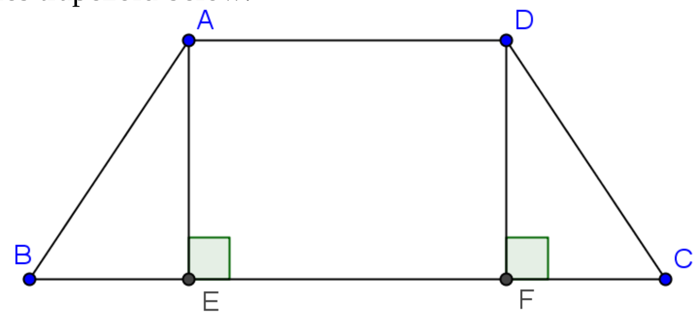
b. Diagonal $\overline{BD}$ bisects $\angle ABC$ *and* $\angle ADC$.

c. $\overline{AC} \perp \overline{BD}$.

d. $\Delta EAD \cong \Delta ECD \cong \Delta EAB \cong \Delta ECB$.

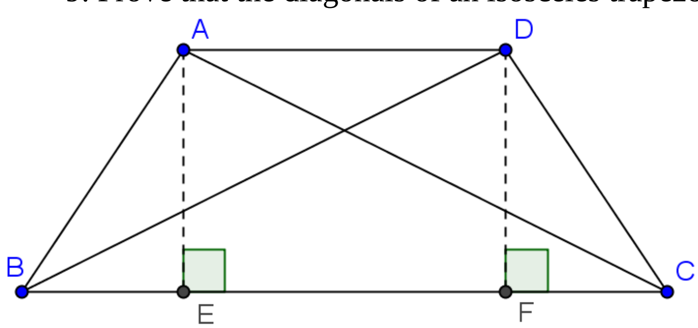
8 Altitudes from A and D have been drawn in the isosceles trapezoid below.

a. Prove that AEFD must be a rectangle.

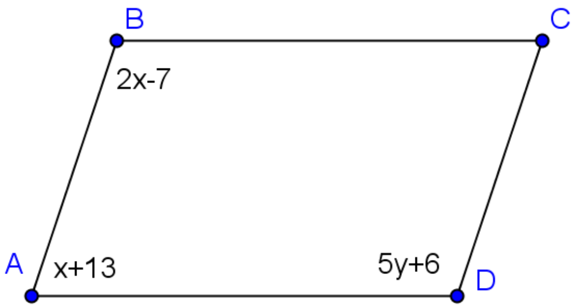


b. Prove that the base angles (B and C) of the isosceles trapezoid are congruent.

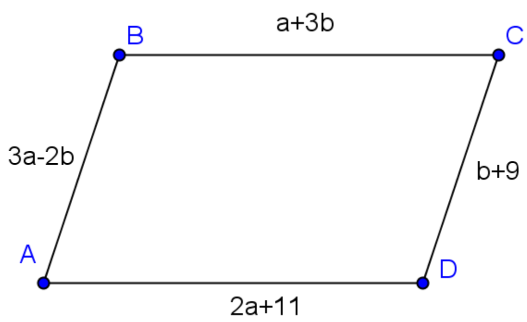
9. Prove that the diagonals of an isosceles trapezoid are congruent. Hint: use the results of the prior proof!



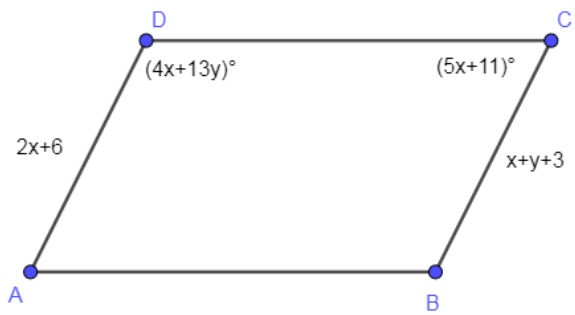
10. There is only one set of values for *x* and *y* that make the figure below a parallelogram. What values must *x* and *y* have?



11. What are the values of *a* and *b* such that the quadrilateral below is a parallelogram?



12. ABCD below is a parallelogram. Find the measure of angle A. Don't be afraid of fractions!



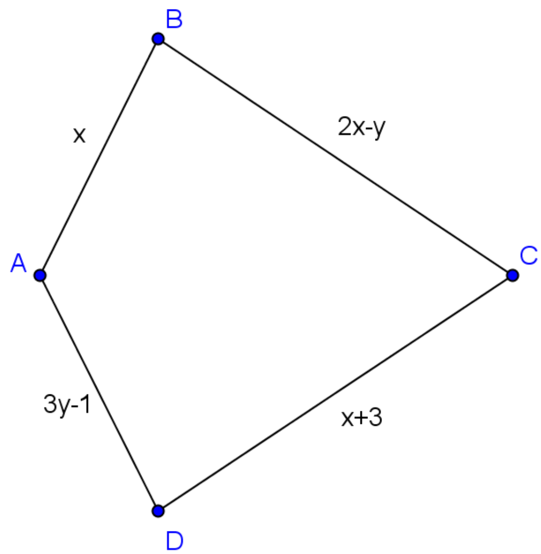
13. In the kite below, $\overline{AB} \cong \overline{AD}$.

a. Find the values of x and y.

b. Must it be the case that $\Delta ADC \cong \Delta ABC$? Explain briefly (no formal proof necessary).

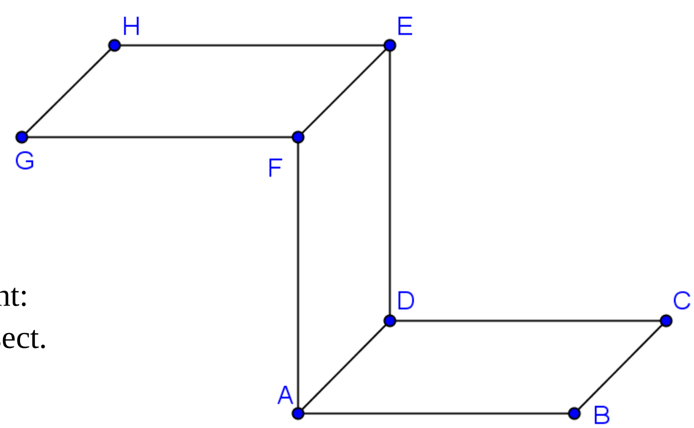
c. What is the longest possible measure of diagonal $\overline{AC}$?

d. Must angle A's measure exceed the measure of angle C? Explain. Hint: draw $\overline{AC}$.



14. In the (two-dimensional) diagram, HEFG and DCBA are congruent parallelograms and $\overline{EF} \parallel \overline{AD}$.

a. Must ADEF also be a parallelogram? Explain.

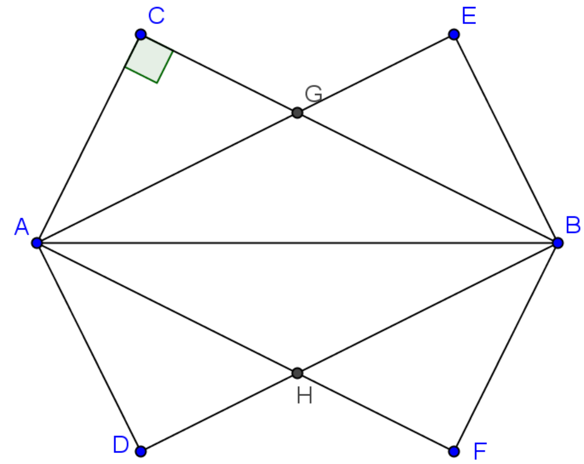


b. Must ABFG also be a parallelogram? Explain. Hint: extend $\overline{AB}$ to the left and $\overline{EF}$ down until they intersect.

c. Must ACEG also be a parallelogram? Explain.

15. In the diagram below, ACBD and BEAF are congruent kites and angle GBE measures 10° more than angle GBA.

a. Find the measure of $\angle AGC$.



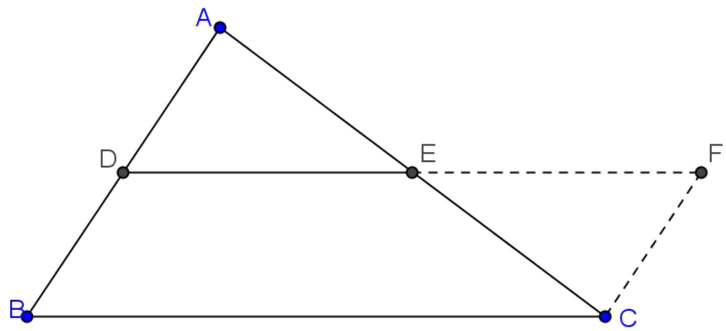
b. Is AGBH a rhombus? Justify your answer.

16. Two kites both have two sides of 8 and two sides of 10. Must they be congruent? Explain.

17. ***Triangle Midsegment Theorem***: In a triangle, the line segment connecting the midpoints of any two sides must be parallel to the third side and half of the length of the third side.

In the diagram below, D and E are midpoints of sides $\overline{AB}$ *and* $\overline{AC}$. Continue $\overline{DE}$ to point F such that $\overline{DE} \cong \overline{EF}$.

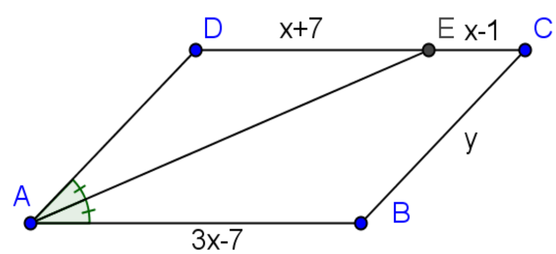
a. Show that $\Delta ADE \cong \Delta CFE$.



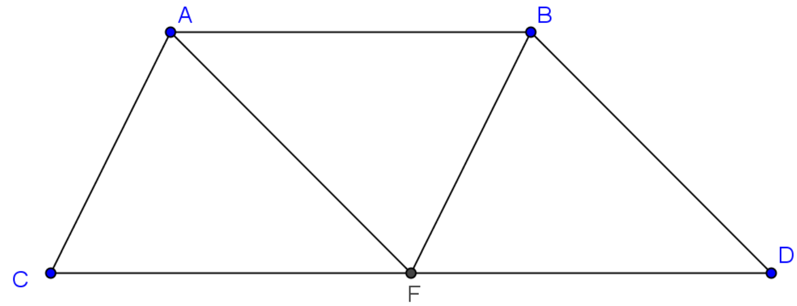
b. Explain why BCFD must be a parallelogram.

c. Now prove the theorem, using parallelogram properties.

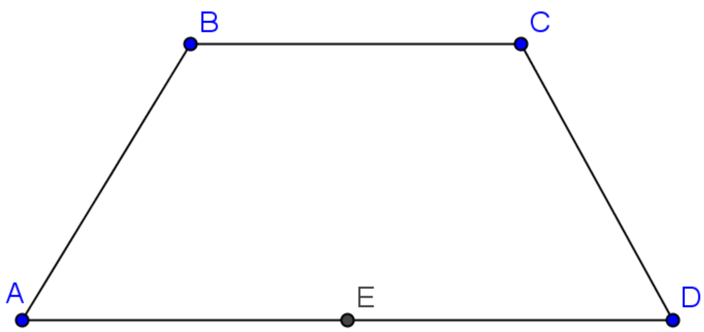
18. In the diagram below, $\overline{AE}$ bisects angle DAB in parallelogram ABCD. Find the value of *y*.



19. Given trapezoid ABDC where the lower base is twice the length of the upper base and the midpoint of $\overline{CD}$ is F. Prove that segments $\overline{AF}$ *and* $\overline{BF}$ divide ABDC into three congruent triangles. Use properties of quadrilaterals!

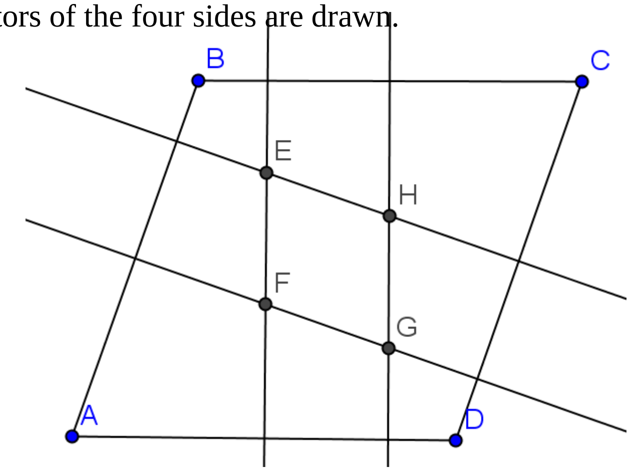


20. In the trapezoid below, E is the midpoint of $\overline{AD}$, and AB=BC=CD=DE. Show that ABCE is a rhombus. Then explain why $\overline{AC} \perp \overline{CD}$.



21. In parallelogram ABCD below, the perpendicular bisectors of the four sides are drawn.

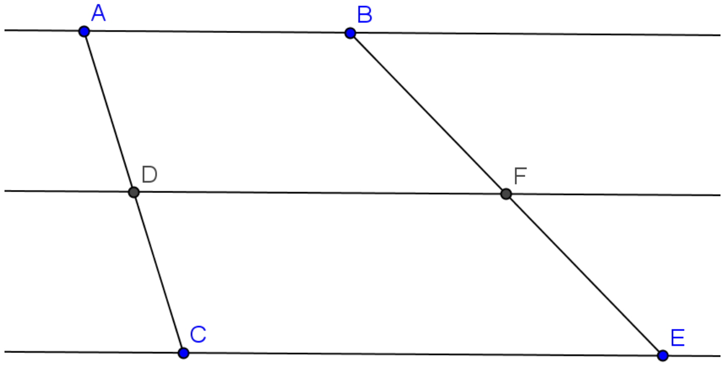
a. Explain why EFGH must be a parallelogram.



b. Explain why angles FGH and HEF must be congruent to angle A.

c. Must E, F, G, and H lie on the diagonals $\overline{AC}$ *and* $\overline{BD}$? Justify your answer

22. There's a theorem that states that if three parallel lines cut off congruent segments on one transversal, then they do so on all transversals. In others words, in the diagram below, if $\overline{AD} \cong \overline{CD}$, then $\overline{BF} \cong \overline{EF}$. Use the properties of parallelograms to prove this. Since there are no parallelograms in the diagram below, you'll need to draw a line segment or two!



Answers

2. squares! 10. x=58 and y=20.6 11. a=10 and b=7 12. x=65/11 so angle A =446/11

13a. x=5 and y=2 b. yes by SSS c. must be below 13 d. yes; draw AC and each ½ of A is larger than each ½ of B using the triangle inequality

14a. yes; opposite sides are // and congruent b. yes (same logic) c. yes (same logic)

15a. 160/3 b. yes since triangle AGB is isosceles and the figure is symmetric about segment AB

16. not necessarily; like two triangles with sides of 8 and 10 do not have to be congruent; and a kite can be thought of as congruent triangles sharing a side (and oriented properly…)

17a. by SAS since AE=EC (midpoint); DE=EF, and vertical angles b. FC=DB and angle F=angle ADE so two sides = and // make it one c. DF=BC, so DE=half BC and opp sides of parallelogram are //

18. x=13 and y=20 (note that ADE is isosceles)

19.ABFC is a parallelogram since opposite sides are // and congruent so AC=BF; same logic makes AF=BD; then SSS … (there are other ways to do this)

20. ABCE must be a parallelogram since it has two congruent and parallel sides (AE and BC); thus EC must be congruent to AB so it is a rhombus. Then CED is equilateral….

triangles ABE, BCE, and DCE are all equilateral so angle BCE measures 60°. Diagonal AC of parallelogram ABCE bisects angle BCE so ACE=30°. Add that to 60° for ECD and get 90°

21a. since EH and FG are perpendicular to parallel lines, they must be parallel; same with EF and GH b. in quadrilateral where A and F are opposite vertices there are two right angles so A and the obtuse F must be supplementary; thus A and EFG are also supplementary. Adjacent angles in a parallelogram are supplementary so the statement is true by transitivity. C. Only if it is a rhombus or rectangle

22. Draw segment through F parallel to AC; let it hit CE at G and line AB at H. because AHFD and DFGC are parallelograms, HF=AD and FG=DC. Then by ASA we know ΔBFH is congruent to ΔEFG…

Unit 4 Handout #3: Quadrilaterals on the Coordinate Plane

Tools of Coordinate Geometry

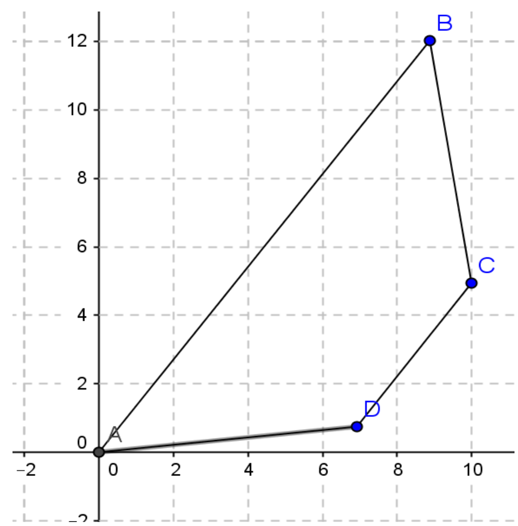
1. To find the midpoint of a line segment, ***average*** the *x* coordinates and average the *y* coordinates. So the midpoint of the segment connecting (x_1, y_1) *and* (x_2, y_2) *is* $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$. Midpoints are particularly useful when asked to prove that a segment is bisected.

2. The distance formula is an application of the Pythagorean Theorem. To find the distance between any two points not on the same horizontal or vertical line, just make the segment the hypotenuse of a right triangle and draw the legs. You will see that the distance between (x_1, y_1) *and* (x_2, y_2) is $\sqrt{(\Delta x)^2 + (\Delta y)^2}$ where $\Delta x = x_1 - x_2$ *and* $\Delta y = y_1 - y_2$.It is useful for showing that line segments have equal lengths. Note: the distance between two points on the same horizontal line is just the absolute value of the difference of their *x*-coordinates. So the distance between (7,3) and (19,3) is |7-19|=12.

3. Slopes are good for showing segments are perpendicular or parallel. Remember that parallel lines have the same slope and perpendicular lines have slopes that are negative reciprocals of each other.

4. Intersections of lines can be found by solving the system of equations.

Example: A quadrilateral's vertices are A(0,0), B(9,12), C(10,5), and D(7,1). Show that ABCD is an isosceles trapezoid.



Solution

Three legs are necessary:

1. Bases are parallel:

Slope of $\overline{AB} = \frac{12-0}{9-0} = \frac{4}{3}$; slope of $\overline{CD} = \frac{5-1}{10-7} = \frac{4}{3}$.

2. Legs are congruent:

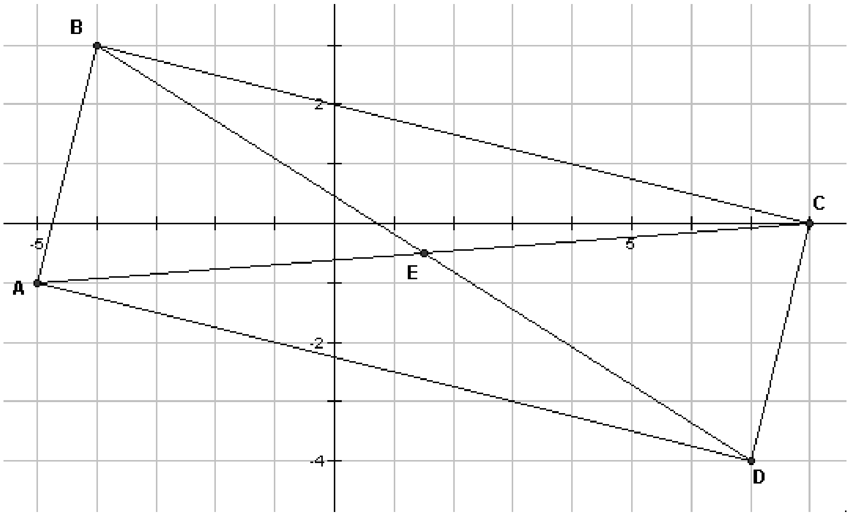
$BC = \sqrt{(9-10)^2 + (12-5)^2} = \sqrt{50}$

$AD = \sqrt{(0-7)^2 + (0-1)^2} = \sqrt{50}$

3. Legs are not parallel:

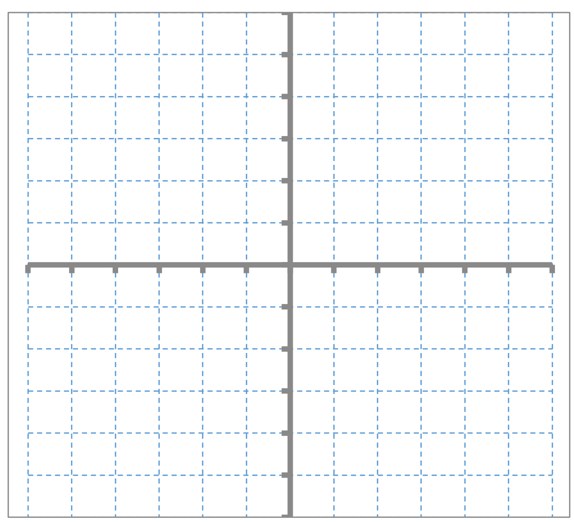
Slope of $\overline{BC}$ is -7; slope of $\overline{AD}$ is 1/7 so they are not parallel (note: if continued, they would meet in a right angle since they are negative reciprocals).

1. Show that figure ABCD below is a rectangle but not a square. You will likely need slopes and distances! Note that each grid line represents one unit.

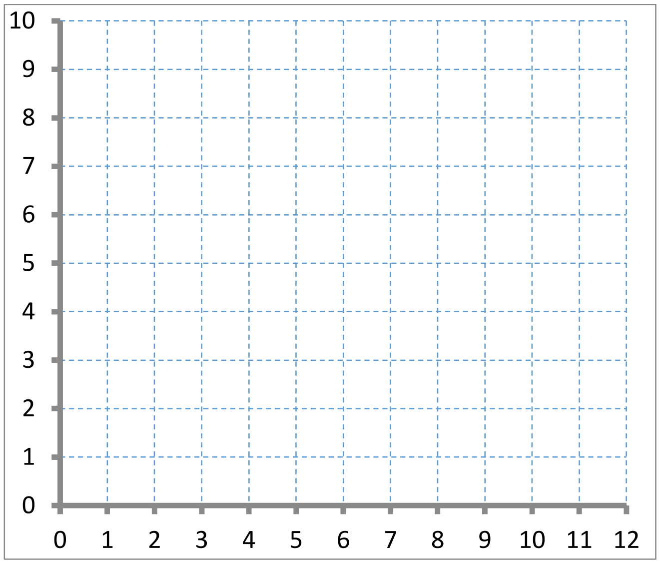


2. Find the exact coordinates of point E in the diagram above. While you may write equations of lines and find their intersection algebraically, you can also try another approach!

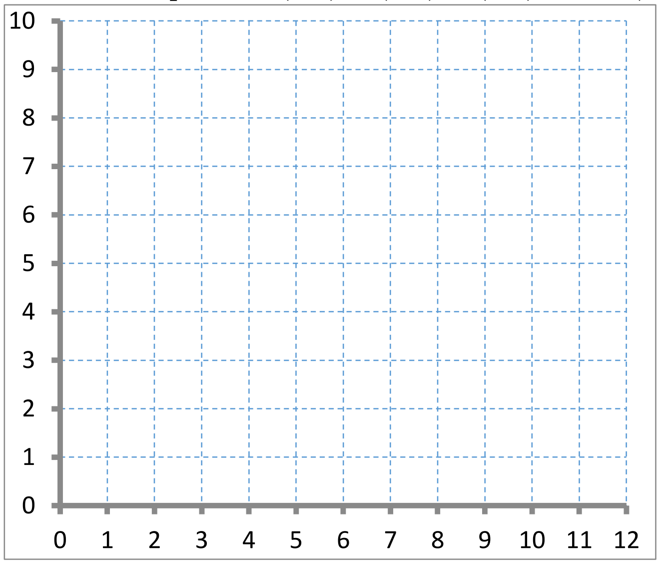
3. Three vertices of a parallelogram are (3,-4), (0,2), and (-4,-2). Find all possible coordinates of the fourth vertex.



4. The vertices of ΔABC are A(2,10), B(6,4), and C(12,8). Explain why ABC is an isosceles right triangle.



5. Given the points A(0,0), B(5,0), C(8,4), and D(3,4) do the following:

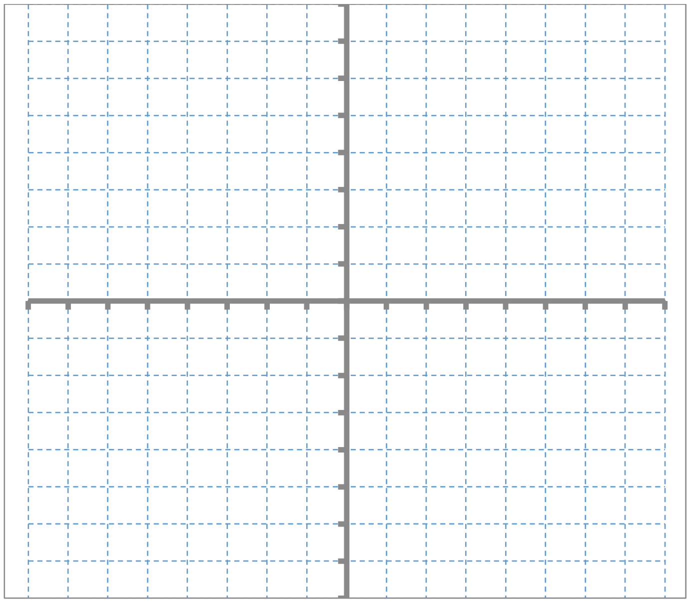


a. Show that ABCD is a rhombus by demonstrating that all sides are congruent.

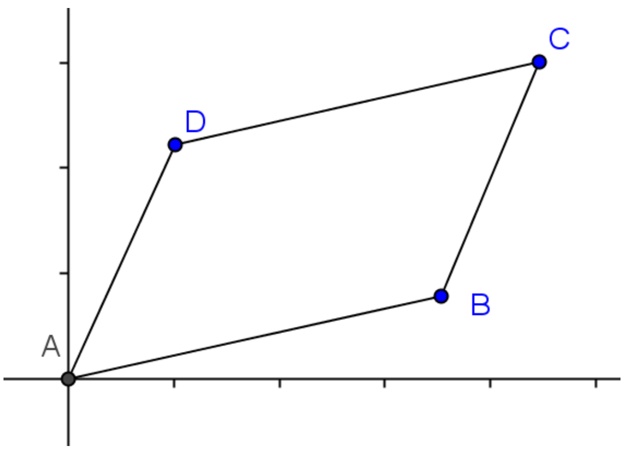
b. Show that the diagonals of the rhombus are perpendicular.

c. Show that the diagonals bisect each other by showing that their midpoints are the same point.

6. Draw a rhombus where two opposite vertices are (0,0) and (6,4). How many are possible? Challenge: can you find the exact coordinates of a possible set of the other two vertices?

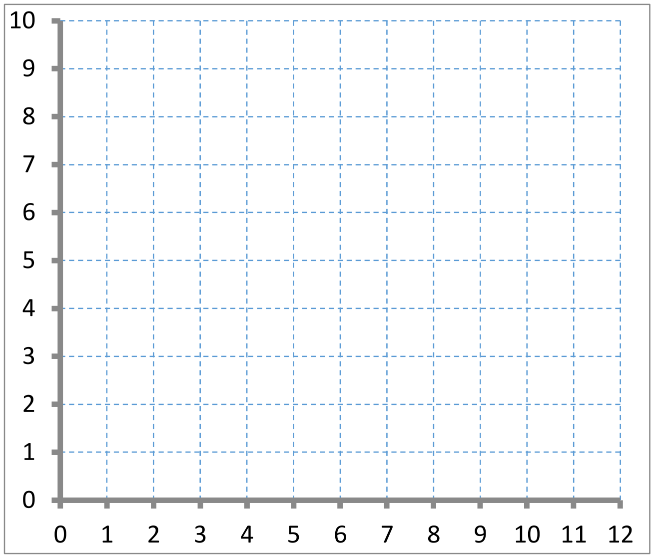


7. Given that ABCD is a parallelogram and the coordinates of A(0,0), B(*a*,*b*), D(*c*,*d*). Find the coordinates of C in terms of *a*, *b*, *c*, and *d*. (pay no attention to the tick marks on the axes)



8. Three vertices of a rhombus are A(0,1), B(8,2), C(12,9), and D.

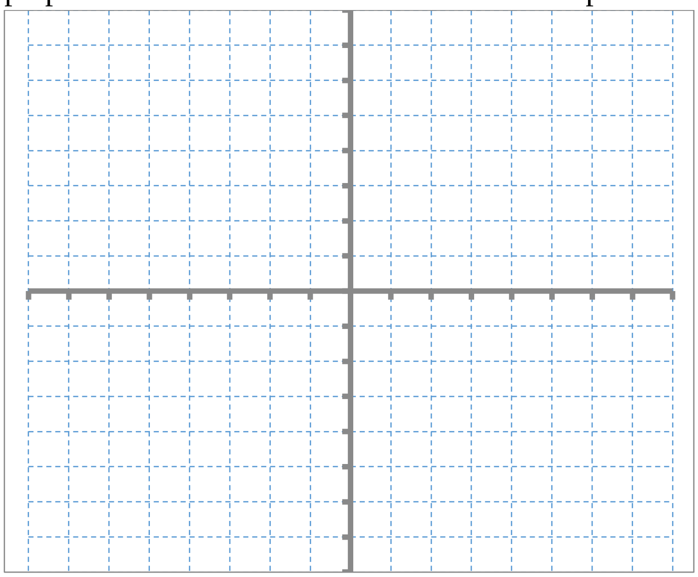
a. Verify that $\overline{AB} \cong \overline{BC}$.



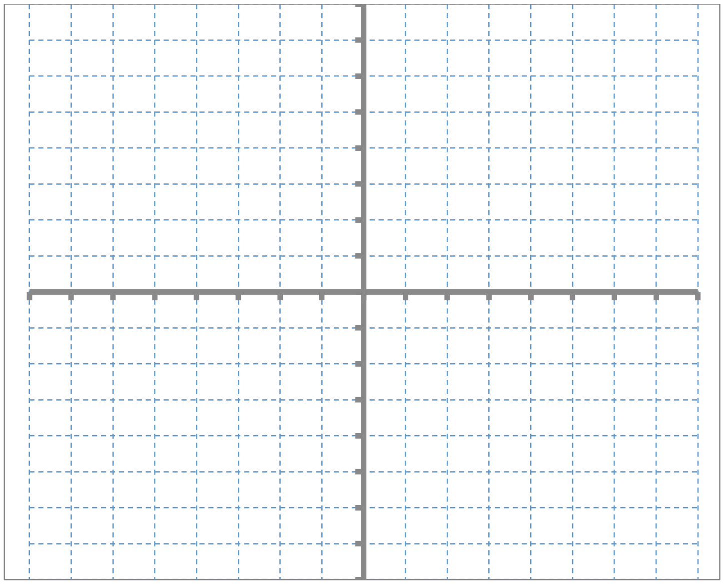
b. Find all possible coordinates of point D.

c. Find the coordinates where the diagonals intersect.

9. Show that quadrilateral K(-2,-5), I(7,-3), T(8,0), E(5,1) is a kite. Then show that its diagonals are perpendicular and find the coordinates of the point where they intersect.



10. What shape is ABCD, with A(0,4), B(-3,0), C(-2,-4), and D(4,4). Verify your conclusions!

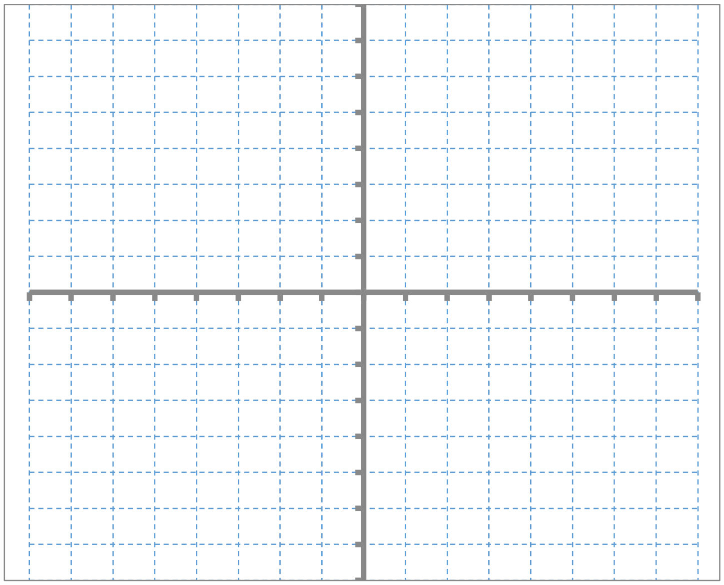


11. Create a rhombus on the coordinate grid below that satisfies all of the following three conditions. Give the coordinates of all vertices.

-The origin is not a vertex.

-No side is parallel to a coordinate axis.

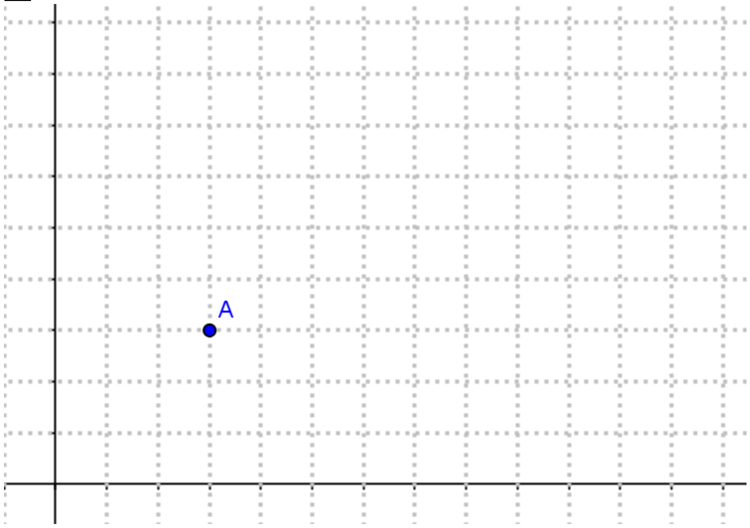
-The figure is not a square.



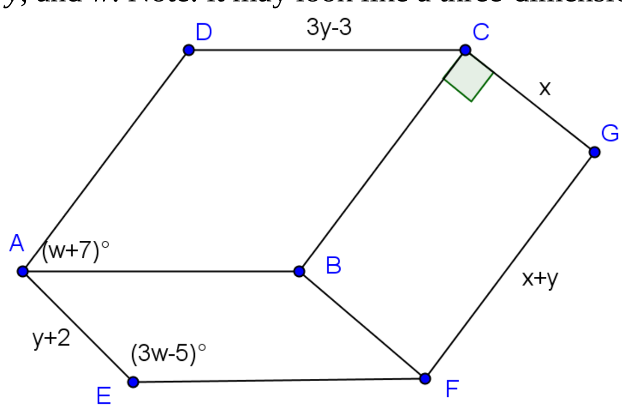
12. Triangle ABC has vertices A(2*a*,2*b*), B(2*a*+2*c*,2*b*), and C(2*a* +*c*,2*b*+2*d*).

a. Show that triangle ABC is isosceles for all values of *a*, *b*, *c*, and *d*. (well, not really all, since if *cd*=0 then there's no triangle!). Hint: find the lengths of the sides of the triangle in terms of the variables given.

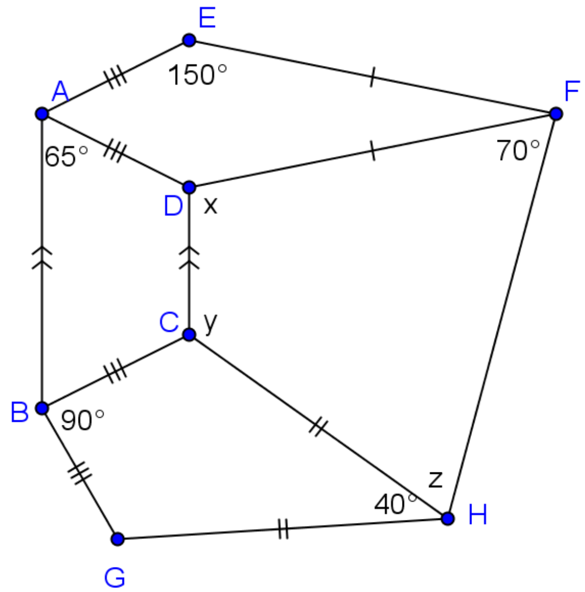
b. What must be true of *c* and *d* for the triangle to be equilateral? Find *c* in terms of *d*.



13. The object below consists of a rectangle, a rhombus (ABCD) and a parallelogram. Find the values of *x*, *y*, and *w*. Note: it may look like a three-dimensional object, but it is not!

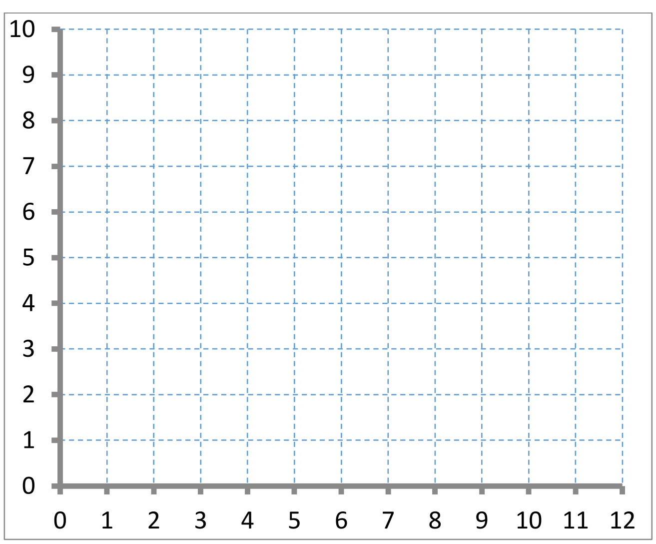


14. Find the values of *x*, *y*, and *z* in the diagram below.

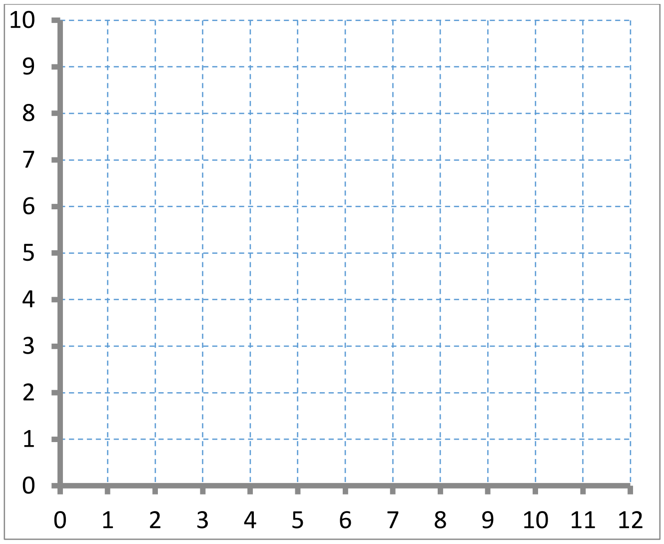


15. Now assume that C, D, and E are collinear in previous problem Find the measures of $\angle DAE$ and $\angle EFD$.

16. Three vertices of kite ABCD are A(0,8), B(3,8), and C (10,3). Find the exact coordinates of D.



17. Isosceles trapezoid ABCD has vertices A(10,5) B(2,1), and C(3,6). $\overline{AB} // \overline{CD}$. Find the exact coordinates of D. Solve in as many ways as you can!



Answers

1. opposite sides // and at least one right angle 2. Diagonals bisect each other so midpoint is (1.5,-0.5)
3. (7,0) or (-1,-8) or (-7,4) 4. AB=BC and AB is perp to BC

6. an infinite #… endpoints are on the perp bisector of (0,0) and (6,4) → one set is (0,6.5) and (6,-2.5)

7. (c+a,d+b) 8a. Both are $\sqrt{65}$ b. only (4,8) c. (6,5)

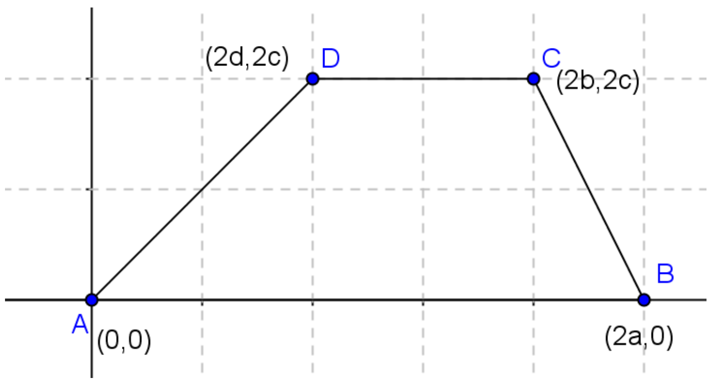
9. $\overline{KI} = \overline{EK} = \sqrt{85}$, $\overline{IT} = \overline{TE} = \sqrt{10}$; slope of KT=1/2 and slope of IE=-2 so they are perpendicular; the diagonals meet at the midpoint of IE which is (6,-1)
10. trapezoid since AB//CD and other sides are neither congruent nor parallel
11. easiest is to draw diagonals that are perp bisectors of each other…
12a. $AB = |2c|$; $AC = BC = \sqrt{c^2 + 4d^2}$ b. $4c^2 = c^2 + 4d^2$ $so\ c^2 = \frac{4}{3}d^2$ or $c = \pm\frac{2d}{\sqrt{3}}$
13. x=7; y=5; w=51 14. x=95; y=130; z=65 15. DAE=50° and EFD=10°
16. (1.8,5.6) → the diagonals are perp and meet at (2.4, 6.8) and B&D are symmetric around this point
17. (5.4, 7.2).(one way is to drop an altitude; another involves the Perp bisector of AB- which must go thru the intersection of lines AD&BC)….or perp bisector of AB must also be perp bisector of CD

Unit 4 Handout #4: Coordinate Proofs

Tips for Placing a Generic Shape on the Coordinate Plane

-Plot points as generally as possible, but use the origin and points on the coordinate axes when possible.
-Have either sides or diagonals parallel to the coordinate axes when possible. Diagonals parallel to the axes may make sense for rhombi or kites.
-Do not use the origin when you need symmetry, such as for an isosceles triangle or an isosceles trapezoid.
-When you expect to use midpoints, it may be best to define coordinates as things such as *2a* or *2b*, instead of *a* or *b*.

Example: Place a generic trapezoid on the coordinate plane. Use it to show that the segment connecting the midpoints of the legs is parallel to the bases and equal to the average of their lengths.



Solution

Given that it is not an isosceles trapezoid (so there's no need for symmetry), it makes the most sense to place the one vertex at the origin and another on the positive *y*-axis at (2*a*,0).

The other two vertices need to be along a horizontal line, so make their y-coordinates 2*c*. Their *x*-coordinates need to be different from each other and different from those of vertices *A* and *B* to be as generic as possible, so place vertex *C* at (2*b*,2*c*) and vertex *D* at (2*d*,2*c*).

The midpoints of the legs are *E* and *F*. *E*'s coordinates are $\left(\frac{2b+2a}{2}, \frac{2c}{2}\right)$ *or* $(a+b, c)$.

And *F*'s are $\left(\frac{2d}{2}, \frac{2c}{2}\right)$ *or* (d, c).

The slope of $\overline{EF}$ is 0, so it is parallel to the bases.

Distance along a horizontal is simply the absolute value of the difference of the *x*-coordinates, so (*a*+*b*-*d*).

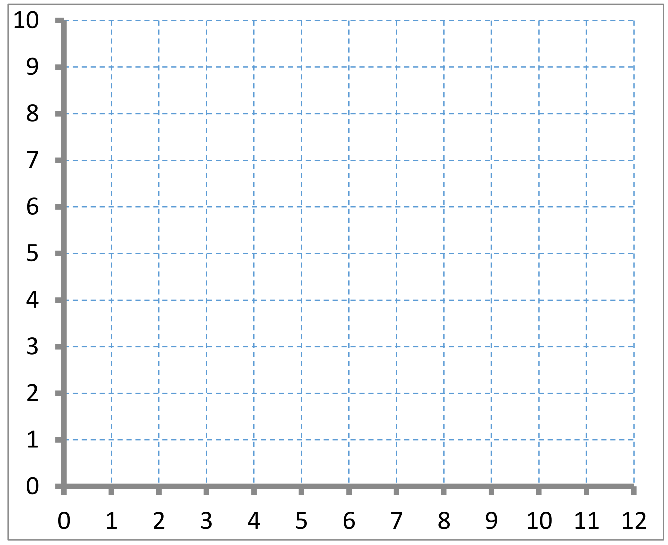
The length of base $\overline{AB}$ is 2*a* and the length of base $\overline{CD}$ is 2*b*-2*d*. The average of these is also (*a*+*b*-*d*).

1a. Sketch the quadrilateral whose vertices are (0,0), (6,0), (10,8), and (4,10).

b. Find the coordinates of the midpoints of all four sides.

c. Draw four line segments connecting the midpoints of the adjacent sides.

d. Show that the line segments are the sides of a parallelogram. Use the definition of parallelogram!

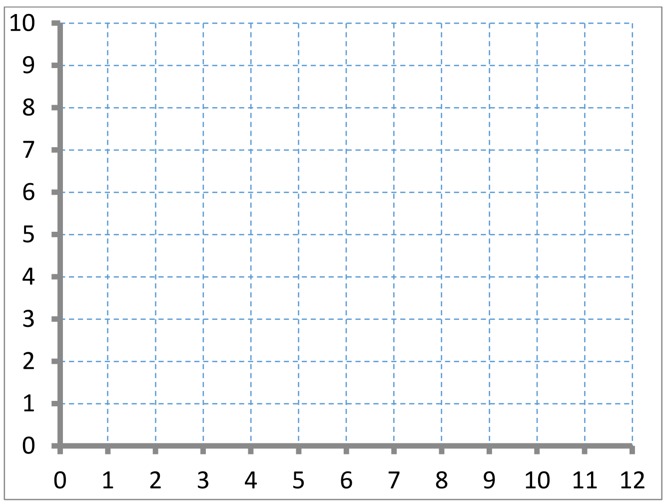


2. (continuation). Was that a lucky set of coordinates, or is it always the case that the segments connecting the midpoints of adjacent sides of a quadrilateral form a parallelogram? Draw a convex quadrilateral whose vertices are (0,0), $(2a,0)$, $(2b, 2c)$, and $(2d, 2e)$ and see if the same result holds!

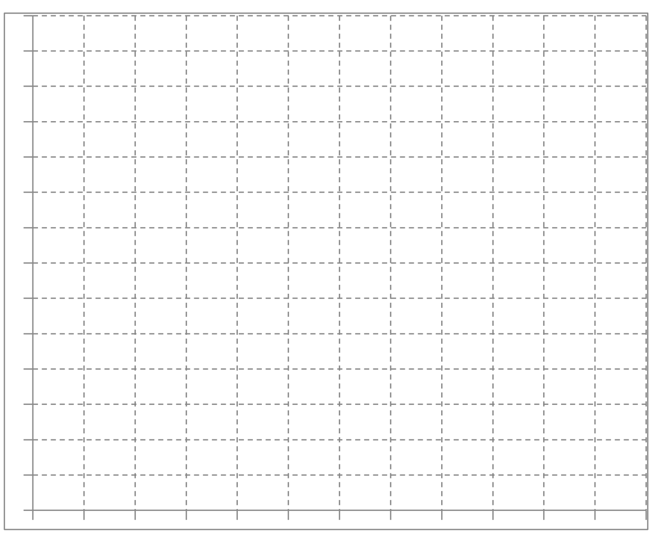
3. Why do you think it is the case that the quadrilateral formed by joining midpoints of any quadrilateral is a parallelogram? Hint: remember that theorem a few handouts ago where the line connecting the midpoints of two sides of a triangle…

4. The midpoints of the sides of a rectangle form a rhombus! Do the following:

a. Verify that this is the case for the rectangle whose vertices are A(0,0), B(10,0), C(10,6), and D(0,6). Note: you need the definition of a rhombus for this!

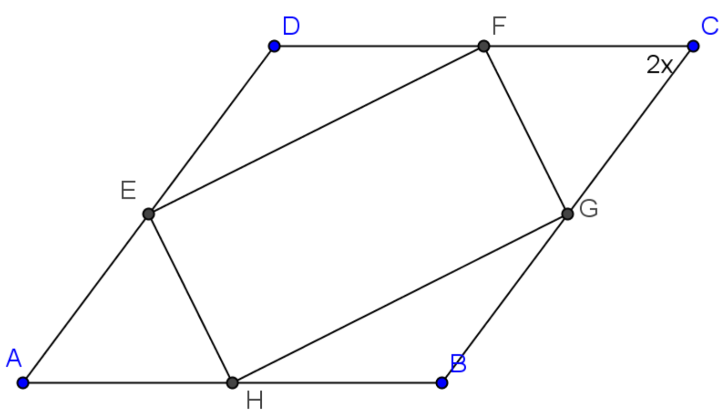


b. Do a coordinate proof for the rectangle whose vertices are A(0,0), B(2a,0), C(2a,2b), and D(0,2b).

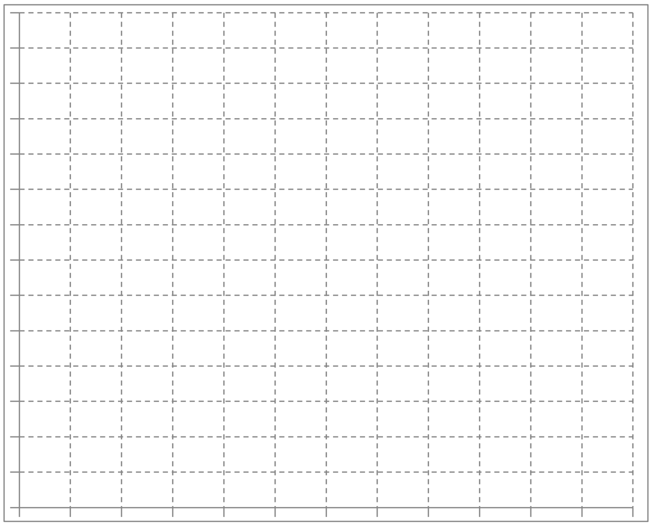


c. Do a two-column proof to prove the same thing. You may use the properties of a rectangle.

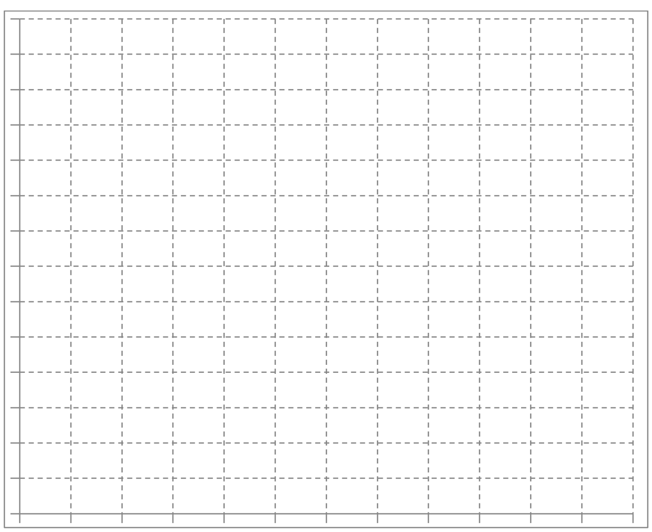
5. Show that the shape whose vertices are the midpoints of a rhombus's sides is a rectangle. Hint: you should be able to write all 16 angles below in terms of *x*!



6. Prove that the medians to legs of an isosceles triangle are equal in length. Remember, the median connects the vertex to the midpoint of the opposite side. I recommend placing your vertices at (2a,0), (-2a,0), and (0,2b).



7. Show the quadrilateral whose vertices are A(a,0), B(b,c), C(-b,c), and D(-a,0) is an isosceles trapezoid (what three things are needed for this?) and then show that its diagonals are equal in length.

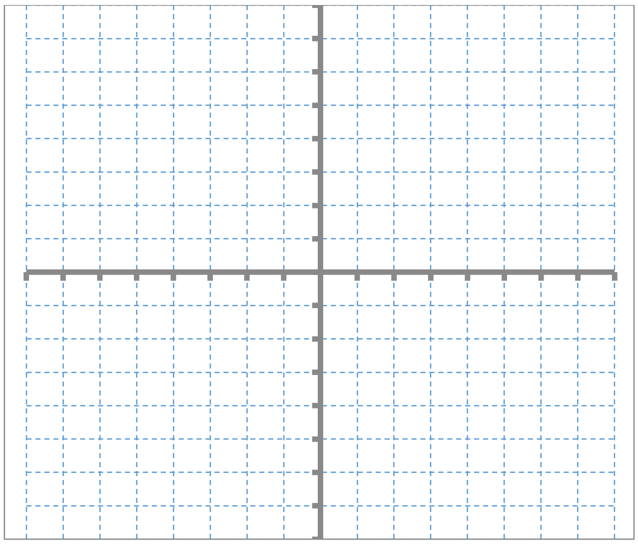


8. A line joining midpoints of adjacent sides of a parallelogram divides the diagonal it crosses into two segments, one three time the length of the other.

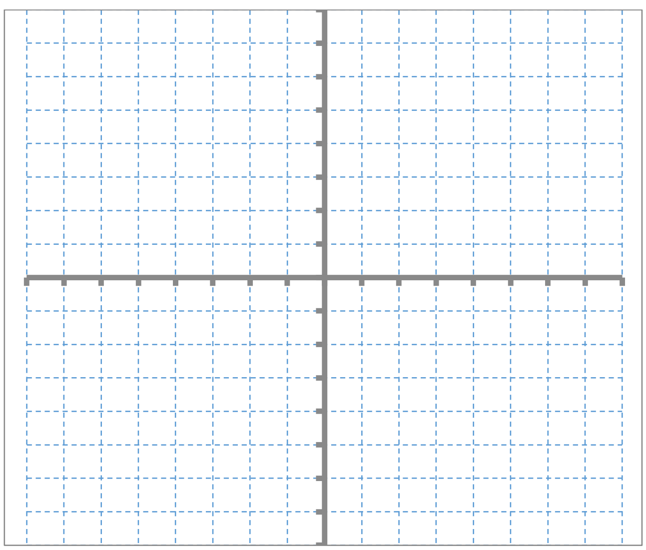
a. Draw the parallelogram whose vertices are A(0,0), B(0,-4), C(8,0), and D(8,4). (note: by defining the vertices this way we have a nice horizontal line as the diagonal!)

b. Find the midpoints of $\overline{CD}$ and $\overline{CB}$.

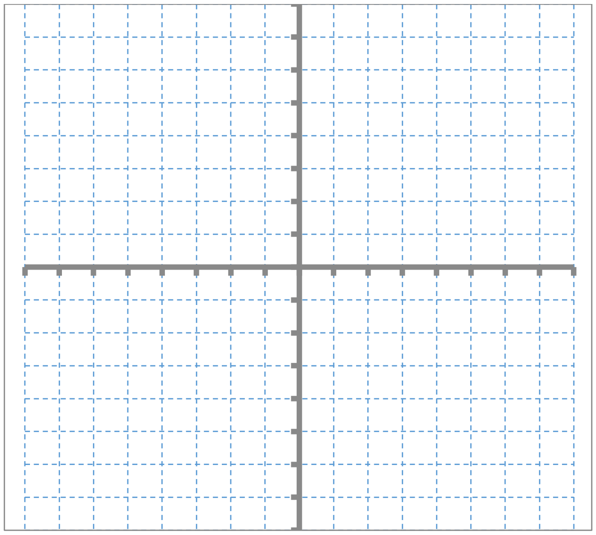
c. Show that the segment connecting these midpoints breaks diagonal $\overline{AC}$ into two parts; one three times the length of the other.



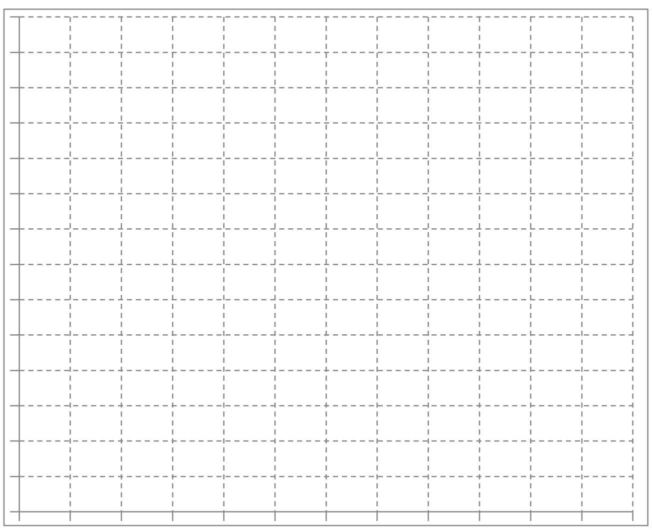
d. Now do the same thing with the parallelogram whose vertices are (0,0), (0,-2a), (2b,0), and (2b, 2a) where *a* and *b* are positive numbers.



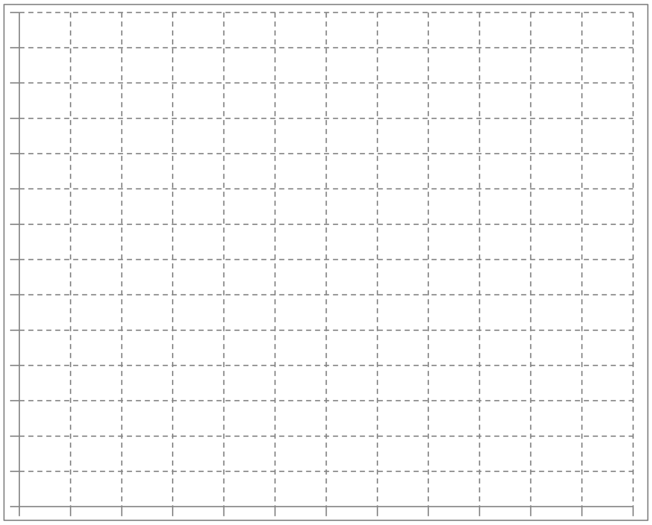
9. Three vertices of a kite are (-1,-3), (7,-1), and (4,-6). The fourth vertex is on the *y*-axis. Find its coordinates.



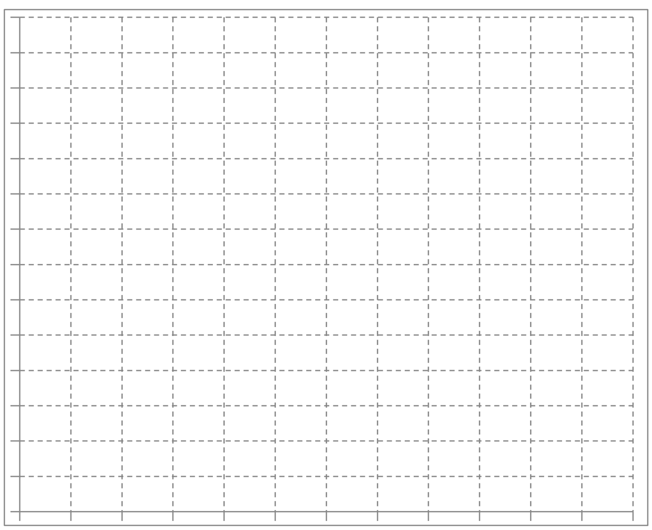
10. Place a generic right triangle on the coordinate plane. Use it to show that the midpoint of the hypotenuse of a right triangle is equidistant from the three vertices. I recommend that you place your right angle at the origin (0,0). Then chose reasonable generic coordinates for the other two vertices.



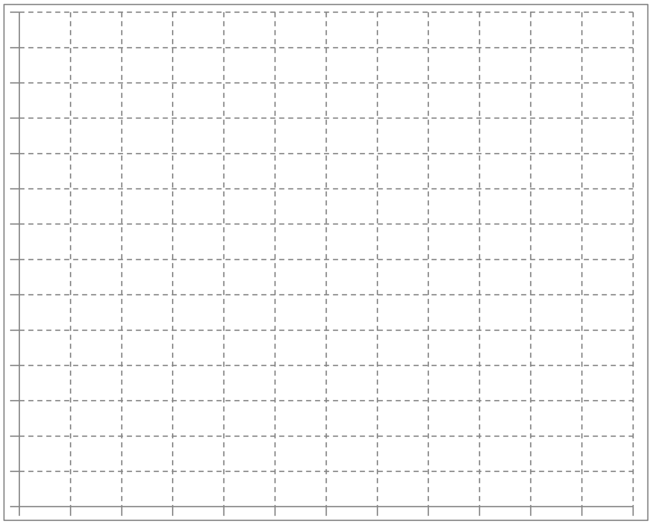
11. A few handouts ago, we proved that the line segment connecting the midpoints of two sides of a triangle is parallel to the third side and half the length of the third side. Prove this with a coordinate proof. Place a generic triangle on the coordinate plane; place one side along the *x*-axis.



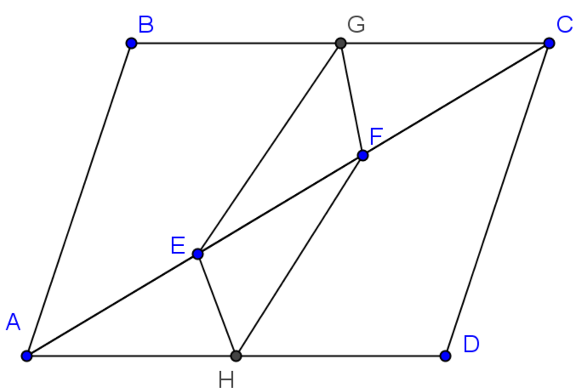
12. We did a two-column proof to establish that, if a pair of opposite sides of a quadrilateral are congruent and parallel then the quadrilateral must be a parallelogram. Do a coordinate proof now. Make three vertices (0,0), (*a*,0), and (*b*,*c*). Then find the fourth in terms of *a*, *b*, and *c*. And show that the other pair of sides is also parallel.



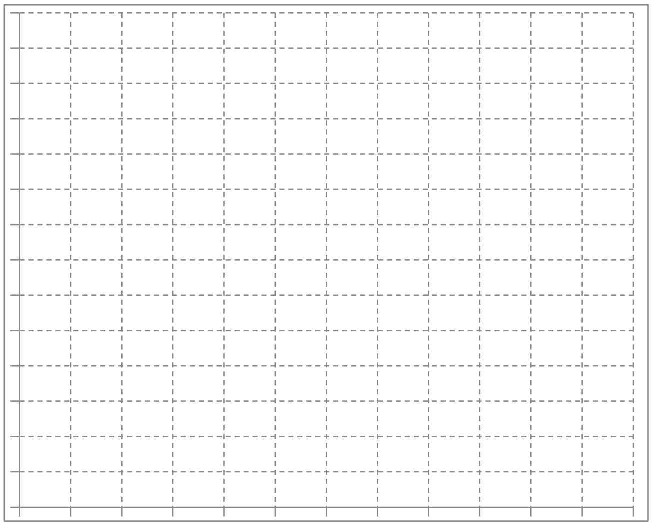
13. Put a generic parallelogram on the coordinate plane. Prove that the sum of the squares of the four sides is equal to the sum of the squares of the two diagonals.

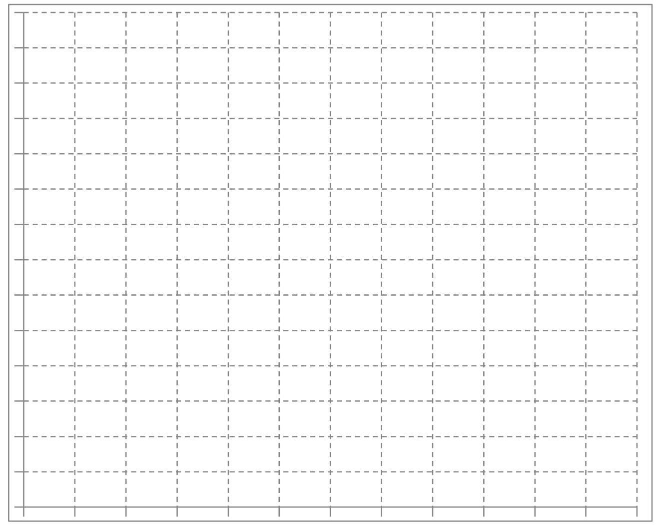


14. A parallelogram has a diagonal trisected. Join the trisection points to the midpoints of a pair of opposite sides. Prove this is a parallelogram. See diagram below



15. Prove that the altitudes to the sides of any triangle intersect in a single point. Hint: place a "generic" triangle on the coordinate plane and find where two altitudes meet. Then show that the line through the third vertex and that intersection is perpendicular to the third side.





16. In a right triangle, find the point where the altitude to the hypotenuse hits the hypotenuse. From this point, draw line segments to the midpoints of both legs. Prove they are perpendicular to each other. It is very messy with coordinate geometry; maybe try something else!

Answers

1b. (3,0), (8,4), (7,9), (2,5) d. slopes are 4/5, -5, 4/5, and -5 so opposite sides are //.. cool!

3. draw a diagonal of the original quadrilateral and look at the two triangles created. In each on the line connecting the midpoints of the sides is // to the base and ½ as long; so the two segments are // and =

4a. midpoints are (5,0), (10,3), (5,6), and (0,3) → opp sides are // and all sides are =

b. midpoints are (a,0), (2a,b), (a,2b), and (0,b) → opp sides are // and all sides have length $\sqrt{a^2+b^2}$

c. using SAS, 4 triangles are congruent so all sides are = and a quad with 4 = sides is a rhombus

5. since CFG is isosceles, angle CGF=90-x; B measures 180-2x so BGH=x so FGH must be 90°

7. one pair of // bases; sides = and not //; diagonals are both $\sqrt{(a+b)^2+c^2}$

9. (0,10) → where perp bisector of segment connecting (-1,-3) and (7,-1) hits y-axis.

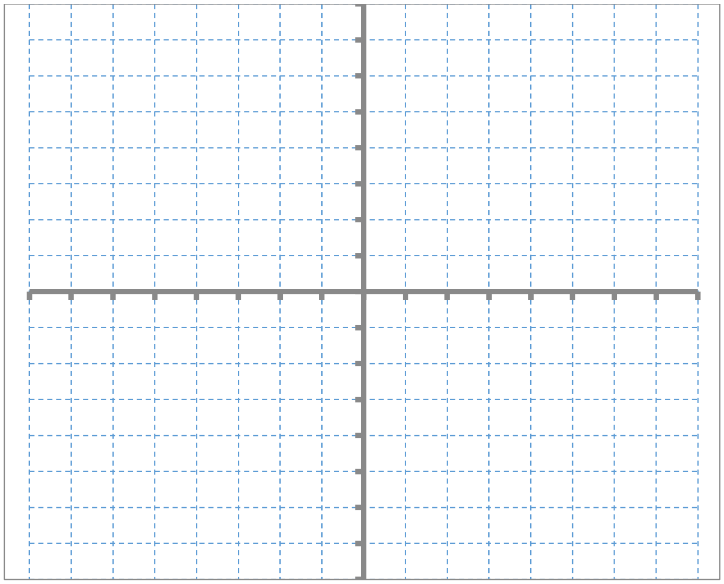
10. I used (2a,0) and (0,2b) and found that the midpoint (a,b) is $\sqrt{a^2+b^2}$ from each vertex

12. last vertex is (a+b,c) so the slopes of the non-horizontal sides are both c/b

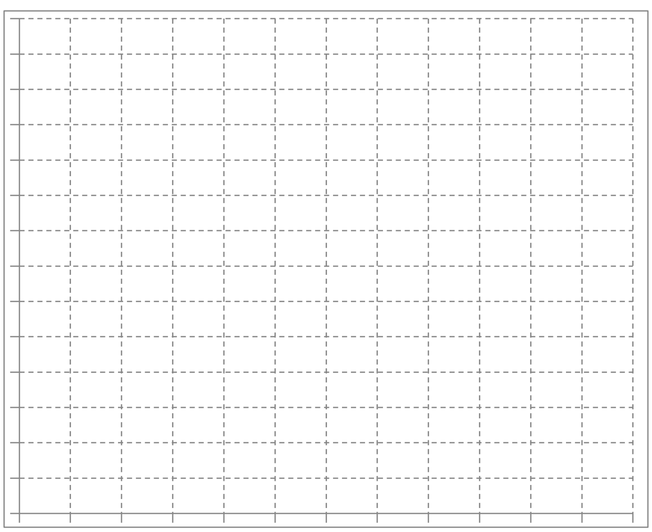
16. using the fact that the midpoint of the hypotenuse is equidistant from the three vertices on the two smaller triangles created by the altitude, you can see that a kite is created, where the angle in question is opposite the original triangle's right angle—so it must also be 90°

Unit 4 Handout #5: More Coordinate Proofs

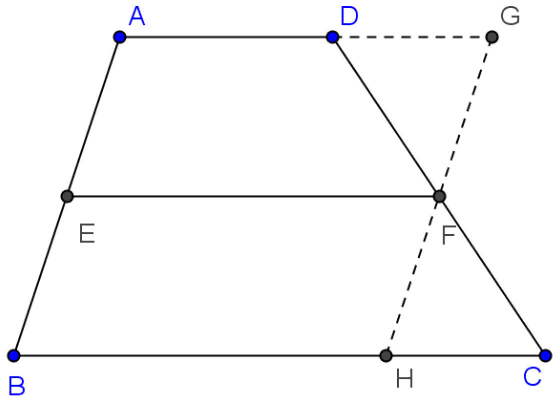
1. Show that the points (a,0), (-a,0), (0,b), and (0,-b) form a rhombus. And then use them to show that the midpoints of adjacent sides form a rectangle.



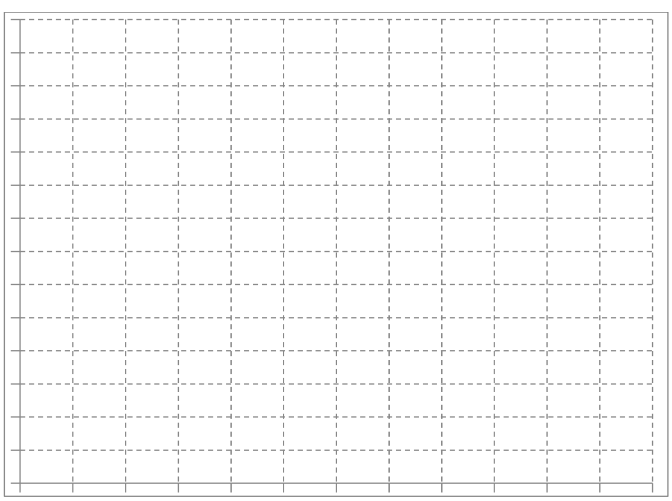
2. Draw a generic trapezoid (not isosceles and not right)—using letters as coordinates. Find good coordinates for all of the points. Show that the line connecting the midpoints of the two non-parallel sides is parallel to the bases and equal to the average length of the bases.



3. Try to prove this in a two-column way using the diagram below. You may use all parallelogram properties! Note: GH is parallel to AB through F and E and F are the midpoints of the sides of the trapezoid. So you want to prove that $\overline{EF} \parallel \overline{AD} \parallel \overline{BC}$ and that the length of EF is the average of the lengths of AD and BC.

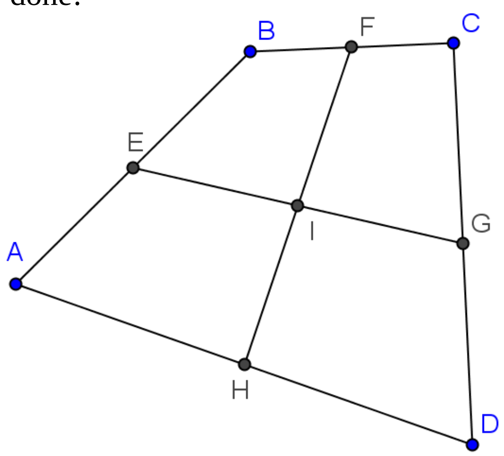


4. Place a "generic" kite on the coordinate plane. Write the coordinates of its vertices using letters (*a*, *b*, *c*, etc…). How few different letters can you use?



5. Prove that when you connect the midpoints of adjacent sides of a kite, you get a rectangle. There are many ways to try this, including coordinate proof.

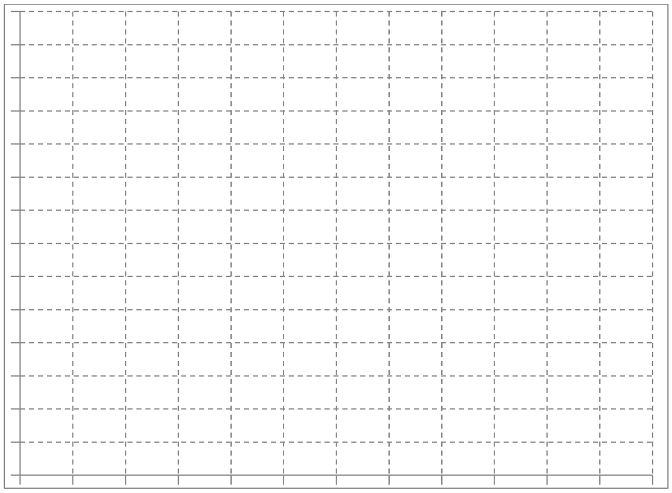
6. A quadrilateral with no two sides equal or parallel has line segments drawn connecting the midpoints of the opposite sides. Prove that these line segments bisect each other. Do a coordinate proof and think of the easiest way to prove segments bisect each other (hint: it does not involving finding the intersection algebraically!) You may use a coordinate proof or you may rely on some proofs that we have previously done!



7. A trapezoid has two right angles.

a. Write "generic" coordinates for its vertices. Use as few letters as possible.

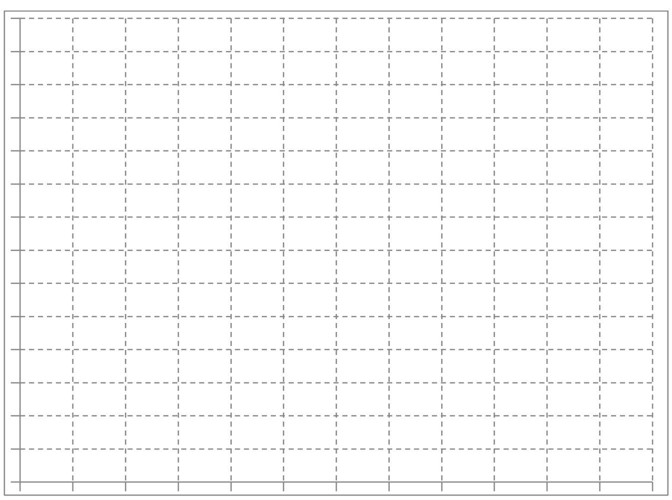
b. Show that the line segments connecting the two vertices with right angles to the midpoint of the opposite leg are congruent.



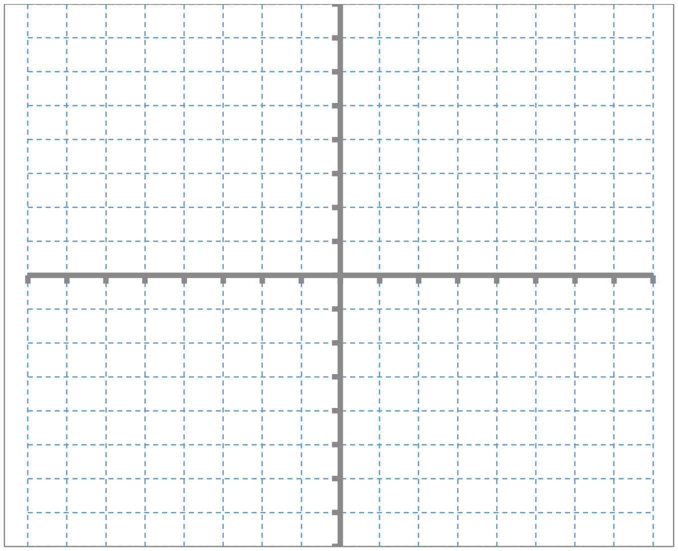
8. Write the coordinates of a "generic" isosceles trapezoid with as few "letters" as possible.

a. Use this to prove that the diagonals of an isosceles trapezoid are congruent.

b. Now prove that the quadrilateral formed by connecting midpoints of adjacent sides is a rhombus.

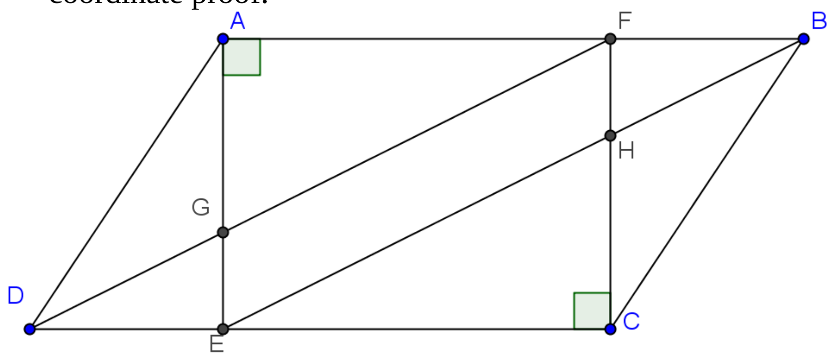


9. Show that the segments connecting the midpoints of an isosceles triangle form another isosceles triangle. Use coordinate proof.

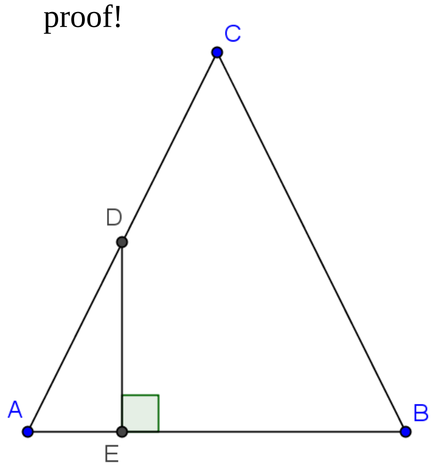


10. Use coordinate proof to show that if the diagonals of a parallelogram are congruent, then it must be a rectangle. Hint: use a generic parallelogram and see what must be true about its coordinates.

11. Drop altitudes to the top and bottom bases of parallelogram and connect the ends of altitudes to the opposite corners, as shown in the diagram below. Prove that EGFH is a parallelogram. You may do it as a coordinate proof.



12. In isosceles triangle ABC below, a segment is drawn from the midpoint of leg $\overline{AC}$ perpendicular to the base $\overline{AB}$, intersecting it at point E. Prove that AE = ¼ of AB. You may consider doing a coordinate proof!



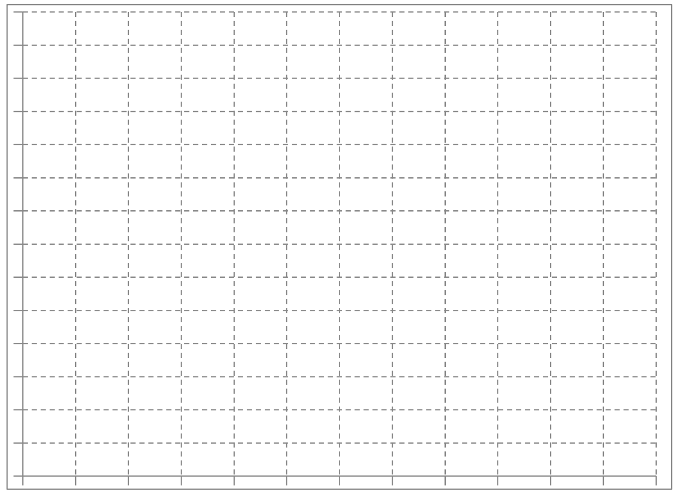
13. Prove that the medians of a triangle are concurrent and the point at which they intersect (the centroid) is two-thirds of the way from each vertex to the midpoint of the opposite sides. Here are some steps:

a. Make vertices A(0,0), B(2*a*,0), and C(2*b*,2*c*).

b. Find the equations of two of the medians and find the coordinates of their intersection.

c. Show that the third median goes through this point.

d. Now show that this intersection point is two-thirds of the way from each vertex to the opposite median.



Unit 4 Handout #6: Regular Polygons

A regular polygon is a polygon that is both equilateral and equiangular.

Finding the measure of an interior angle of a regular polygon:

Method #1: Find the sum of the interior angles, which is $(n-2)180$, and then divide by *n*.

Method #2 (more efficient!). Find the exterior angle of the polygon—which is just 360/*n*. Then subtract this from 180°. [The exterior angle is 360/*n* because "walking around" the polygon, turning at each exterior angle, one would turn 360° by the time one made *n* turns and turned to the starting point]

Finding the number of diagonals in a convex polygon of *n* sides.

-From each vertex, there are (*n*-3) diagonals, because there are no diagonals to itself or to adjacent vertices.

-There are *n* vertices, each with (*n*-3) diagonals, so *n*(*n*-3).

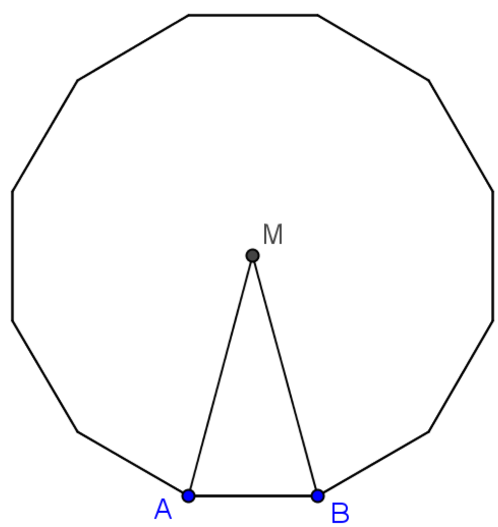
-But this double-counts all diagonals since each end is counted once. So divide by two and get *n*(*n*-3)/2.

Example: A regular polygon has 12 sides.

a. How many diagonals does it have?

b. What is the measure of each interior angle?

c. What is the measure of angle M?



Solution

a. Using the formula, number of diagonals equals *n*(*n*-3)/2, we plug *n*=12 in and get 54.

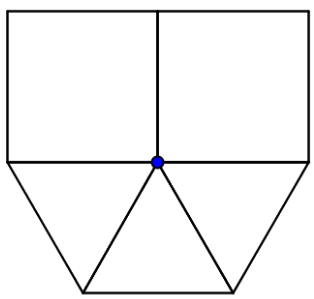
b. Each exterior angle is 360/12=30°, so each interior angle must be the supplement of 30°, or 150°.

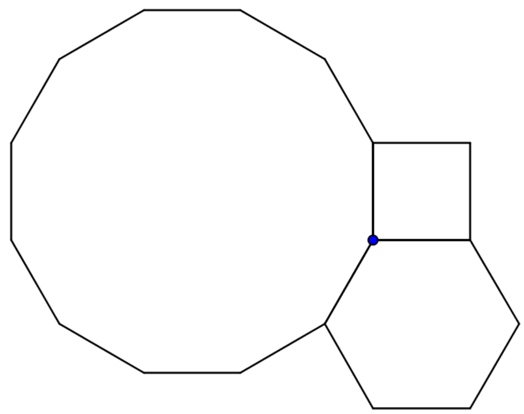
c. Angle M is one-twelfth of the full circle so 30°.

Activity: Surrounding a point with regular polygons

In both diagrams below, a point is surrounded by regular polygons, in that the angles of the polygons sum to 360°. On the left, two quadrilaterals and three triangles surround the point. On the right it is surrounded by a quadrilateral, a hexagon, and a 12-sided polygon (dodecagon). We can use shorthand to say that, on the left, we have 4-4-3-3-3 (4-sided, another 4-sided, and three 3-sided polygons) and on the right we have 4-6-12. Your goal is to find all possible ways of surrounding a point with regular polygons.

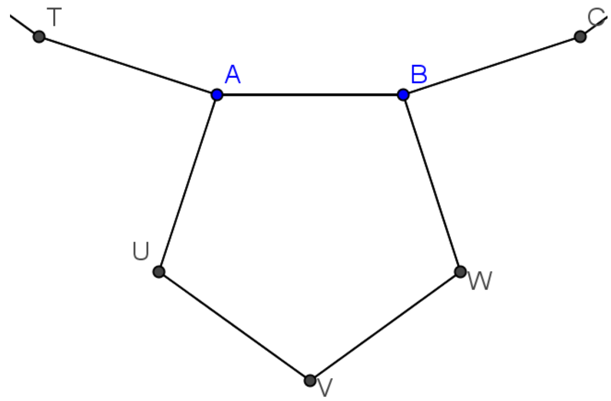
First fill in the table below, showing the measure of each internal angle for regular polygons with different numbers of sides.





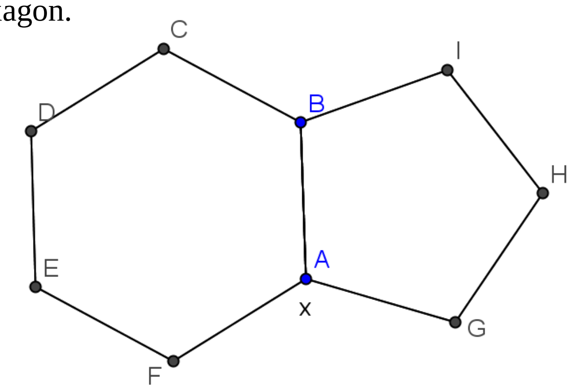
Sides	Angle
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
15	
18	
20	
24	
30	

1. TABC is part of a regular 20-sided polygon ("icasogon") and ABWVU is a regular pentagon. Angle CBW appears to be a right angle. Is it?



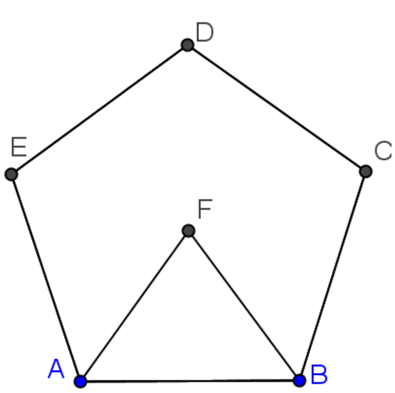
2. Segment $\overline{AB}$ is a side of both a regular pentagon and a regular hexagon.

a. Find the value of *x*.

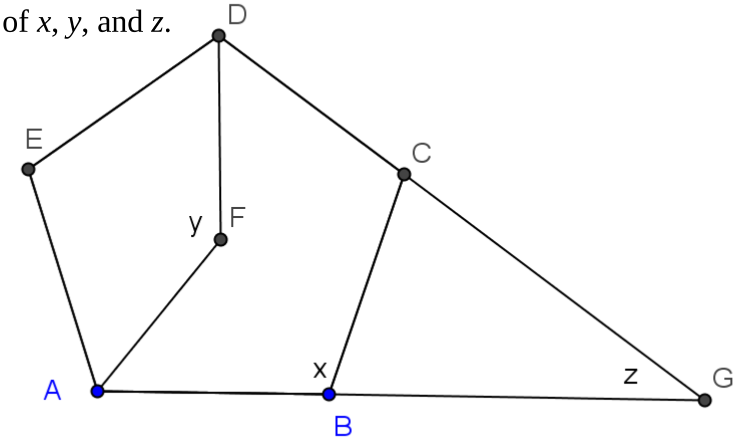


b. Can GA and AF be adjacent sides of a regular polygon? If so, how many sides does it have?

3. Given the regular pentagon below whose center is F (define the center of a regular polygon as the point equidistant from all vertices) Determine which is longer, $\overline{AB}$ or $\overline{AF}$. Explain.



4. In the diagram below, ABCDE is a regular pentagon. Sides $\overline{AB}$ and $\overline{CD}$ have been extended to meet at G. Point F is the center of the polygon (one way to think of it: if you drew a circle through A, B, C, D, and E, then F would be its center). Find the values of *x*, *y*, and *z*.

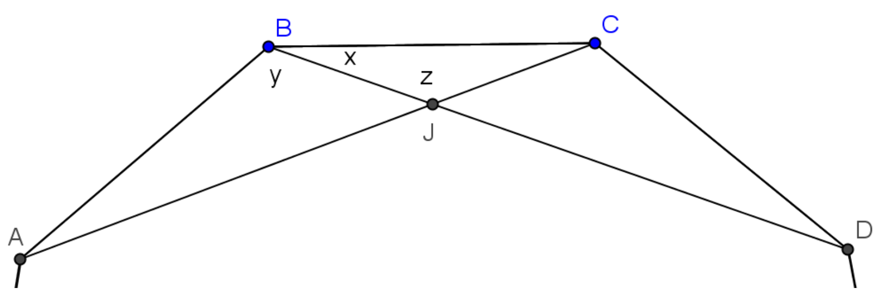


5a. Can a quadrilateral be equilateral and not equiangular?

b. Can a quadrilateral be equiangular and not equilateral?

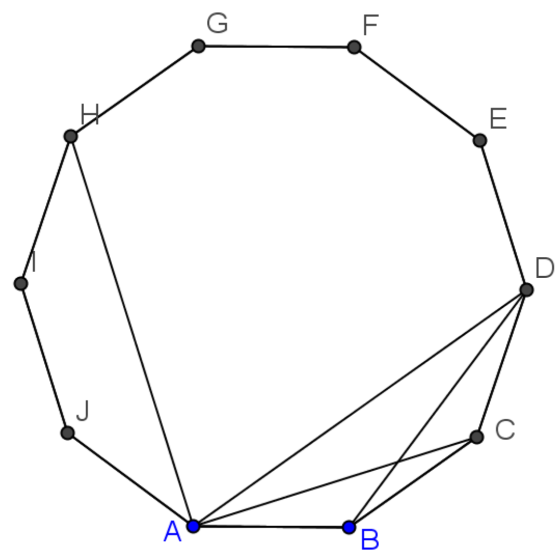
c. Can a triangle be equiangular and not equilateral?

6. ABCD is part of a regular nine-sided polygon (nonagon). Find the values of *x*, *y*, and *z*.



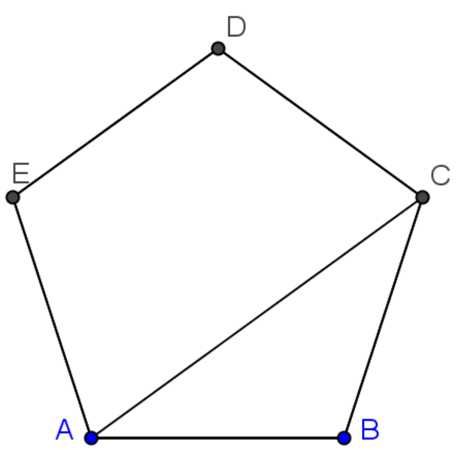
7. Given a regular 10-sided polygon (decagon), explain why….

a. Diagonals joining vertices two apart are congruent (ie: $\overline{AC}$ *and* $\overline{BD}$)

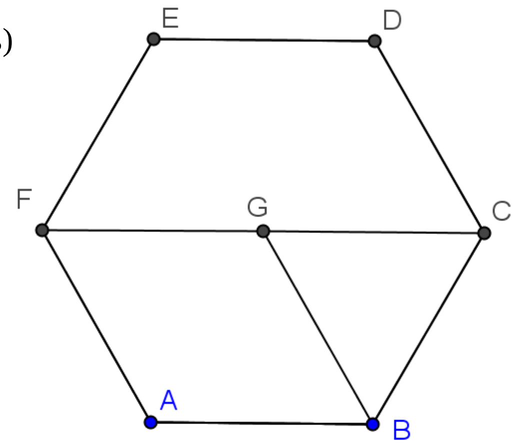


b. Diagonals joining vertices three apart are congruent. (ie: $\overline{AD}$ *and* $\overline{AH}$)

8. Show that diagonal $\overline{AC}$ divides the regular pentagon below into an isosceles triangle and an isosceles trapezoid.



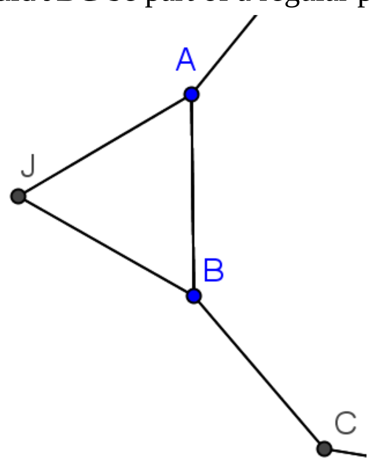
9. Given regular hexagon ABCDEF with center G, do the following:
(again, the center is the center of the circle through all of the vertices)



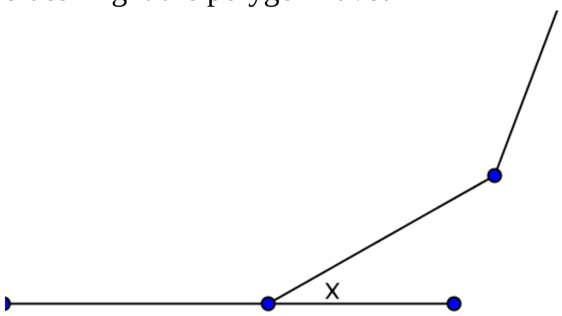
a. Explain why ABGF must be a rhombus.

b. Explain why ABCF must be an isosceles trapezoid.

10. In the diagram below, ABC is part of a regular nonagon (9 sides) and JAB is an equilateral triangle. Could JBC be part of a regular polygon? If so, how many sides does it have?

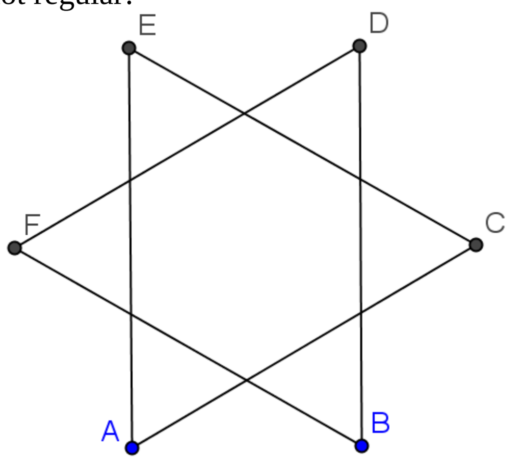


11. In the diagram below, x is an exterior angle of a regular polygon. If $20^\circ \le x \le 25^\circ$, then how many sides might the polygon have?

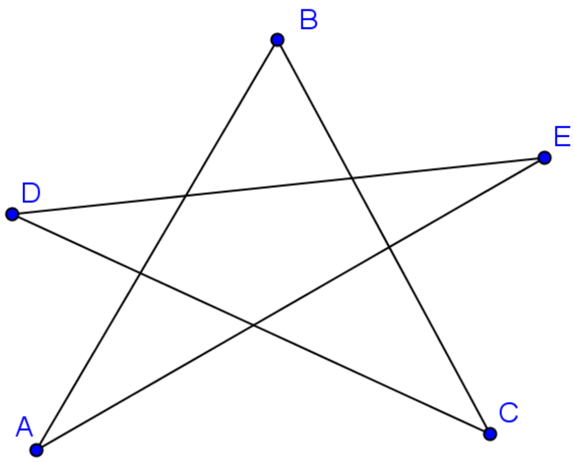


12. For the two stars below, find the sum of the vertex angles (by this I mean angles A, B, C, D, E, and F in the first one and A, B, C, D, and E in the second). Note that the pentagon in the center of part *b* is clearly not regular!

a.



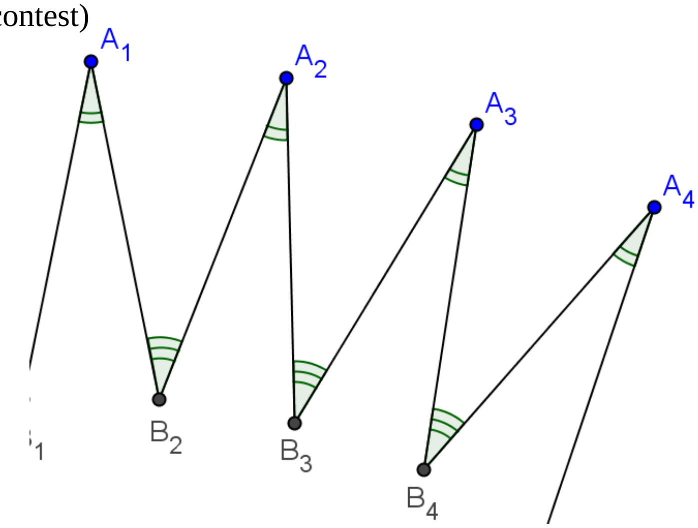
b.



13. The diagram below is part of a large "star". All segments are one unit long. Also, the acute angles at all of the A's are equal. The acute angles at all of the B's are also equal, and are 10° different than the angles at the A's. (Adapted from the 1992 AHSME math contest)

a. Are the angles at A's or B's larger? Why?

b. What is the perimeter of the object?

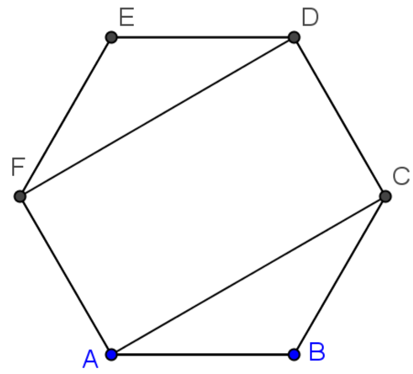


Answers

1. yes; ABC measures 162° and ABW measures 108° so CBW measures 90°
2a. 132° b. no; no regular polygon has vertex angles of 132° (it would have to be a 7.5-agon!)
3. F measures 72° so other angles in ABF measure 54° so AB is longer 4. x=108°; y=144°; z=36°
5a. yes; most rhombi b. yes; most rectangles c. no 6. x=20°; y=120°; z=140°
7a. by SAS triangles ABC and BCD are congruent b. now by SAS triangles ACD and AIH are congr.
8. ABC is isos since AB=BC; angles CAB and ACB measure 36° so ACD measures 72° and CDE measures 108° so AC // DE and AE=CD so ACDE is an isos trapezoid
9a. AF=AB and AGC & AGF are equilateral so all sides are congruent b. GFE and FED are supplementary (60 and 120°) so CF // DE and EF=CD 10. JBC measures 160°; so an 18-sided one
11. 15-18 sides 12a. 360° b. 180° 13a. B's are larger or it would bend the other way b. 72

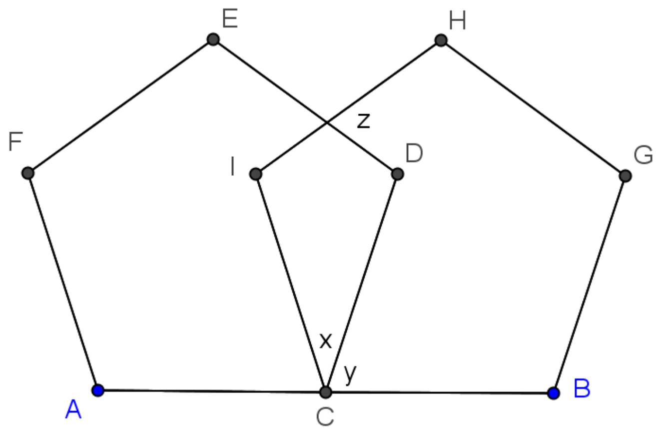
Unit 4 Handout #7: More Regular Polygons

1. Given the regular hexagon below, explain why ACDF must be a rectangle.

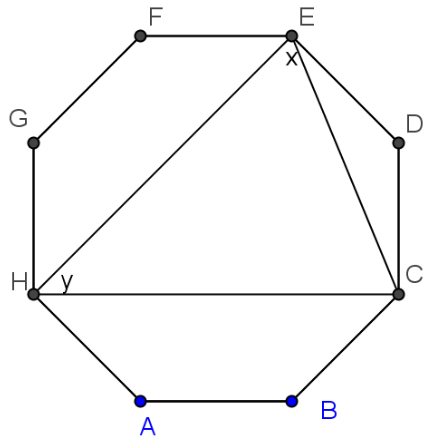


2. The ratio of an internal to an external angle of a regular polygon is 4:1. How many sides does it have?

3. Two congruent regular pentagons share vertex C. Points A, B, and C are collinear. Find *x*, *y*, and *z*.

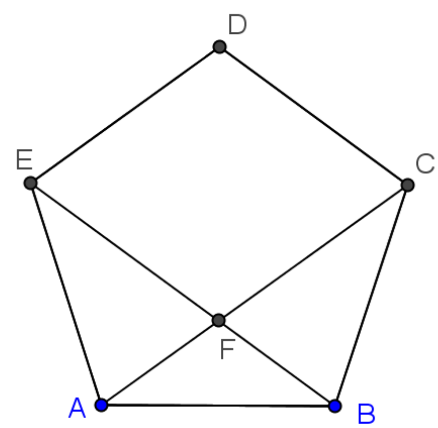


4. Find the measures of *x* and *y* in the regular octagon below. You may assume $\overline{GH} \perp \overline{CH}$.



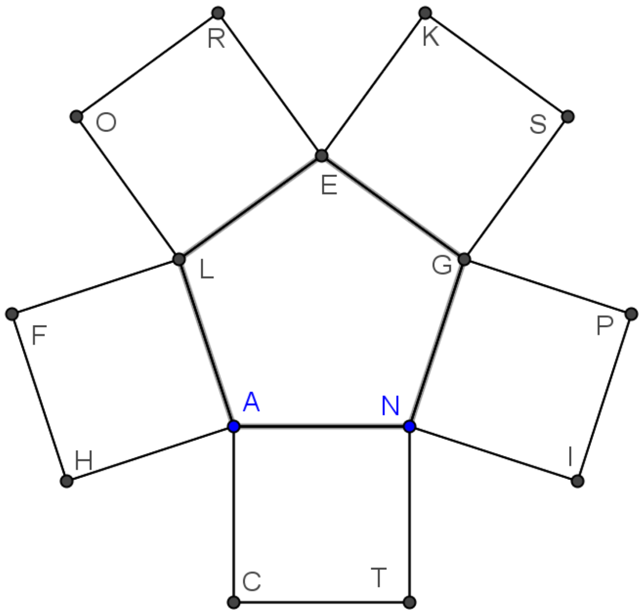
5. ABCDE is a regular pentagon.

a. Find the measure of angle BFC.



b. Explain why EFCD must be a rhombus.

6. Suppose that *ANGEL* is a regular pentagon, and that *CANT*, *HALF*, *ROLE*, *KEGS*, and *PING* are squares attached to the outside of the pentagon. Show that decagon *PITCHFORKS* is equiangular. Is this decagon equilateral? (Exeter)



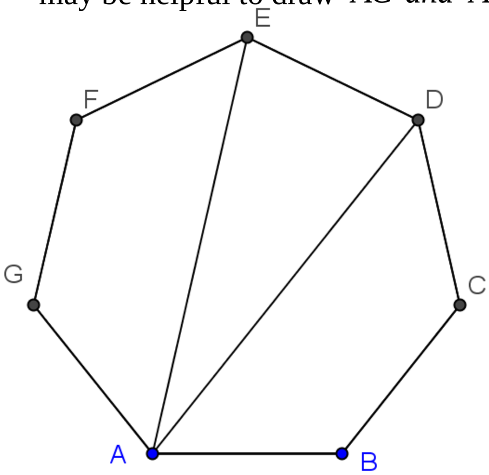
7. A cube has 8 vertices, 12 edges, and 6 square faces. A soccer ball (also known as a *buckyball* or *truncated icosahedron*) has 12 pentagonal faces and 20 hexagonal faces.

a. Without any calculations, do you think sum of angles at a vertex is equal to 360°, greater than 360°, or less than 360°. Explain briefly.

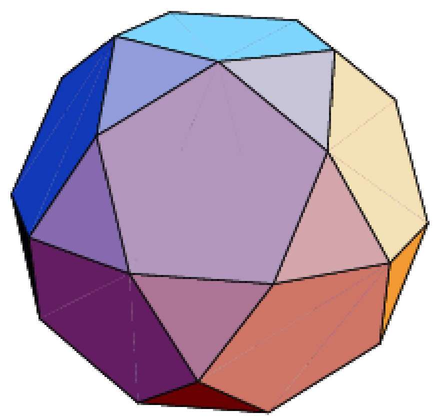
b. How many vertices does a soccer ball have?

c. How many edges does a soccer ball have?

8. A regular heptagon is shown below. Carefully explain why triangle ADE must be isosceles. Hint: it may be helpful to draw $\overline{AC}$ *and* $\overline{AF}$.



9. An icosidodecahedron (pictured below) is a solid composed of pentagonal and triangular faces. It has 12 pentagonal faces.

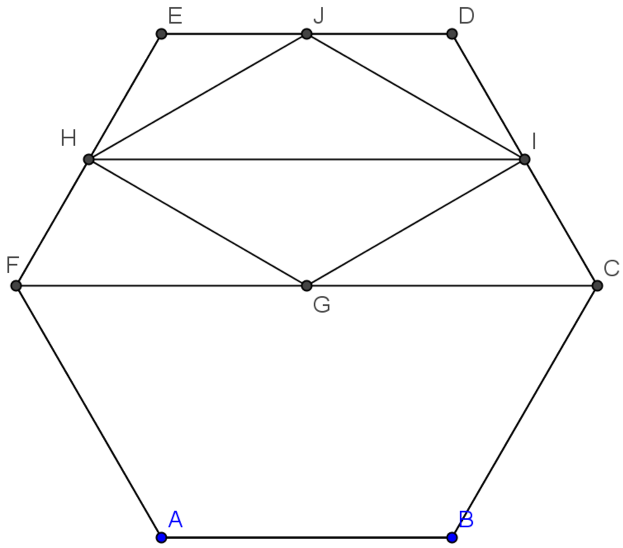


a. How many triangular faces does it have?

b. How many vertices?

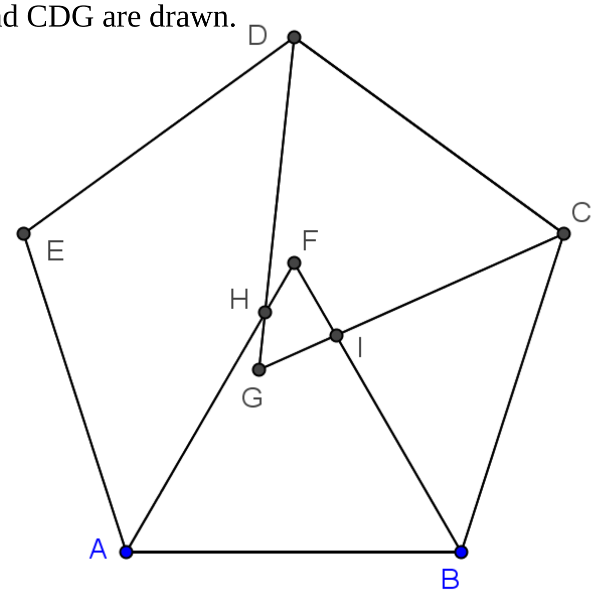
c. How many edges?

10. In the regular hexagon below, G is the center and H, I, and J are the midpoints of three sides. Is GHJI a rhombus? Explain.



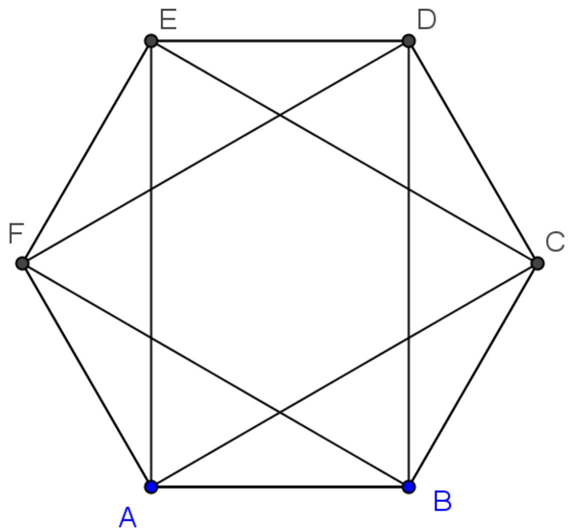
11. From regular pentagon ABCDE, equilateral triangles ABF and CDG are drawn.

a. Find the measures of the angles of quadrilateral FIGH.

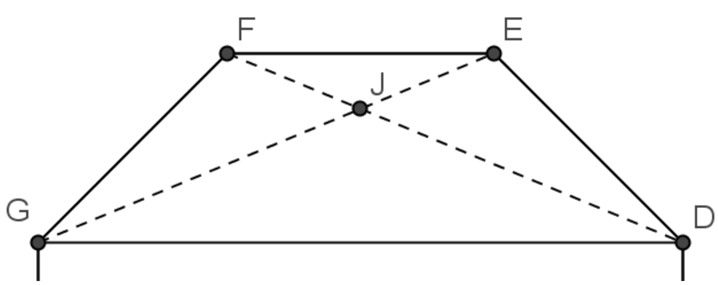


b. FIGH appears to be a kite. Is it? Justify your answer.

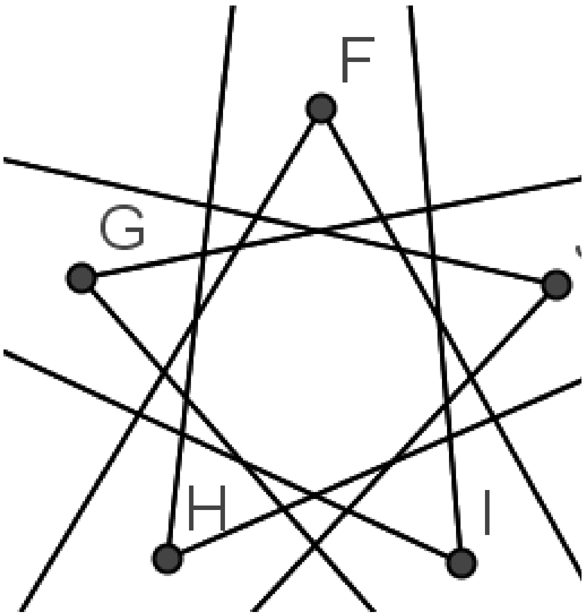
12. ABCDEF is a regular hexagon. Show that $\overline{DF}$ and $\overline{DB}$ trisect $\overline{EC}$.

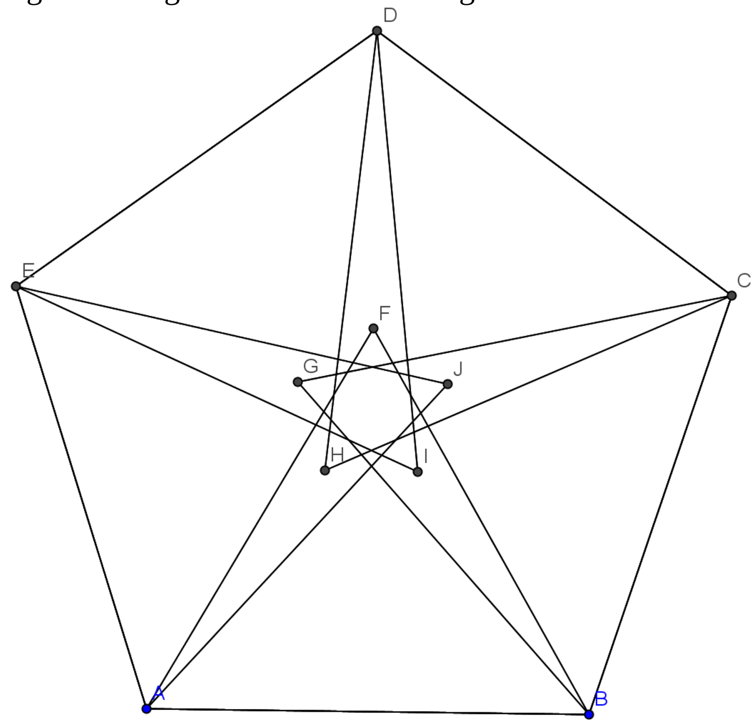


13 D, E, F, and G are vertices of a regular octagon and $\overline{DG}$ is a diagonal. It certainly appears that $\overline{DG} \parallel \overline{EF}$. Prove it! (no necessarily in a 2-column format)



14. Pentagon ABCDE on the right has equilateral triangles sharing each side. Is the decagon in the center regular? A close-up of the center is at the left!





Answers

1. FED and ABC are isosceles triangles; 120-30-30; so the other angles must be 90° 2. 10
3. x=36°; y=72°; z=72° 4. y=45° and x=67.5° 5a. 72° b. it is a parallelogram b/c ACD and CDE are supplementary… and thus opposite sides are equal… since CD=DE all sides are equal
6. look at triangle GSP; it is isos with G measuring 72° and the other angles 54°; so all angles of PITCHFORKS measure 144°. It is not equilateral since GSP is not equilateral and KS=GS.
7a. less than 360° since it "folds in"—if it were 360° then the "ball" would be flat! b. 60 c. 90
8. $\Delta ABC \cong \Delta AGF$ by SAS so $\overline{AC} \cong \overline{AF}$ and $\angle GFA \cong \angle BCA$ by CPCTC. Then we know that $\angle AFE \cong \angle ACD$ since subtract equals from (equal) vertex angles. Thus by SAS $\Delta ACD \cong \Delta AFE$ so $\overline{AD} \cong \overline{AE}$, thus ΔADE is isosceles. 9a. 20; b. 30 c. 60
10. Yes: Draw DF: GI and HJ are parallel to it and ½ its length by triangle midsegment thm, so GI is congruent to HJ. Draw CE. By same logic JI is congruent to GH. Since triangles HEJ and IDJ are congruent, JH is congruent to IJ so all four sides of GHJI are congruent
11a. I=84°; F=G=60°; H=156° b. yes; ΔIBC is isosceles so BI=CI and CG=BF so FI=GI.. draw AD and you can see that AHD is isosceles (D & H both measure 108-60-36) so DH=AH and DG=AF so GH=FH
13. GFE is 135 and FG=FE so FEG=FGE=22.5… and tri GFE = tri DEF so DFE=22.5 so FJE=135 so GJD = 135 FJ=JE (isos) and GE=DF so JG=JD by subtraction so JGD=JDG and both must be 22.5 so lines are parallel… there must be an easier way! 14. no

Unit 4 Handout #8: Practice Problems on Polygons and Special Quadrilaterals

<u>What's on the test?</u>
-Regular polygons: diagonals and angles
-Quadrilaterals and their properties
-Coordinate geometry: plotting quadrilaterals on the coordinate plane
-Proofs: two-column and coordinate proofs
-Problem-solving and algebra: using quadrilateral properties, polygon angles, etc.

1. Given a regular decagon (10-sides):

a. What is each vertex angle?

b. What is each exterior angle?

c. How many diagonals does it have?

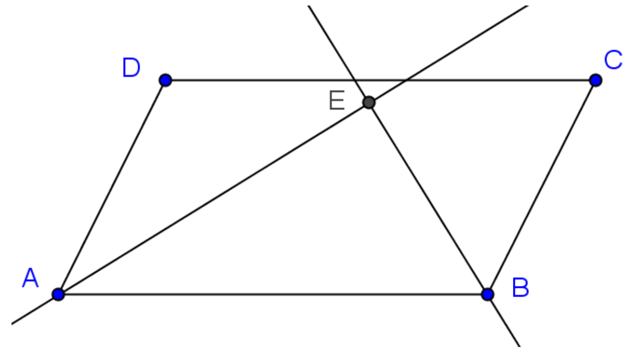
2. Draw each of the following, if it is possible!

a. A hexagon that is equiangular and not equilateral.

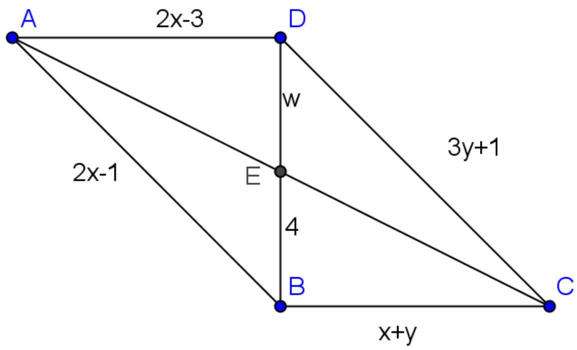
b. A pentagon that is equiangular and not equilateral.

c. A quadrilateral that is equilateral but not equiangular.

3. In parallelogram ABCD, the angle bisectors of A and B are drawn, meeting at E. Must AEB be a right angle? Explain briefly.



4. Find the values of *w*, *x*, and *y* given that ABCD is a parallelogram. Then determine if $\overline{DB} \perp \overline{BC}$.

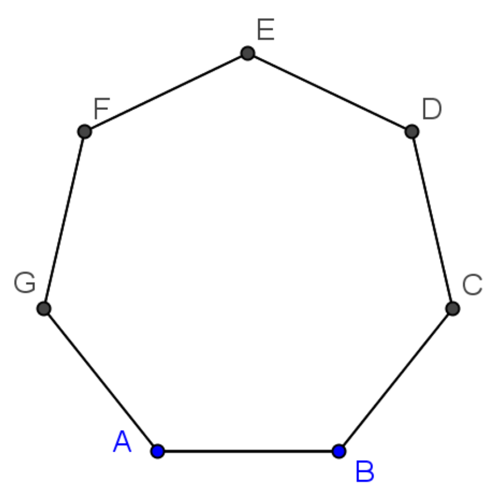


5. Given the regular heptagon below, do the following:

a. Draw the perpendicular bisectors (approximate) of sides $\overline{AB}$ and $\overline{BC}$. Call their intersection point K.

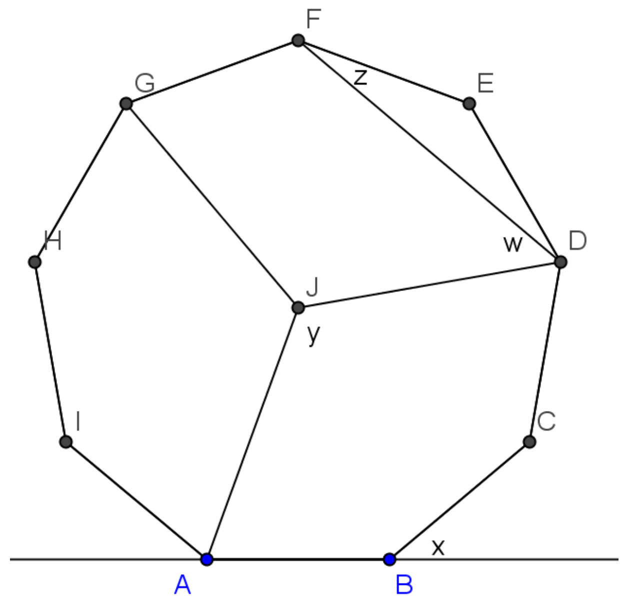
b. A quadrilateral's vertices are B, K, and the midpoints of sides $\overline{AB}$ and $\overline{BC}$. Explain why it must be a kite.

c. Is K equidistant from all vertices of the heptagon? Explain why or why not.



6. J is the center of the regular nonagon. It is thus equidistant from all vertices.

a. Explain why $\overline{AJ}$ must bisect $\angle BAI$.



b. Find the measures of angles *x*, *y*, *z*, and *w*.

c. Explain why DFGJ cannot be a parallelogram.

7. True or false:

a. SSSS is a congruence relationship for quadrilaterals.

b. The diagonals of a kite bisect each other.

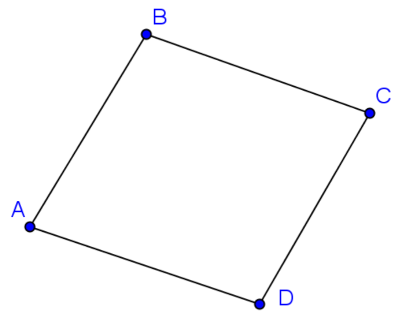
c. The diagonals of a parallelogram must bisect each other.

d. The diagonals of a rhombus divide the rhombus into four congruent right triangles.

8. Prove that if the diagonals of a quadrilateral bisect each other, then the quadrilateral must be a parallelogram. No using properties; stick to the definition!

9. Prove that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and one half its length. (coordinate proof recommended!)

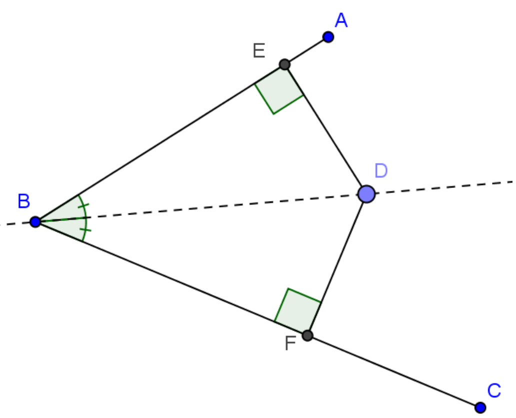
10. Prove that if a quadrilateral has four congruent sides then it must be a parallelogram.



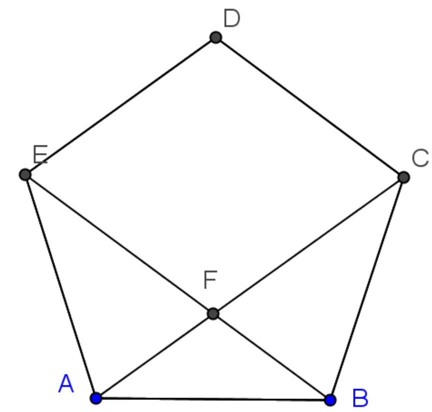
11. Prove that if a diagonal of a parallelogram bisects a vertex angle then the parallelogram must be a rhombus. Use the definitions only; no properties!

12. A regular polygon has an exterior angle that is one 11th of the interior angle. How many diagonals does this polygon have?

13. Segment $\overline{BD}$ bisects angle ABC below. From point D on the bisector, perpendicular segments to the sides of the angle ABC are drawn. Show that BEDF is a kite.



14. Given the regular pentagon at the right, do the following:



a. Find the measure of $\angle BAF$.

b. Find the measure of $\angle EFA$.

c. Is $CDEF$ a parallelogram? Explain.

d. If your answer to *c* was yes, then determine whether *CDEF* is a rhombus. Explain.

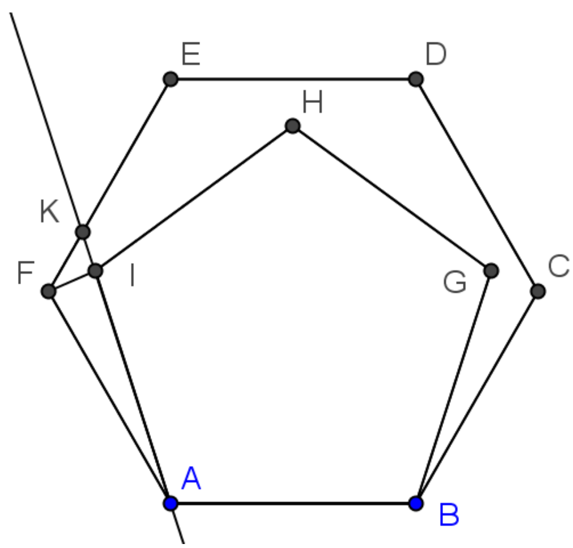
15. A regular hexagon and a regular pentagon share side $\overline{AB}$. Side $\overline{AI}$ is extended and meets $\overline{EF}$ at K. Segment $\overline{IF}$ is drawn. Do not assume that F, I, and H are collinear.

a. Find $\angle FAI$.

b. Find $\angle FKI$.

c. Find $\angle AFI$. (hint: AB is a side of both polygons)

d. Are F, I, and H collinear? Justify.



16. Show that connecting the midpoints of adjacent sides of a rectangle forms a rhombus. And, if you then connect the midpoints of adjacent sides of the rhombus, what shape is formed?

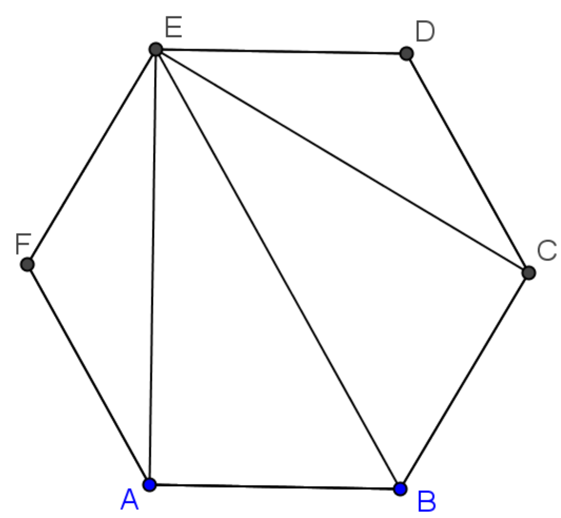
17. ABCDEF is a regular hexagon.

a. Show that $\overline{EA} \perp \overline{AB}$.

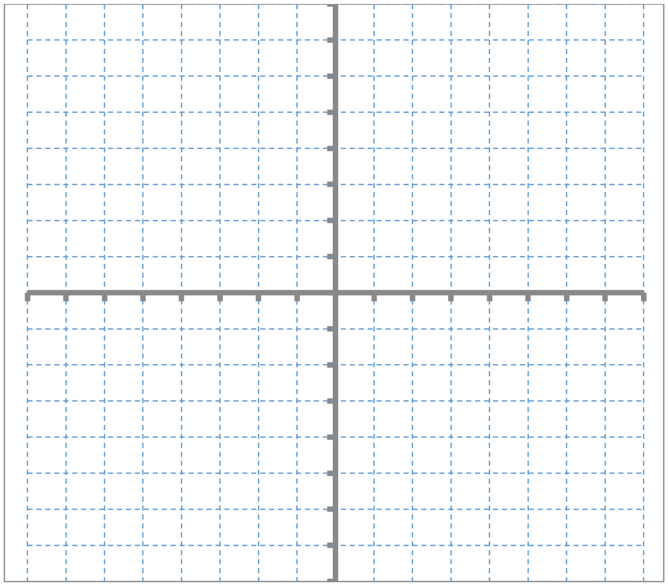
b. Explain why ABCE must be a kite.

c. Explain why $\overline{EB}$ must bisect angle FED.

d. Explain why BCDE must be an isosceles trapezoid.



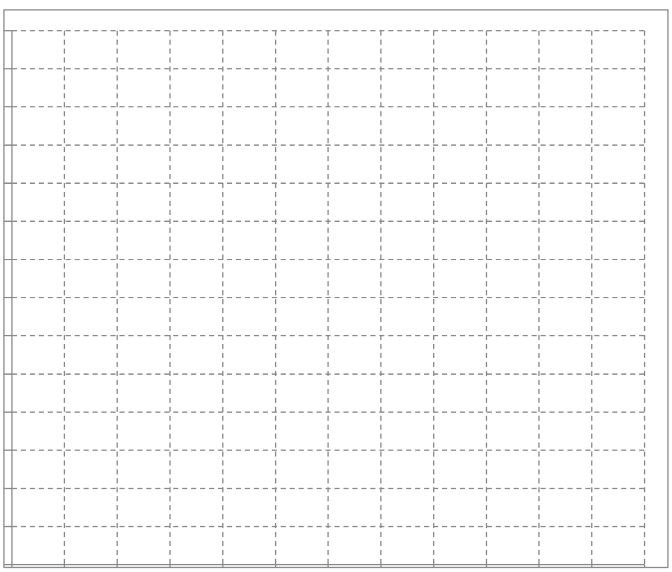
18. Show that A(0,0), B(8,1), and C(4,8) can be three vertices of a rhombus. Now find the coordinates of the fourth vertex.



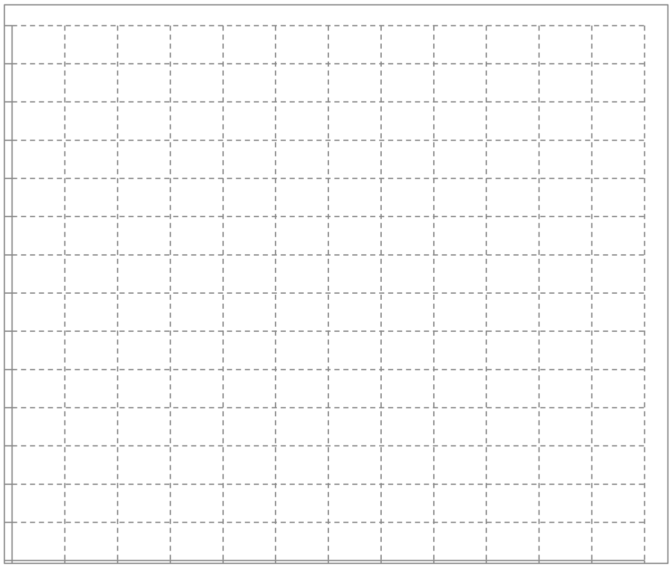
19. Put a generic parallelogram on a coordinate plane, using as few letters as possible (three!)

a. Draw the line segment joining the midpoint of the top to the lower-right corner. Now draw the line segment joining the midpoint of the bottom to the upper-left corner.

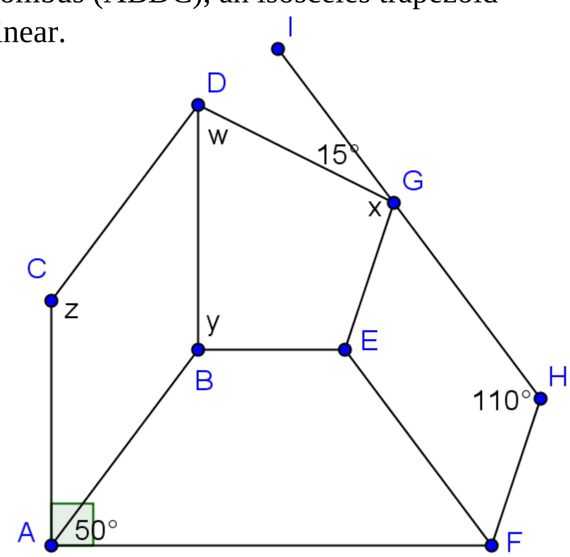
b. Show that these two segments are parallel and congruent.



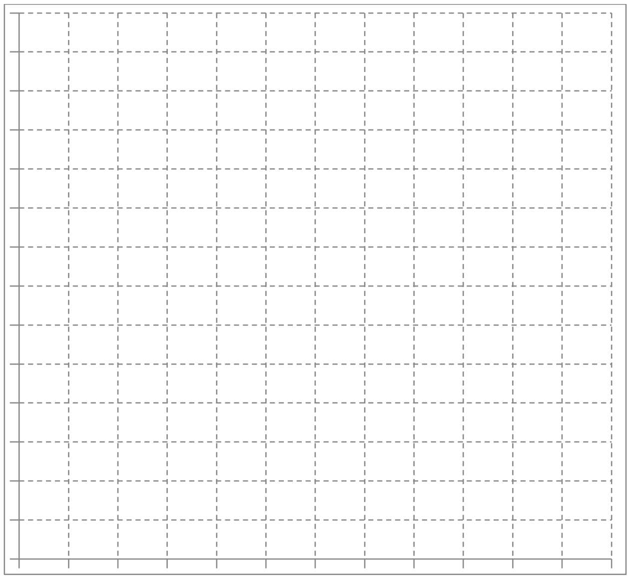
20. Put a generic isosceles trapezoid on a coordinate plane, using as few letters as possible (three!) Show that the segments joining the ends of the smaller base to the midpoint of the longer base are congruent.



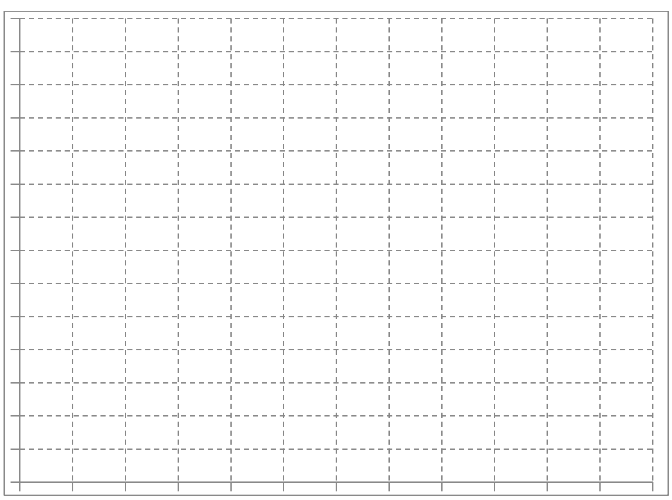
21. Find the values of *w*, *x*, *y*, and *z* below given that there is a rhombus (ABDC), an isosceles trapezoid (ABEF), and a parallelogram (EFHG). Also, H, G, and I are collinear.



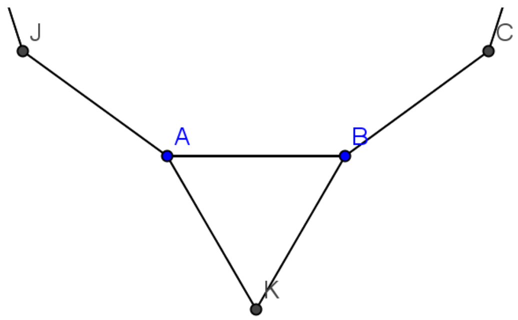
22. Given that three vertices of kite ABCD are A(3,0), B(10,4), and C(8,10). Find the fourth.



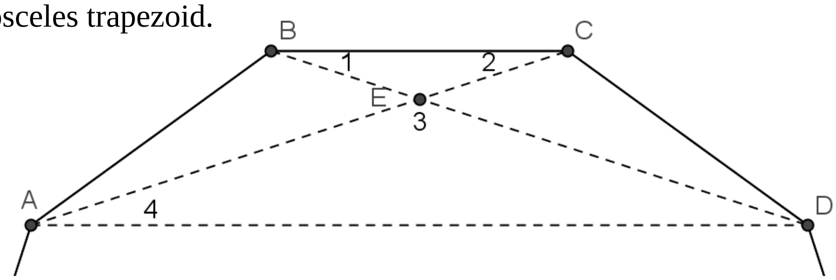
23. Put a generic right triangle on the coordinate plane and that the midpoint of the hypotenuse is equidistant from the three vertices.



24. In the diagram below, JABC is part of a regular decagon (10 sides) and ABK is an equilateral triangle. Could JAK be part of a regular polygon? If so, how many sides does it have?



25. Prove that in any regular polygon with more than four sides, four consecutive vertices form an isosceles trapezoid.



Answers

1a. 144° b. 36° c. 35

2a. stretch a pair of opposite sides equally b. shift 2 sides in parallel c. rhombus

3. angles DAB and CBA add to 180° so EAB and EBA add to 90°, so AEB must be 90°

4. x=7, y=4, and w=4… can't be because Pythagorean Thm does not hold in ΔCBD

5b. KB splits the quad into 2 right Δ's congruent by HL; so both pairs of adjacent sides are congruent
 c. KC=KB by def of perp bisector; then ΔKAB ≅ ΔKBC by SSS then $\Delta KBC \cong \Delta KCD$ by SAS and can keep working way around showing each triangle is congruent to the previous one..

6a. $\Delta JBA \cong \Delta JIA$ by SSS b. x=40°; y=120°; z=20°; w=50° c. $\angle GJD$ =120° so not supplem to $\angle JDF$

7a. F b. F c. T d. T

8. Top and bottom Δ's are congr by SAS; CPCTC → angles cong that are alt interior to both sets of sides

9. Use coordinate with (0,0), (2a,0) and (2b,2c) 12. 24 sides so 252 diags

14a. 36° b. 72° c. EFC=108° and FCD=FED=72° so consecutive angles are supplement so sides are //
d. EA=EF b/c ΔAAEF is isos; same with EC and CB; so all four sides are congruent

15a. 12° b. 48° c. 84° (b/c ΔAFI is isosceles) d. no: FIA=84° & AIH=108° so they don't sum to 180°

16. 4 sides congruent by SAS; a rectangle (call one vertex angle 2x; adj one is 180-2x; isos triangles so quadrilateral must have right angles…)

17a. F=120° and ΔAFE is isosceles so FAE=30°; but FAB=120° so EAB=90°
b. AB & BC are congruent since regular hexagon; $\Delta ABE \cong \Delta CBE$ by HL so AE=EC and has 2 prs of congruent adjacent sides
c. angles FEA & CED are congruent (both 30°) and AEB & CEB are cong by CPCTC so FEB=DEB
d. since EB bisects angle FED, BED=60°. D = 120° so they are supple and BE//CD; also ED=BC

18. AB=BC=$\sqrt{65}$; 4th vertex is (-4,7)

19. A(0,0), B(2a,0), C(2b,2c), D(2a+2b,2c) b. top midpoint is (a+2b,2c) to lower-right has slope of $\frac{2c}{2b-a}$ and distance of $\sqrt{(2b-a)^2+(2c)^2}$ bottom midpoint is (a,0) and to upper-left has same slope and same distance; thus congruent and parallel

20. A(2a,0), B(-2a,0), C(2b,2c), and D(-2b,2c) ends of smaller base are C and D; midpoint of longer base is (0,0); distance from each to (0,0) is $\sqrt{4b^2+4c^2}$

21. x=95°; y=90°; z=140°; w=55° 22. diagonals meet at (6,6) and fourth vertex is (2,8)

23. A(0,0), B(2a,0), and C(0,2b)… midpoint of hypotenuse is (a,b) is $\sqrt{a^2+b^2}$ from each vertex

24. yes, a fifteen-a-gon!

25. ΔABC = ΔBCD by SAS so angles 1 and 2 are congruent…. So angle BCE is equal to a vertex angle, and so is angle 3. BD is congruent to AC by CPCTC and BE is congruent to EC by isos triangle theorem. Thus by subtraction properry, DE = AE so angles 4 and EDA are congruent (isos triangle).. since 3 is a vertex angle, 4 must be ½ of its supplement, which is the same as angle 2. Therefore BC // AD by alt interior angles… so a trap and AB = CD since regular polygon…

Unit 5 Handout 1: Review of Ratios and Proportions

In the similarity unit, we will frequently use ratios. To solve equations with one fraction on each side, you can either cross-multiply or multiply both sides of the equation by the lowest common denominator of the fractions.

Example #1: Solve each equation for *x*.

a. $\frac{5}{x}=\frac{7}{9}$ **b.** $\frac{x}{3}=\frac{x+2}{7}$ **c.** $\frac{x+4}{3}=\frac{7}{x}$

a. Cross multiply to get $7x = 45$ *and* $x=\frac{45}{7}$

b. Cross-multiply to get $7x = 3x + 6$. Thus $4x = 6$ *so* $x = 1.5$.

c. Cross-multiply to get $x^2 + 4x = 21$. To solve this quadratic equation by factoring, rearrange to make one side equal to zero and then factor: $x^2 + 4x - 21 = 0$ *so* $(x+7)(x-3) = 0$. Now set each expression to zero, so $x + 7 = 0$ *or* $x - 3 = 0$ *thus* $x = -7$ *or* 3.

Example #2: The sides of a triangle are in the ratio of 3:5:7.

a. If the longest side is 35 then find the shortest side.

b. Instead, if the perimeter is 360, then find the longest side.

a. The ratio of shortest side to longest side is 3:7. Set up a proportion, where short:long equals short:long. $\frac{3}{7}=\frac{x}{35}$. Then solve by cross-multiplying to get 7*x*=105 and *x*=15.

b. Let the sides measure 3*x*, 5*x*, and 7*x*. The perimeter is the sum of the side lengths, so $3x + 5x + 7x = 360$. Thus 15*x*=360 and *x*=24. The longest side is 7*x*, so it is 7(24)=168.

Example #3: On a scale model of a city, one cm corresponds to 3 meters. If a building is 150 meters high, how tall is its model?

Solution: The ratio of model to real is 1:300, since there are 300 cm in one meter. Set up a proportion, where model:real = model:real. So $\frac{1}{300}=\frac{x}{15000}$. Solve it by cross-multiplying to get *x*=50 cm tall.

1. Solve the following for x:

a. $\frac{3}{x} = \frac{12}{20}$

b. $\frac{5}{x} = \frac{3}{7}$

c. $\frac{x}{5} = \frac{4}{9}$

d. $\frac{x}{5} = \frac{x+3}{10}$

e. $\frac{3}{x} = \frac{7}{x+2}$

2. Solve the following for x. You will need to either solve using square roots or factoring.

a. $\frac{4}{x} = \frac{x}{9}$

b. $\frac{x}{3} = \frac{8}{10-x}$

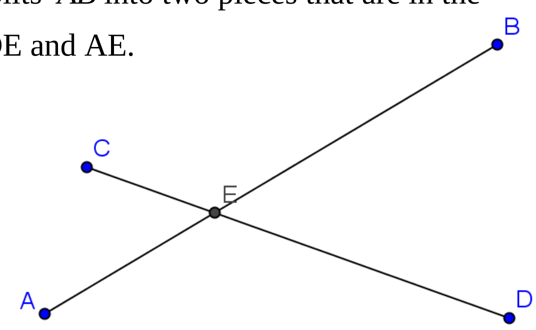
c. $\frac{x}{10} = \frac{8}{18-x}$

3. Two numbers sum to 80 and they are in the ratio of 3:2. What is the smaller one?

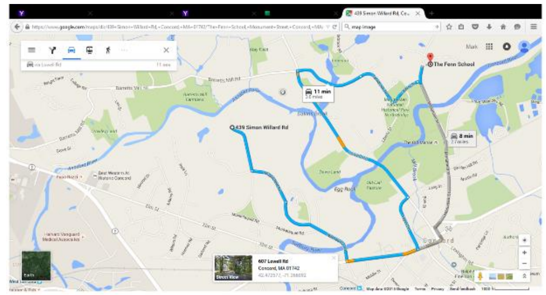
4. The ratio of an angle to its supplement is 7:2. What is the angle?

5. The ratio of the length of a rectangle to its width is 5:3 and its perimeter is 80. Find its area.

6. $\overline{AB}$ splits $\overline{CD}$ into two pieces that are in the ratio 2:5. $\overline{CD}$ splits $\overline{AB}$ into two pieces that are in the ratio 5:12. If $\overline{AB}$ is 15 units long and $\overline{CD}$ is 13 units long, find DE and AE.



7. On a map, once inch corresponds to 800 feet. If two points are 5.25 inches apart on the map, then what is the distance between them?

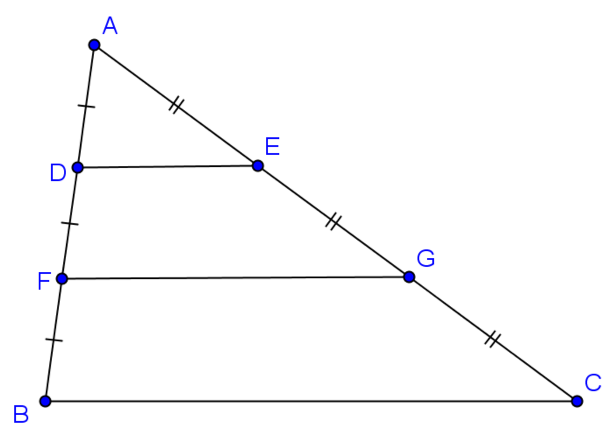


8. The remote-controlled Mini Cooper model is on a scale of 1:15 with the actual car.

a. If the model is 10.5 inches long, then how long is the car?

b. If the steering wheel of the actual car has a diameter of 18 inches, then what is the diameter of the model car's steering wheel?

9. Find the ratios below given that $EC = 3 \cdot BF$



a. $\dfrac{AD}{DB}$

b. $\dfrac{BD}{AB}$

c. $\dfrac{AG}{AE}$

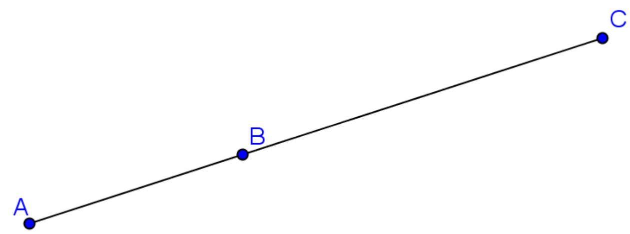
d. $\dfrac{EG}{AC}$

e. $\dfrac{DF}{EG}$

f. $\dfrac{AC}{AD}$

10. In class, the teacher described a triangle with a perimeter of 462 and said the ratios of its side lengths and its angles were 6:10:17 and 3:5:6. But Kelly forgot which one was the ratio of side lengths and which was the ratio of the angles! Find the measures of the sides and the angles of the triangle.

11. In the diagram below, B is on $\overline{AC}$ and $\frac{AB}{AC} = \frac{3}{8}$



a. If AB=4 then find BC and AC.

b. Instead, if CB=7 then find AC.

c. Instead, if AC=19 then find AB.

d. Instead, if $\overline{BC}$ is three units longer than $\overline{AB}$, then find the length of $\overline{AC}$.

12. Show algebraically that if $\frac{a}{b}=\frac{c}{d}$ then $\frac{a}{b}=\frac{a+c}{b+d}$.

Answers

1a. 5 b. 35/3 c. 20/9 d. 3 e. 1.5 2a. 6 or -6 b. 4 or 6 c. 8 or 10

3. 32 4. 140° 5. 375 6.DE=65/7 and AE=75/17 7. 4200 feet

8a. 157.5 inches which is 13 feet 1.5 inches b. 1.2 inches

9a. 1:2 b. 2:3 c. 2:1 d. 1:3 e. 2:3 f. 9:2

10. Sides' ratio is 3:5:6 since 6:10:17 can't form a triangle so sides are 99, 165, and 198 and angles are about 33°, 54°, and 93°.

11a. BC=20/3 and AC=32/3 b. 56/5 c. 57/8 d. 12

12. cross-multiply both, and cancel the ab terms in the 2nd expression.

Unit 5 Handout #2: Identifying and Establishing Similarity

Similar figures have the same shape but may have different sizes. Two polygons are similar if and only if:

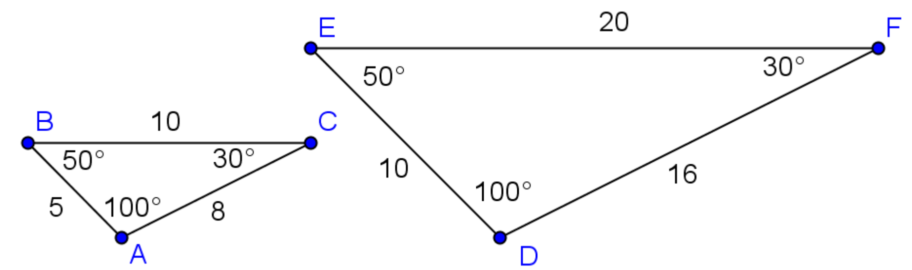
a. Corresponding angles are congruent.

b. Corresponding sides are proportional.

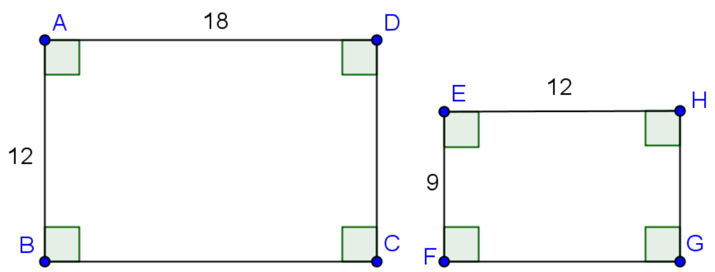
Note: by this definition, congruent figures are similar!

Example #1: In each part below, determine whether the two polygons are similar or not.

a.



b.



a. Corresponding angles are congruent: $\angle A \cong \angle D,\ \angle B \cong \angle E,\ and\ \angle C \cong \angle F$.

Corresponding sides are proportional: $\frac{5}{10} = \frac{10}{20} = \frac{8}{16}$.

We can conclude that ΔABC~ΔDEF ("~" means "is similar to").

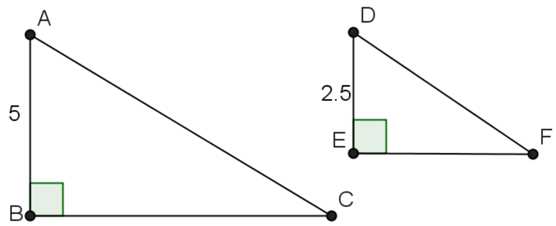
b. Corresponding angles are congruent. But corresponding sides are not proportional, since $\frac{12}{9} \neq \frac{18}{12}$.

Thus the rectangles are not similar.

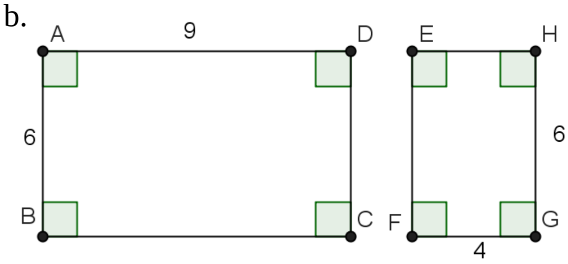
Similar Triangles: Triangles can be shown to be similar if corresponding angles are congruent. Since the sum of the angles of any triangle is 180°, we only need to show that two corresponding angles are congruent (***called "AA~"***). Triangles can also be shown to be similar if the lengths of corresponding sides are proportional (***called "SSS~"***). As with congruency, these similarity "short-cuts" do not apply to polygons with more sides. For example, two rectangles with congruent angles are not necessarily similar.

1. Must the pairs of objects in each part be similar? Explain briefly.

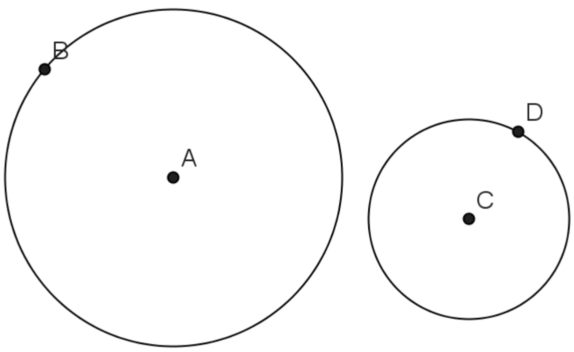
a.



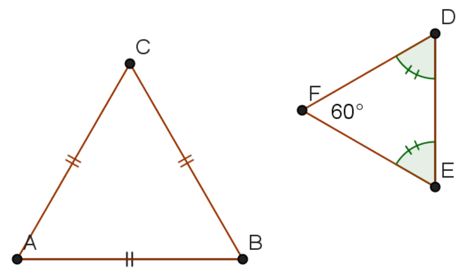
b.



c.



d.



2. True or false. Explain your answers briefly.

a. All regular pentagons are similar.

b. All right triangles are similar.

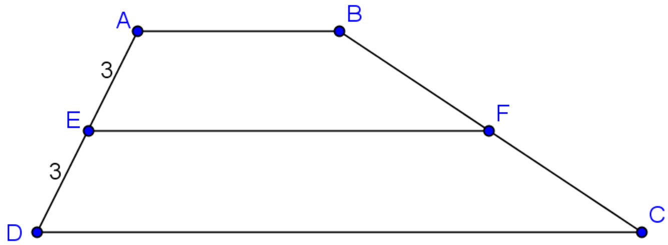
c. All squares are similar.

3. Each side of quadrilateral EFGH is twice as long as the corresponding side of quadrilateral ABCD. Must the quadrilaterals be similar? Explain.

4. Is the statement "if congruent then similar" true? Write its inverse, converse, and contrapositive and determine whether each must always be true.

5. Based on this photo of an adult labrador with her puppy, would you say that puppies are similar to adult dogs, using the geometric definition of similarity? If not, what body parts seem inconsistent with the definition of similarity?

6. In the diagram below, $\overline{AB} // \overline{CD} // \overline{EF}$. There are three trapezoids (ABCD, ABFE, and EFCD). Are any two similar to each other? Explain.



7. Is similarity transitive? If ΔA~ΔB and ΔB~ΔC does that mean that ΔA~ΔC? Explain.

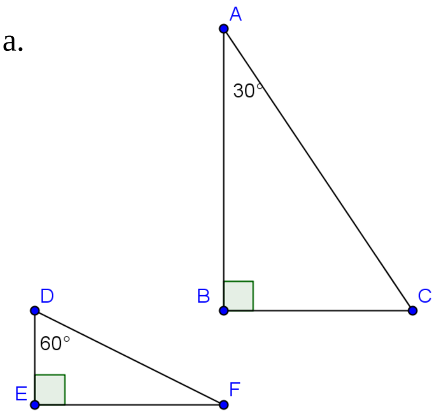
8. Explain what "AA similarity" means, and also what "SSS similarity" means, in the context of similar triangles.

9a. Two isosceles triangles each having an angle of 80° must be similar: true or false?

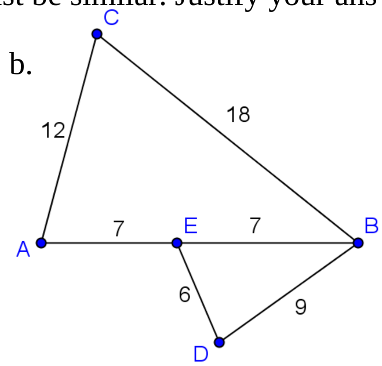
b. Two isosceles triangles each having an angle of 100° must be similar: true or false?

10. In each part below, determine whether any triangles must be similar. Justify your answer.

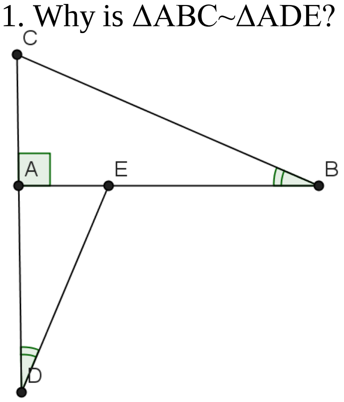
a.



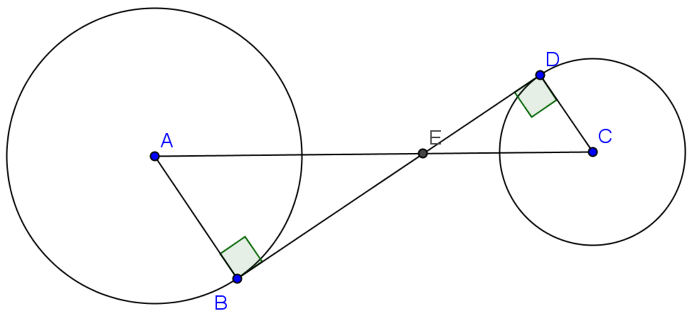
b.



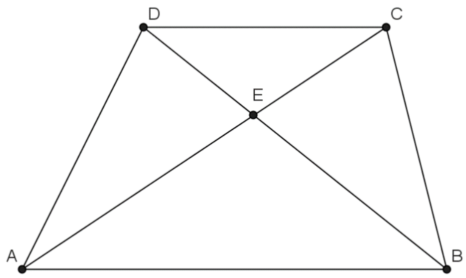
11. Why is ΔABC~ΔADE?



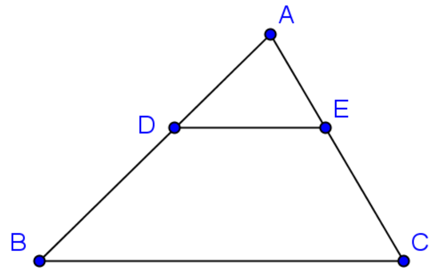
12. Is ΔABE~ΔCDE? Why or why not?



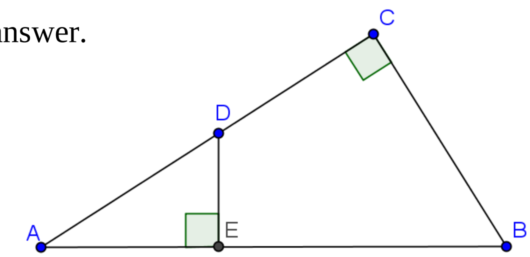
13. Are there any similar triangles in the trapezoid below? If so, name all pairs of similar triangles that you find.



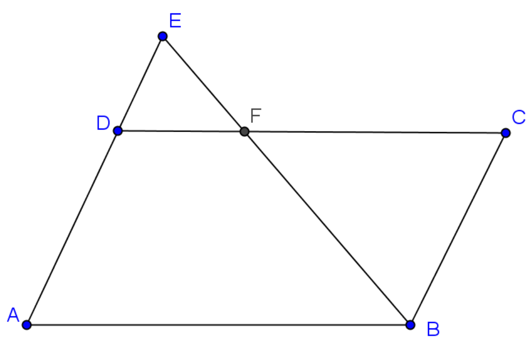
14. Given that $\overline{BC} \parallel \overline{DE}$, explain why $\Delta ABC \sim \Delta ADE$.



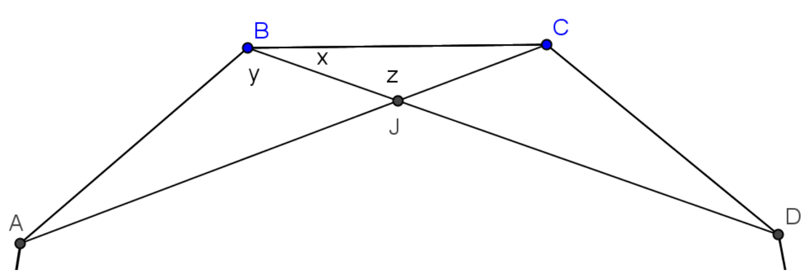
15. Identify the similar triangles in the diagram below. Justify your answer.



16. In the parallelogram below, side $\overline{AD}$ is extended to E. Find two different triangles similar to ΔDEF.



17. ABCD is part of a regular nine-sided polygon (nonagon). Is ΔJBC~ΔCBD? Explain.



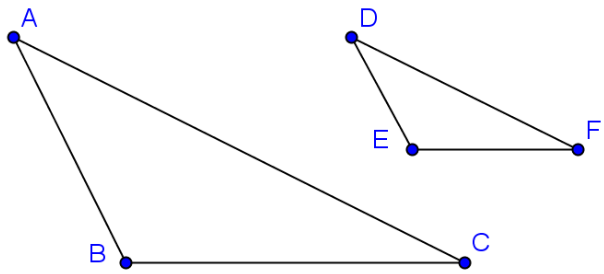
18. In the diagram below, ΔABC~ΔDEF. What must be true? Circle all that apply.

a. $\frac{BC}{AB}=\frac{EF}{DE}$

b. $\frac{BC}{AC}=\frac{EF}{ED}$

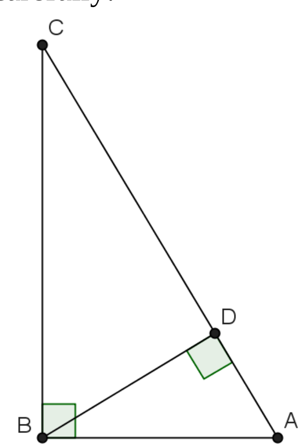
c. $\frac{AC}{AB}=\frac{DF}{DE}$

d. $\frac{DE}{AB}=\frac{perim\ of\ \Delta DEF}{perim\ of\ \Delta ABC}$

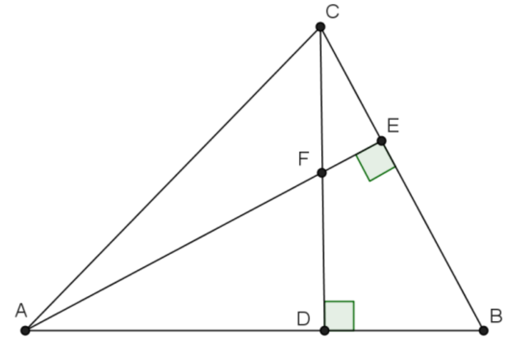


19. Given ΔABC~ΔDEF. The measure of angle A is twice that of angle B. The measure of angle F is 20° greater than the measure of angle E. What is the measure of angle C?

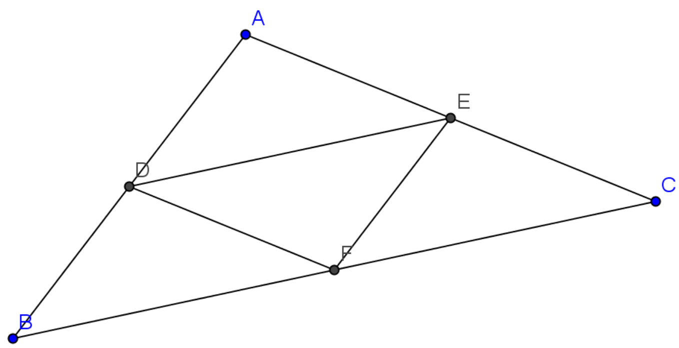
20. Find three triangles similar to each other in the diagram to the right. Name them carefully!



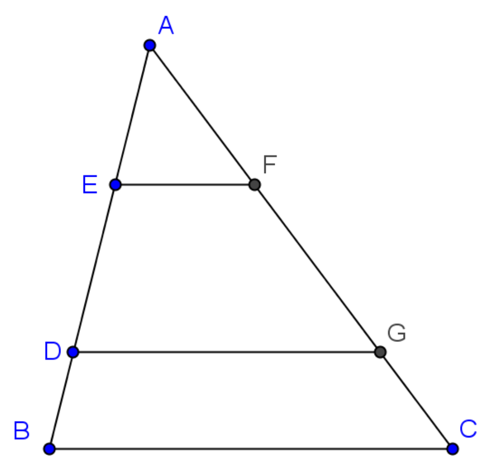
21. Given that $\overline{CD}$ *and* $\overline{AE}$ are altitudes of ΔABC, identify four similar triangles. Justify your answer.



22. In scalene ΔABC, points D, E, and F are the midpoints of its sides. Is ΔABC~ΔFED? Justify.



23. Points D, E, F, and G are on the sides of ΔABC such that $\overline{EF} \mathbin{//} \overline{DG} \mathbin{//} \overline{BC}$. Which of the following must be true. Circle all that apply,



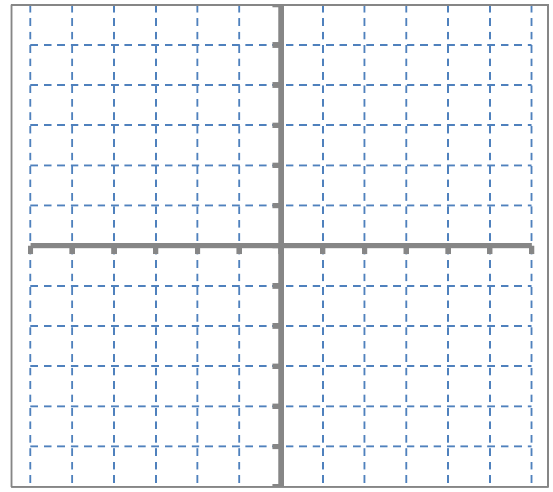
a. $\dfrac{AE}{AB} = \dfrac{EF}{BC}$

b. $\dfrac{AE}{ED} = \dfrac{EF}{DG}$

c. $\dfrac{AE}{AF} = \dfrac{AB}{AC}$

d. $\dfrac{ED}{DB} = \dfrac{DG}{BC}$

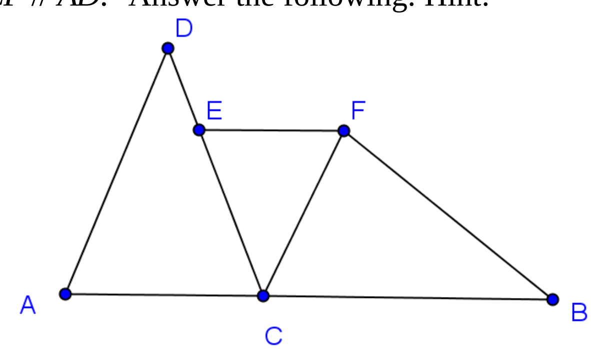
24. Given points A(-4,5), B(1,-5), C(6,-2.5), and D(-1,-1).



a. Point E is on $\overline{AC}$ such that ΔABC~ΔADE. What are E's coordinates?

b. Is there anywhere else E can be such that ΔABC~ΔADE? Explain.

25. In the diagram below, AD=CD. EC=FC and BC=BF. Also $\overline{CF} \parallel \overline{AD}$. Answer the following: Hint: call angle D "2*x*" and write other angles in terms of *x*.

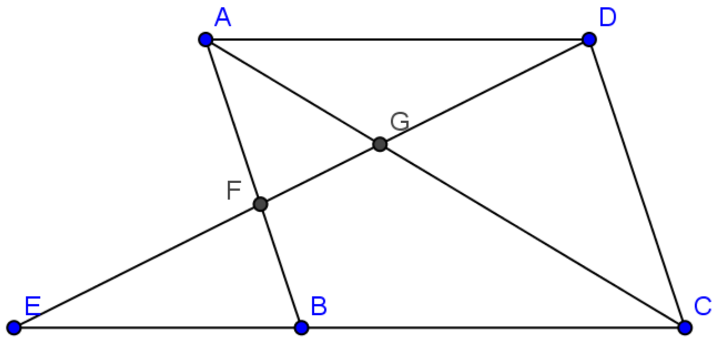


a. Why must ΔDAC~ΔBFC?

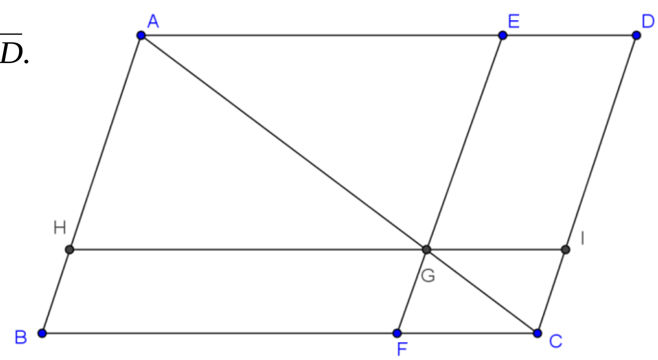
b. Why must ΔDAC~ΔCEF?

c. Must $\overline{EF} \parallel \overline{AB}$?

26. ABCD is a parallelogram, with side $\overline{BC}$ extended to point E. Name three pairs of similar triangles plus one trio of similar triangles.



27. Given parallelogram ABCD with $\overline{EF} \parallel \overline{CD}$ *and* $\overline{HI} \parallel \overline{AD}$.

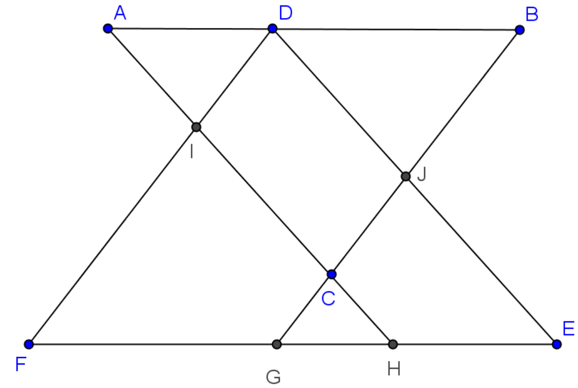


a. List all triangles similar to ΔABC.

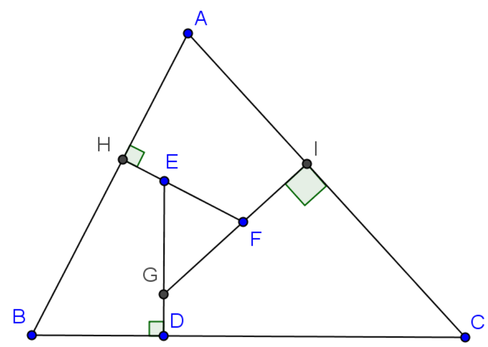
b. Must it be the case that GFCI is similar to ABCD?

c. If it turns out that quadrilateral AHGE is similar to ABCD, then must GFCI also be similar to ABCD?

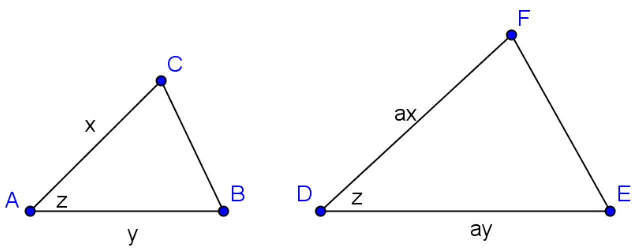
28. Given that, in the diagram below, $\overline{AB} \parallel \overline{EF}$, $\overline{DF} \parallel \overline{BG}$, *and* $\overline{AH} \parallel \overline{DE}$, find all triangles similar to ΔCHG. Be sure to name them correctly.



29. ΔEGF is similar to what other triangle? Why?

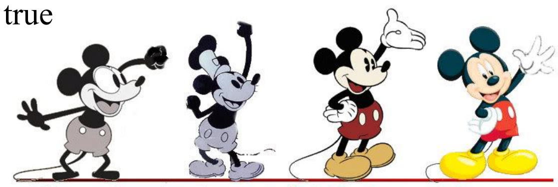


30. SAS similarity: Triangle DEF has two sides in proportion to the corresponding sides of ΔABC, and the angle between them is congruent to the corresponding angle in ΔABC. Explain why ΔABC~ΔDEF.



Answers

1a. no; other sides could be anything b. yes; angles are same and sides are proportional
c. yes, same shape d. yes, same shape 2a. true b. false c. true
3. no, since SSSS does not guarantee quadrilateral congruence… one may be convex and the other not.
4. yes; inverse → "if not congruent then not similar" not necessarily true
Converse → "if similar then congruent" not necessarily true
Contrapositive → "if not similar then not congruent" must be true

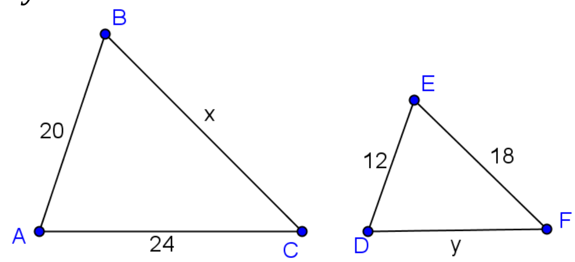

The Evolution of Mickey Mouse

5. Not similar because they are not proportional. For example, the adult dog appears to be more than twice the height of the puppy, but its ears are far less than twice as long. Juveniles in most species have evolved to be cute by having heads, faces, hands, and feet proportionally larger than adults of the same species. Walt Disney used this to change Mickey Mouse long ago to make him cuter by making him look more baby-like. (Images above are copyright by The Walt Disney Company.)
6. ABFE is not similar to ABCD since they have the same base AB but are not congruent
ABFE is not similar to EFCD because they have the same left side but are not congruent
ABCD is not similar to FCDE because they have the same base CD but are not congruent.
7. yes; same shape 8. AA means two triangles with two corresponding angles congruent must be similar since then the third angles are also congruent; SSS~ means two triangles with all sides proportional must be similar since they are the same shape.
9a. no; they can be 80-80-20 or 80-50-50 b. yes; they must be 100-40-40 so AA~ applies
10a. yes; both are 30-60-90 triangles b. yes, sides of smaller one are ½ the length of sides of larger one
11. by AA similarity 12. Yes since both have right angles and vertical angles around E are congruent
13. ΔDCE~ΔBAE since // bases give congruent alternate interior angles
14. by AA since $\angle ADE \cong \angle ABC$ and $\angle AED \cong \angle ACB$
15. ΔACB~ΔAED by AA~ 16. ΔDEF~ΔCBF~ΔAEB 17. angles ABC and BCD are each 140° so x=angle BCJ=20° and z=140° so yes, both are 140-20-20 18. a, c, and d are true; b is not
19. 60°; let A=D=x and B=E=y F=C=180-x-y so x=2y and (180-x-y)=y+20 y=40 and x=80
20. ΔCBA~ΔCDB~ΔBDA 21. ΔADF~ΔAEB~ΔCEF~ΔCDB
22. Yes, since all four small triangles are congruent and ΔADE~ΔABC 23. a and c are true
24a. AD/AB=3/5 so AE/AC=3/5 and E is (2,0.5) b. (-4,-2.5) would seem to work….
25a. by AA; since $\angle DAC \cong FCB$ as they are corresp angles
b. since $\angle ECF + 2 * \angle FCB = 180$ *and* $\angle ECF + 2 * \angle CEF = 180$, $\angle CEF \cong FCB$
c. yes, since $\angle CEF \cong DCA$ and they are alternative interior angles using transversal CD
26: ΔEBF~ΔECD~ΔDAF, ΔAGD~ΔCGE, ΔDGC~ΔFGA, ΔADC~ΔCBA
27a. ΔAHG, ΔGFC, ΔCIG, ΔCDA, and ΔGEA b. yes c. yes
28. ΔJEG, ΔIHF, ΔDEF, ΔCAB, ΔIAD, and ΔJDB. 29. ΔBCA.. sum of angles in quad is 360..
30. drop altitudes from C and F to AB and DE. The triangles to the left of these altitudes must be similar by AA (both have an angle of z and a right angle). So their sides are in proportion and thus the altitudes are in proportion. The triangles to the right of these altitudes are right triangles with legs proportional. So by the Pythagorean Thm the hypotenuses are also in the same proportion. Therefore we have SSS~.

Unit 5 Handout #3: Applying Similar Triangles

Once triangles have been shown to be similar, we can use that fact to find unknown parts. Most commonly, we will want to find side lengths. But we may use side lengths to establish similarity, which then tells us that corresponding angles must be congruent.

Example #1: Given that ΔABC~ΔDEF, find the values of *x* and *y*.



Since similar triangles have proportional sides,

we know that $\frac{12}{20}=\frac{18}{x}=\frac{y}{24}$.

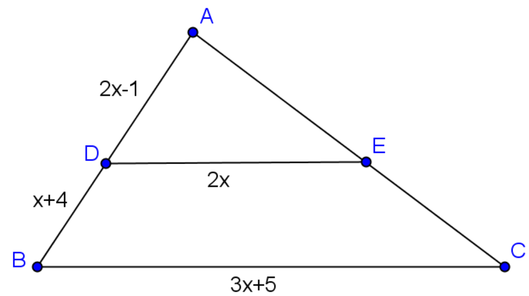
Using the first two, and cross-multiplying, we get *x*=30.

Using the first and third ratios and cross-multiplying, we get $20y=288\ \ so\ \ y=14.4$

Example #2: Given $\overline{DE}\,//\,\overline{BC}$, do the following.

a. Show that ΔABC~ΔADE.

b. Find the value of *x*.

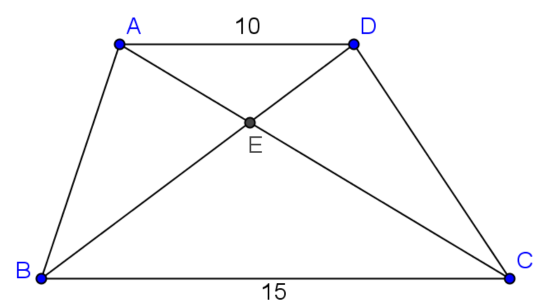


a. Since $\overline{DE}\,//\,\overline{BC}$, we know corresponding angles ADE and ABC are congruent. Since angle A is in both triangles, AA similarity shows that ΔABC~ΔADE.

b. Set up a proportion: $\frac{AB}{BC}=\frac{AD}{DE}$ so $\frac{3x+3}{3x+5}=\frac{2x-1}{2x}$. Be careful that you add AD to DB to get the side length of AB. It can be tempting to use *x*+4 in one ratio, but DB is not the side of any triangle!

Cross-multiplying, we get $2x(3x+3)=(2x-1)(3x+5)\ \ so\ 6x^2+6x=6x^2+7x-5$. Combining like terms yields the solution *x*=5.

Example #3: The diagonals of trapezoid ABCD meet at E. If AE=7 and BD=14, then find the lengths of $\overline{EC}$ *and* $\overline{BE}$.

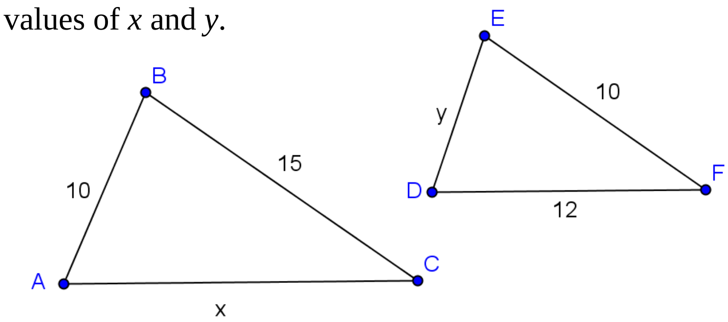


ΔADE~ΔCBE because the parallel lines have congruent alternate interior angles (and also there are vertical angles). Thus, to find EC, $\frac{AE}{EC}=\frac{AD}{BC}$ or $\frac{7}{EC}=\frac{10}{15}$ so EC=10.5.

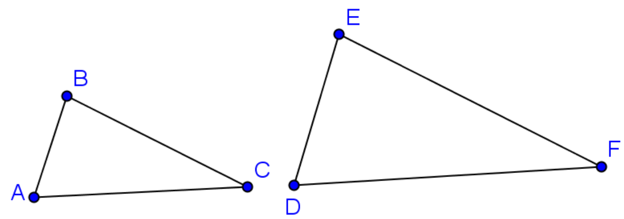
BE: let BE=*x*; then DE=14-*x*. Since $\frac{DE}{BE}=\frac{AD}{BC}$, $\frac{14-x}{x}=\frac{10}{15}$.

Cross-multiplying gives $210-15x=10x$ so *x*=8.4 and thus BE=8.4

1. In the diagrams below, ΔABC~ΔDEF. Find the values of *x* and *y*.

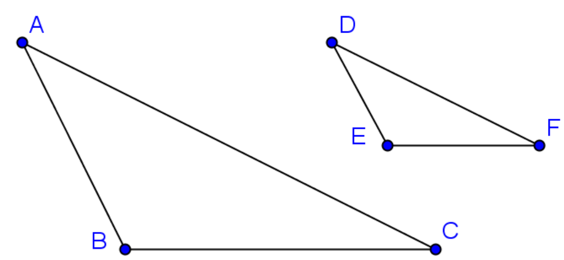


2. Given that $\Delta ABC \sim \Delta DEF$ and AB=10, BC=20, EF=30, and DF=36, find the lengths of sides $\overline{AC}$ and $\overline{DE}$.



3. In the diagrams to the right, ΔABC~ΔDEF. Given that AB=36, BC=40, and DF=30, answer the following:

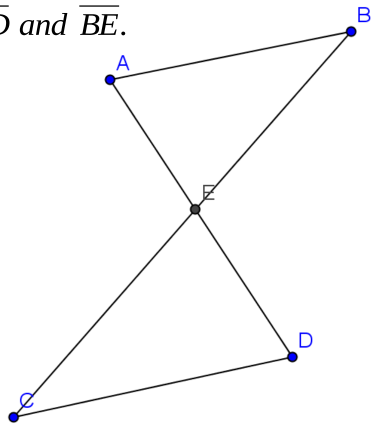
a. If DE = 20 then what is AC?



b. Instead, if AC = 50 then what is EF? (note: $\overline{AB}, \overline{BC}, and\ \overline{DF}$ are still the lengths given above)

c. Instead, if EF = 20 then what is the perimeter of Δ ABC? (note: $\overline{AB}, \overline{BC}, and\ \overline{DF}$ are still the lengths given above)?

4. In the diagram below, $\overline{AB} \parallel \overline{CD}$ and AB=5, AE=4, ED=6, and EC=8. Find the length of segments $\overline{CD}$ *and* $\overline{BE}$.

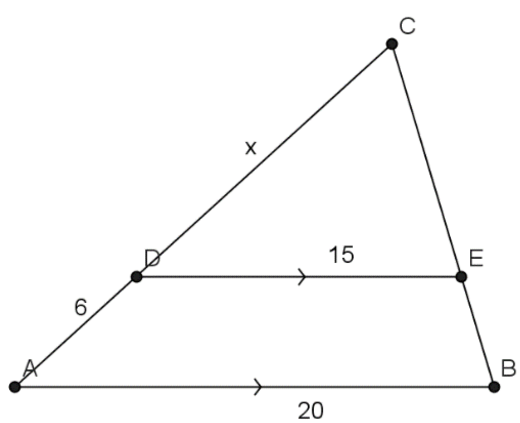


5. In ΔABC, AB=10, BC=8, and AC=14.

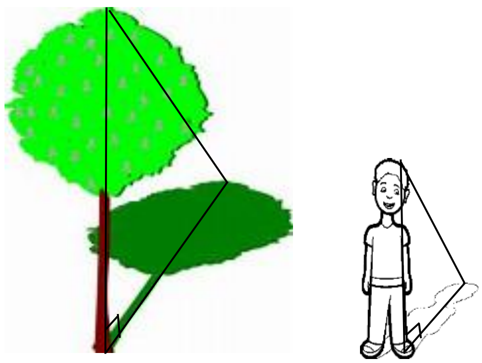
a. Is ΔDEF~ΔABC if $DE = 5, EF = 4,$ *and* $DF = 7$? Explain briefly.

b. Is ΔGHI~ΔABC if HI=20, HG=16, and GI=28? Explain briefly

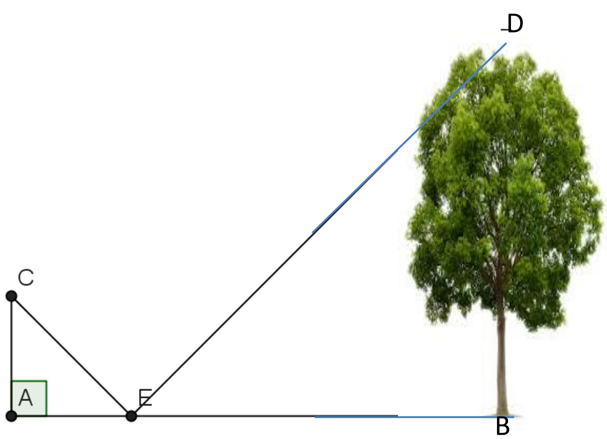
6. Find *x* in the diagram below: (hint: it is ***not*** 4.5)



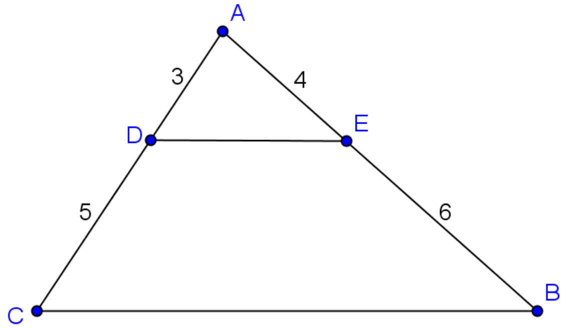
7. A teacher takes her class outside while studying similar triangles. They do an activity to find the height of a tree. A boy 5 feet tall makes a shadow 3.5 feet long. If the tree's shadow is 25 feet long, then how tall is the tree? Why can similar triangles be applied here?



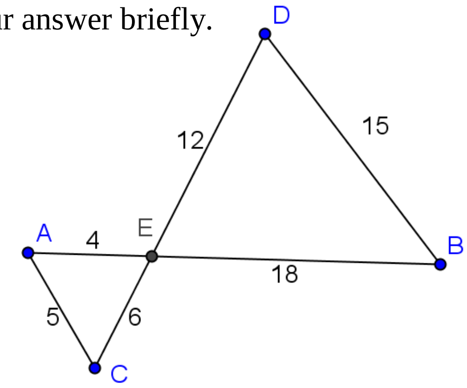
8. Another teacher hears about this idea and decides to the same activity the next day, having her students try to find the height of a different tree. But it is cloudy out, so there are no shadows! Then a student has an idea. She puts a mirror down on the ground (point E) 40 feet from the base on the tree and backs up until she can see the top of the tree in the mirror. She needs to back up 9 feet from E to A. The way a mirror works, the "angle in" equals the "angle out", so $\angle DEB \cong \angle CEA$. If the student's eyes are 5.5 feet above the ground, then how tall is the tree?



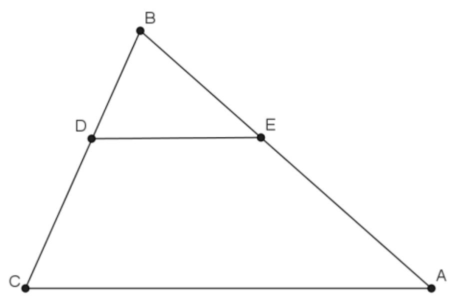
9. It appears that $\overline{DE} \parallel \overline{BC}$. Is it? Justify your answer



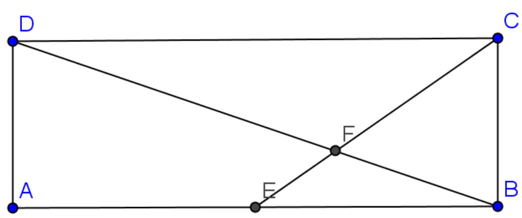
10. In the diagram below, are segments $\overline{AC}$ *and* $\overline{DB}$ parallel? Justify your answer briefly.



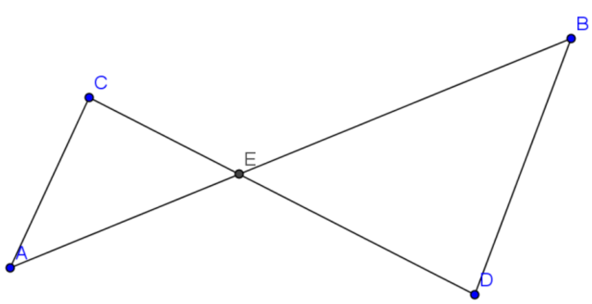
11. In the triangle below, $\overline{DE} // \overline{AC}$. Given that AC=20, DE=8, AE=14, and BC=18, find $\overline{BE}$ *and* $\overline{BD}$.



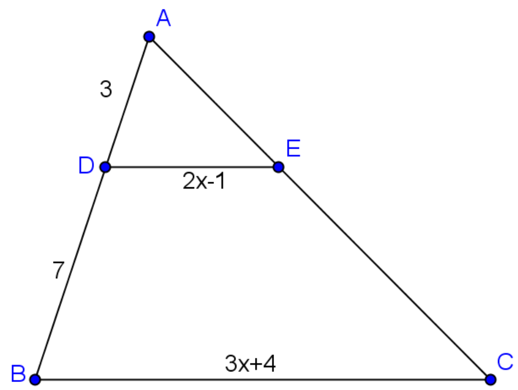
12. In rectangle ABCD below, E is the midpoint of side $\overline{AB}$. If CE=18, then what is the length of $\overline{CF}$?



13. In the diagram below $\angle A \cong \angle B$.. Explain why $CE \cdot DB = AC \cdot DE$.

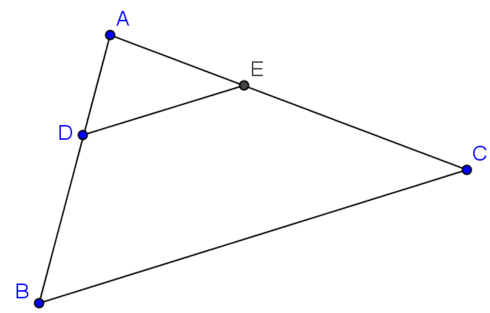


14. In the diagram below, AD=3, DB=7, and $\overline{DE} \parallel \overline{BC}$ What is the value of *x*?



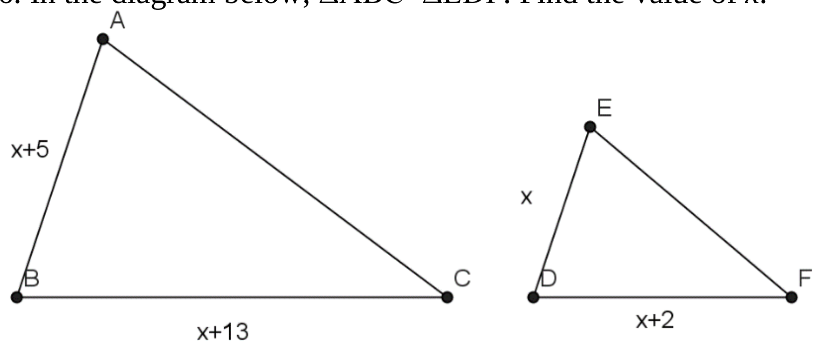
15. Given that $\overline{BC} \parallel \overline{DE}$, AD=4, DB=8, BC=20 and AC=16. Find the following:

a. The length of $\overline{DE}$.



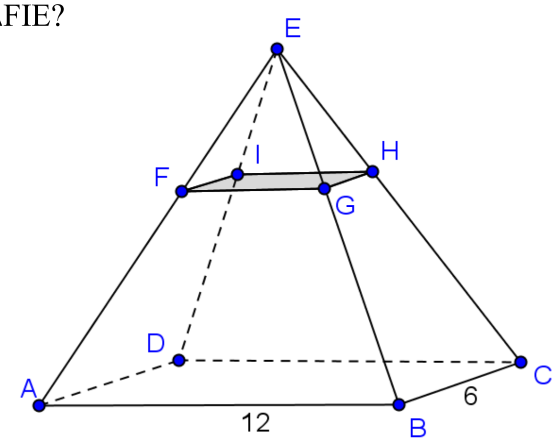
b. The length of $\overline{AE}$.

16. In the diagram below, ΔABC~ΔEDF. Find the value of *x*.



17. The diagram below shows a pyramid. It has rectangular base ABCD and four triangular faces that meet at point E. This pyramid's faces are all isosceles triangles (this is because point E is directly above the center of rectangle ABCD). Rectangle FGHI is drawn; the plane it resides in is parallel to the plane containing base ABCD, so each of its sides is parallel to the corresponding side of ABCD.

a. Must it be the case that ΔABE~ΔFGE? How about ΔADE~ΔFIE?

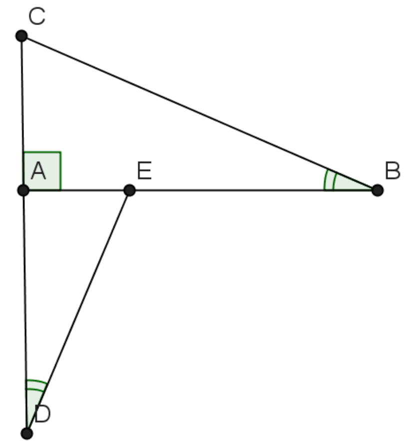


b. Explain why edges $\overline{AE}$, $\overline{BE}$, $\overline{CE}$, *and* $\overline{DE}$ are all congruent.

c. Let BE=15 and GH=2. Find GE, FG, and HC.

d. A rectangle's vertices are the midpoints of edges $\overline{AE}$, $\overline{BE}$, $\overline{CE}$, *and* $\overline{DE}$. What is its perimeter?

18. Use the diagram below to answer the following questions.



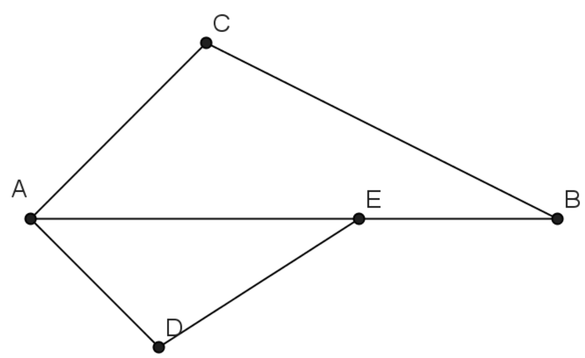
a. If AE=6 and AC=8 and AD=12 then find the length of $\overline{BE}$.

b. Instead, if BC=10 and AC=6 and DE=8 then find the length of $\overline{AE}$.

c. Instead, if EB=5 and AC=4 and AD=6 then find AE. Note: you may have to factor the quadratic equation.

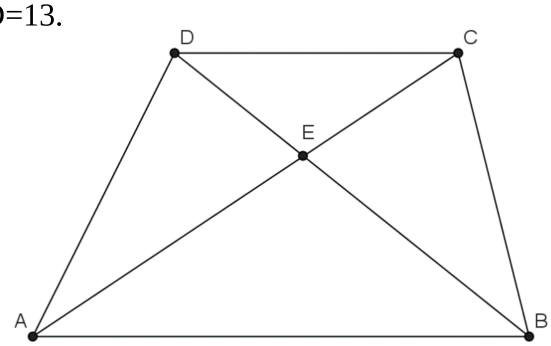
19. In the diagram below, $\overline{AB}$ bisects angle CAD and $\angle C \cong \angle D$

a. Explain why $\Delta CAB \sim \Delta DAE$ and that $AC \cdot DE = BC \cdot AD$



b. If AC=8 and DA=6 and BE=3 then find AE.

20. In trapezoid ABCD below, CD=10, AB=15, EC=6, and BD=13.

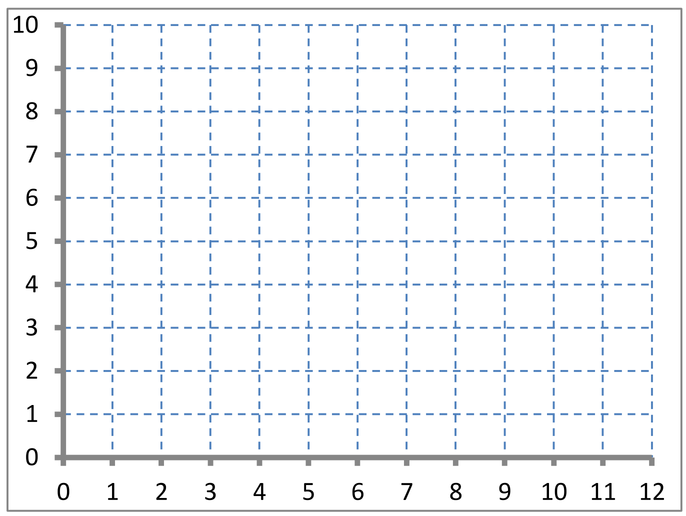


a. Identify a pair of similar triangles. Justify your answer

b. Find the length of $\overline{AE}$.

c. Find the length of $\overline{BE}$.

21. Given points A(5,6), B(1,0), C(9,3).

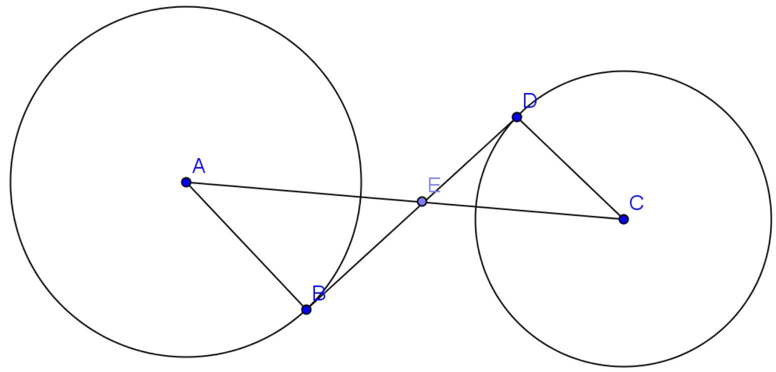


a. A is between points D and C on $\overline{CD}$ such that AD:AC = 1:2. What are D's coordinates?

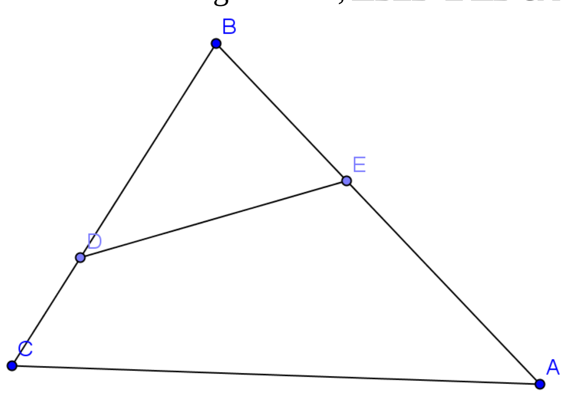
b. ΔACB~ΔADE. Give a possible set of coordinates for E.

c. ΔACB~ΔDAF. Where could point F be located?

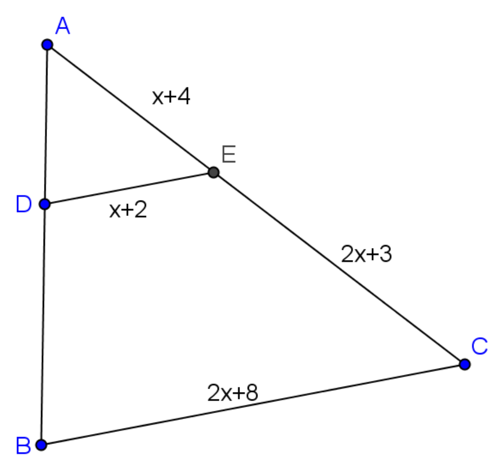
22. In the diagram below, circles centered at A and C have radii of 10 and 8 respectively. Angles B and D are both right angles (tangents to circles always meet the radii at right angles). Segment $\overline{BD}$ is 14 units long. How long is segment $\overline{DE}$?



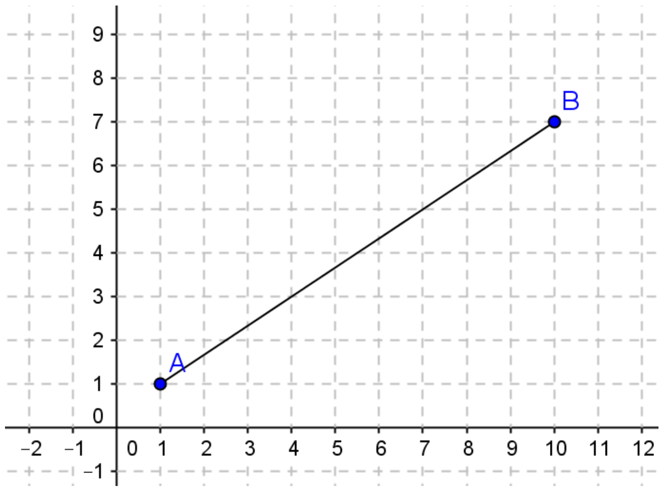
23. In the triangle below, $\angle BED \cong \angle DCA$, BE=8, BC=16, CD=6. Find EA.



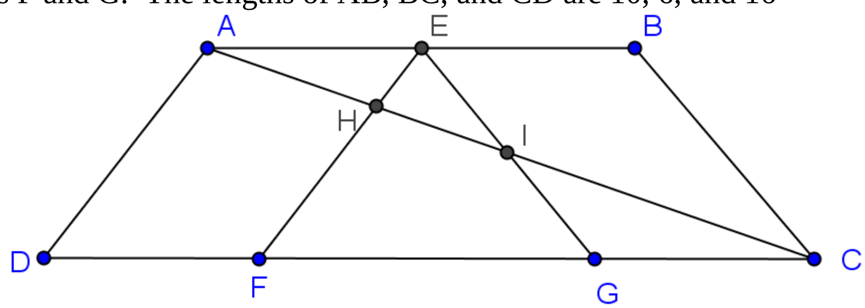
24. In the diagram below, $\overline{BC} \parallel \overline{ED}$. What is the value of x?



25. Points A(1,1) and B(10,7) are plotted below. $\overline{CD}$ is a vertical line segment three units long, intersecting $\overline{AB}$ at point E. ΔCEB~ΔDEA and $AD = 2 \cdot BC$. Find the coordinates of C and D. There are actually two possible answers!



26. In isosceles trapezoid ABCD, lines are drawn from the midpoint E of the upper base parallel to the two legs, hitting the lower base at points F and G. The lengths of AB, BC, and CD are 10, 6, and 16 respectively.

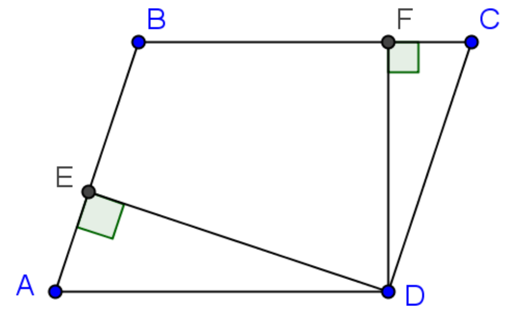


a. Find the length of $\overline{GI}$.

b. Find the length of $\overline{EH}$.

c. Find HI (hint: use the Pythagorean Theorem to find AC first).

27. Parallelogram ABCD has altitudes drawn from D to sides $\overline{AB}$ and $\overline{BC}$. Given that BE=6, BF=9, and the ratio DE:DF = 5:4, find AD.



Answers

1. x=18; y=20/3 2. 24 and 15 3a. 54 b. 24 c. 136 4. CD=7.5 and BE=16/3

5a. yes, corresponding sides are proportional b. no, proportional sides are not corresponding. But it is the case that ΔGHI~ΔCBA. 6. 18 7. 250/7 or about 35.7 feet 8. 220/9 or about 24.4 feet

9. ΔADE is not similar ΔACB ($3/8 \neq 4/10$) so angles ADE ≠ACB … thus DE is not // to BC

10. No. ΔAEC~ΔDEB so $\angle A \cong \angle D$, but then it can't be congruent to angle B, which it would need to be in order for AC and DB to be parallel…

11. BE=28/3 and BD=36/5 12. 12 14. 2 15a. 20/3 b. 16/3 16. 5/3

17a. yes; segments parallel to base of triangle create similar triangles

b. AE=BE since isosceles triangle; BE=CE since isosceles triangle; CE=DE and DE=AE

c. GE=5, FG=4 and HC=10. d. 18

18a. 10 b. 4.8 c.3 19b. 9 20a.ΔDCE~ΔBAE b. 9 c. 7.8

21a. (3,7.5) b. DE//BC so (7,9) c. (1,4.5)

22. 56/9 23. 12 24. 6 25. E must be 2/3 the way from A to B so (7,5) and then C is (7,6) and D is (7,3). But D could be above AB and C below it, in which case D is (7,7) and C is (7,4).

26a. GI=3 since EI=3 and ΔAEI~ΔABC b. HF=33/8 (using ΔCFH and ΔCDA) so EH=15/8

c. AC=14 (drop pero from A to BC; its length is $\sqrt{27}$ …) so HC=77/8 and AI=IC=7 so HI=21/8

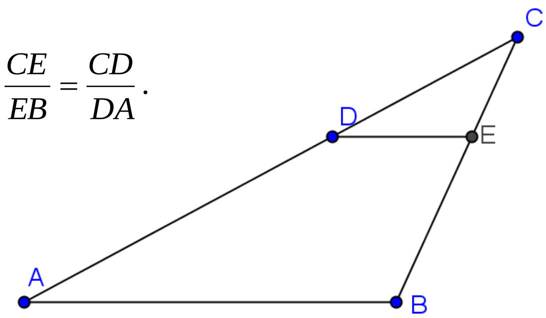
27. ΔAED~ΔCFD and the ratio is 5:4. Let CF=x. then AE=1.25x so $\frac{x+9}{1.25x+6}=\frac{5}{4}$ so x=8/3 & AD=35/3

Unit 5 Handout #4: Two Theorems Involving Proportion

The ***Side-Splitter Theorem*** states that, in any triangle, a segment parallel to one side divides the other sides proportionally.

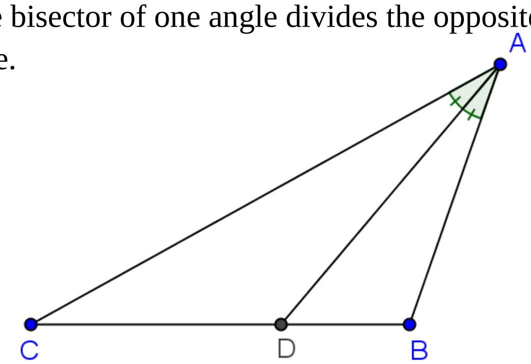
For the triangle at the right where $\overline{DE} \parallel \overline{AB}$, this means that $\frac{CE}{EB} = \frac{CD}{DA}$.

This equation can be rearranged to $\frac{CE}{CD} = \frac{EB}{DA}$.

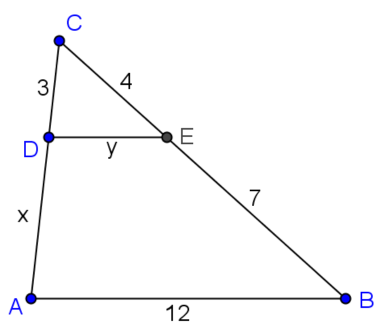


The ***Angle Bisector Theorem*** states that, in any triangle, the bisector of one angle divides the opposite side into pieces proportional to the other sides of the triangle.

Thus, in the triangle at the right, $\frac{DB}{AB} = \frac{CD}{AC}$.



Example #1: Find the values of *x* and *y* in the triangle below, given that $\overline{DE} \parallel \overline{AB}$.

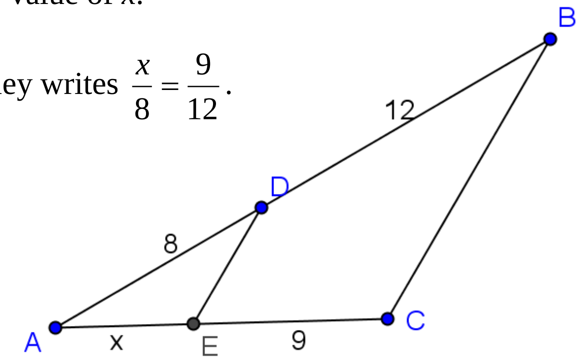


To find *x*, we could use the fact that ΔCDE~ΔCAB. But the side-splitter theorem can help save a step.

Since $\frac{CD}{DA} = \frac{CE}{EB}$, we know $\frac{3}{x} = \frac{4}{7}$ so $4x=21$ and $x=21/4$.

To find *y*, $\frac{4}{11} = \frac{y}{12}$. Thus $11y=48$ so $y=48/11$.

1. Given $\overline{BC} // \overline{ED}$, Angela and Bradley both try to find the value of *x*.

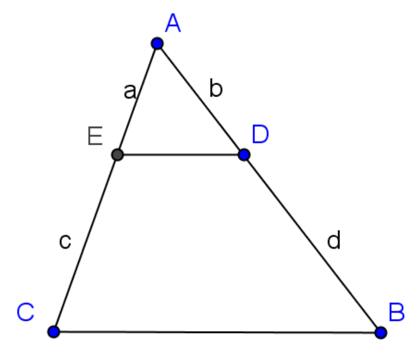


Angela sets up the following proportion: $\frac{x}{8} = \frac{x+9}{20}$. Bradley writes $\frac{x}{8} = \frac{9}{12}$.

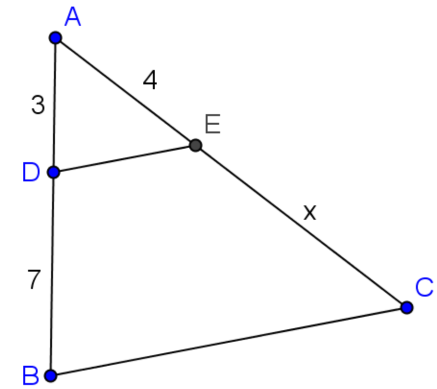
a. Whose approach do you agree with? Why?

b. Solve both equations to see what they each get for *x*.

2. $\overline{BC} // \overline{DE}$ below. We know that $\frac{c+a}{a} = \frac{d+b}{b}$ using similar triangles. Manipulate this expression algebraically to show that it is the same as $\frac{a}{b} = \frac{c}{d}$.



3. ***The Side-Splitter Theorem:*** The side-splitter theorem states that if a line is parallel to one side of a triangle and intersects other two sides of triangle, then it will divide other two sides proportionally. So, in the diagram below, where $\overline{BC} \parallel \overline{DE}$, $\frac{AD}{DB} = \frac{AE}{EC}$.

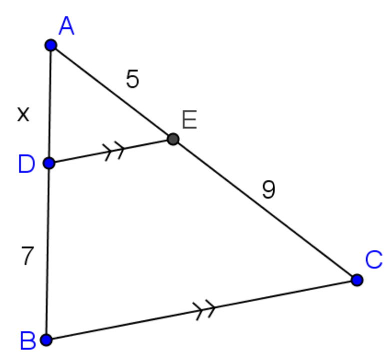


a. Show that this also means that $\frac{AD}{AE} = \frac{BD}{CE}$.

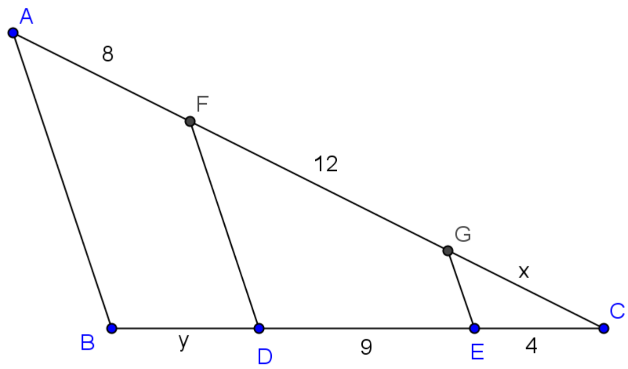
b. Use this to find the value of x in the diagram to the right.

c. Check yourself that this is the same value you would get if you used similar triangles.

4. Use the side-splitter theorem to find x below.

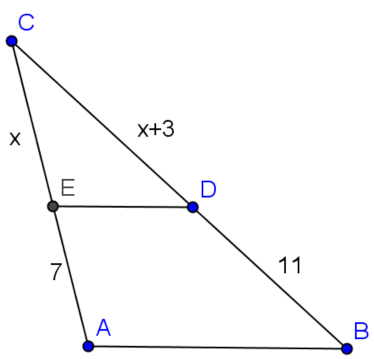


5. Use the side-splitter theorem to find *x* and *y* below given that $\overline{BA} \parallel \overline{DF} \parallel \overline{EG}$.

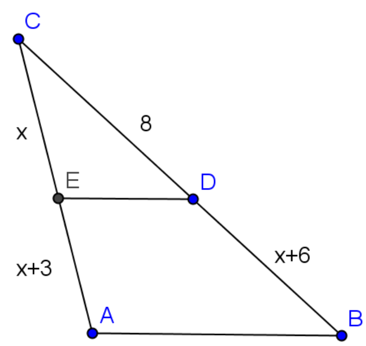


6. In each part below, $\overline{BA} \parallel \overline{DE}$. Find *x*.

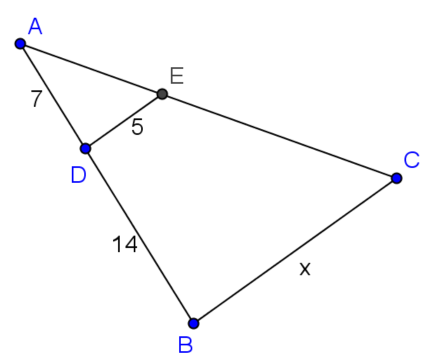
a.



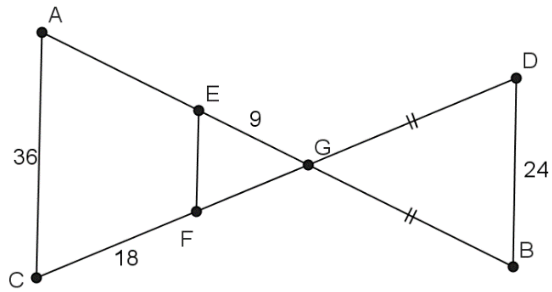
b.



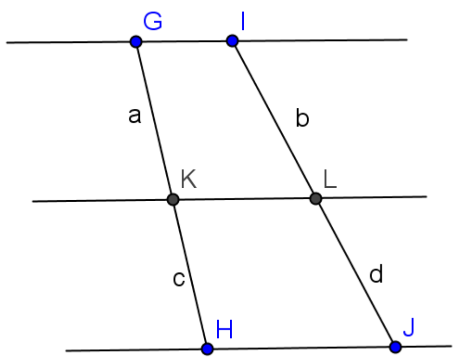
7. In the triangle on the right, $\overline{BC} \parallel \overline{DE}$. To find the value of *x*, Steven uses the side splitter theorem. He sets up the equation $\frac{7}{14} = \frac{5}{x}$ and concludes that *x* is equal to 10. Is Steven correct? Explain. If not, what is the correct answer?



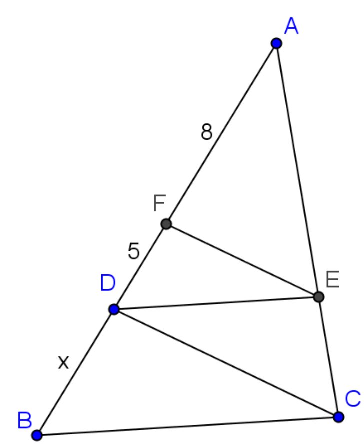
8. In the diagram below, $\overline{AC} \parallel \overline{EF} \parallel \overline{DB}$. $\overline{EG}$ measures 9 units and $\overline{CF}$ measures 18. Find the lengths of $\overline{EF}$ *and* $\overline{DG}$.



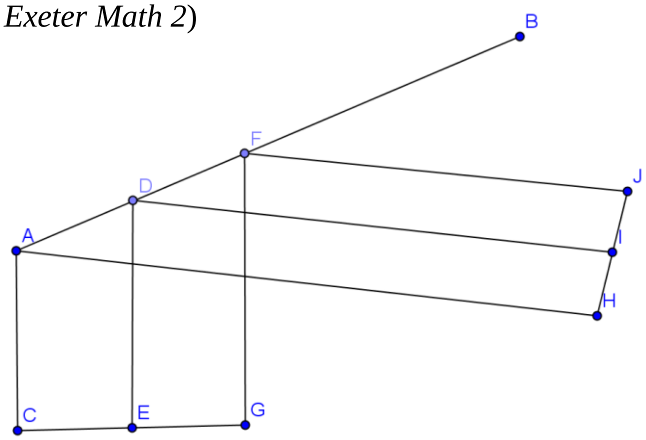
9. Explain how the side-splitter theorem can be applied to show that $\frac{a}{b} = \frac{c}{d}$, given that $\overline{GI} \parallel \overline{KL} \parallel \overline{HJ}$.



10. Find x given that $\overline{EF} \parallel \overline{CD}$ *and* $\overline{DE} \parallel \overline{BC}$. Hint: let EC=5$a$.

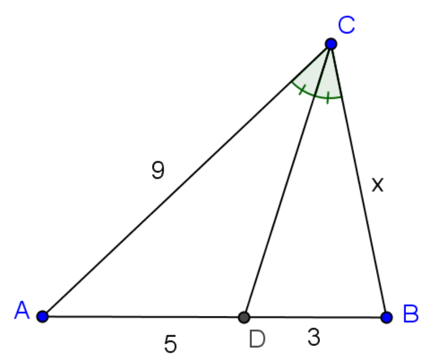


11. In the diagram below $\overline{AH} // \overline{DI} // \overline{FJ}$ and $\overline{AC} // \overline{ED} // \overline{FG}$. If CE=8, EG=7 HI=4 then find IJ. (From *Exeter Math 2*)



12. To find x in the diagram below, do the following:

a. Draw a segment through B upwards that is parallel to $\overline{CD}$ until it intersects the continuation of side $\overline{AC}$ at point E.

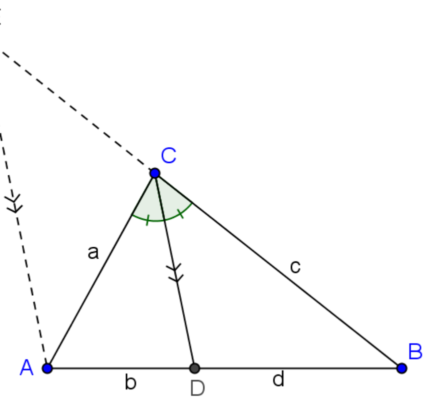


b. Find EC.

c. What kind of triangle is ΔBCE? Why?

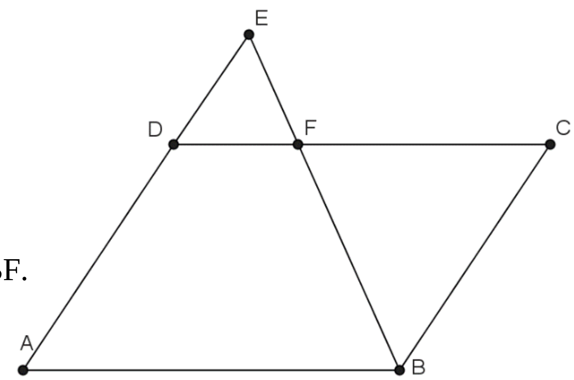
d. Find x.

13. ***The Angle Bisector Theorem***. This theorem states that the bisector of an angle of triangle divides the opposite side into pieces proportional to the other two sides of the triangle—use the diagram below to explain why $\frac{a}{b} = \frac{c}{d}$.



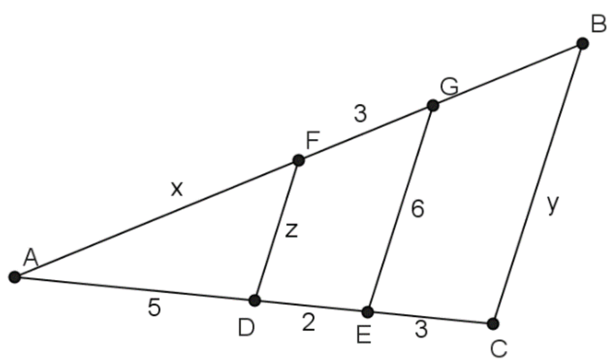
14. In parallelogram ABCD, side $\overline{AD}$ is extended to E.

a. Find two different triangles similar to ΔDEF.

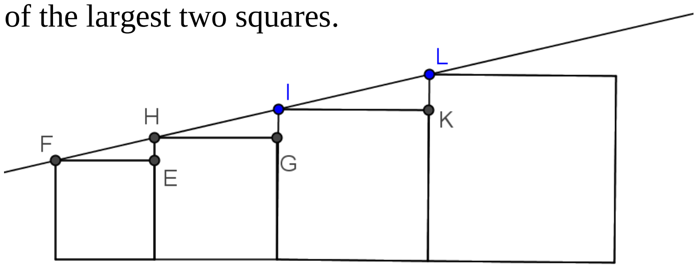


b. If AB=10, ED=5, EB=12, and DF=4, then find AD and BF.

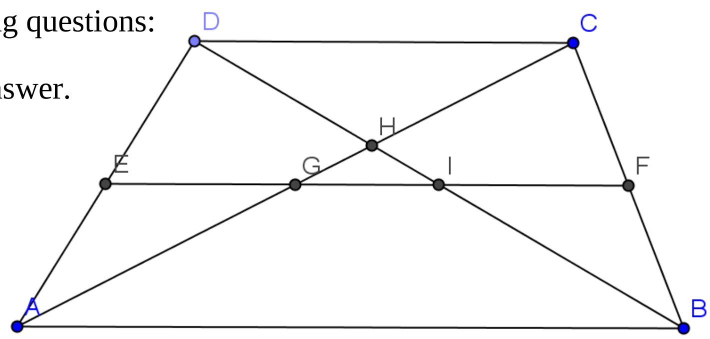
15. Given that $\overline{DF} \parallel \overline{GE} \parallel \overline{CB}$, find the values of *x*, *y*, and *z*.



16. The four shapes below are all squares. Their bases are on the same line and vertical sides of all squares are collinear with sides of adjacent squares. A line goes through the upper-left corner of each square. The smallest square has side of 5 and the next larger square has a side of 7. Find the side lengths of the largest two squares.



17. $\overline{EF}$ is the midsegment of trapezoid ABCD below (it connects the midpoints of the sides). $\overline{CD}$ is 12 units long, $\overline{AB}$ is 18, and $\overline{AC}$ is 20. Answer the following questions:



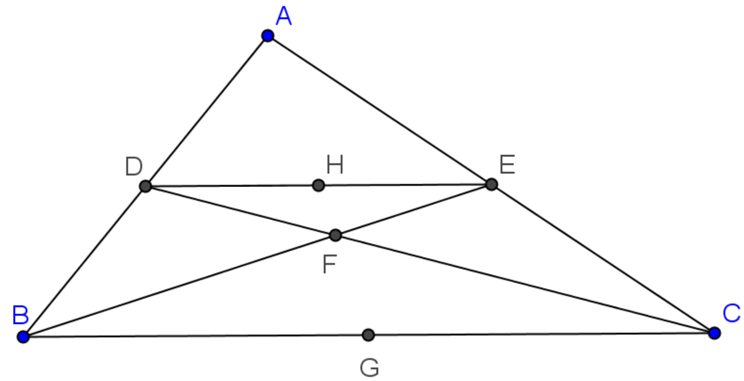
a. What is the length of segment $\overline{EF}$? Justify your answer.

b. What is the length of segment $\overline{CH}$?

c. Find the lengths of segments $\overline{EG}$, $\overline{FI}$, and $\overline{GI}$.

d. Find the length of $\overline{GH}$.

18. In the triangle below, medians from vertices B and C have been drawn and meet at point F. G is the midpoint of $\overline{BC}$ and H is the midpoint of $\overline{DE}$. Explain why the following must be true:



a. DE = 0.5 BC

b. ΔADE~ΔABC

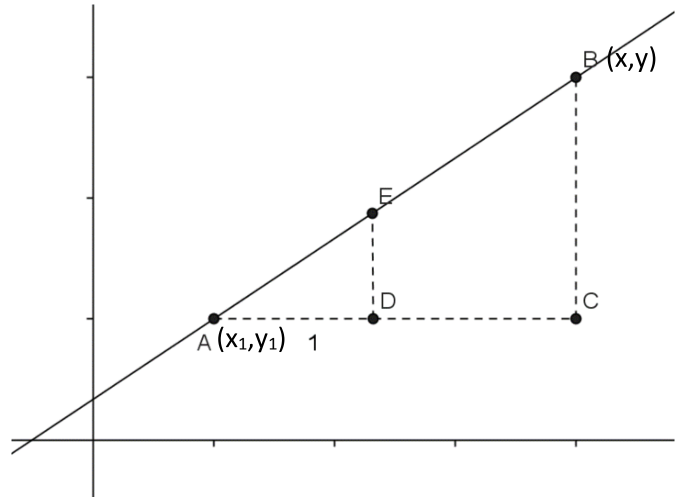
c. AH=0.5AG

d. ΔDEF~ΔCBF

e. FG=2*HF

f. AF is what percentage of AG? Medians intersect what percent of the way through the triangle?

19. A line goes through point A, whose coordinates are (x_1,y_1). Its slope is *m*, some positive number. We can use this to write the equation of the line in point-slope form. One way to do this is using similar triangles. The diagram below shows point A and some other point on the line, B. The dashed segments are horizontal or vertical. AD's length is 1.



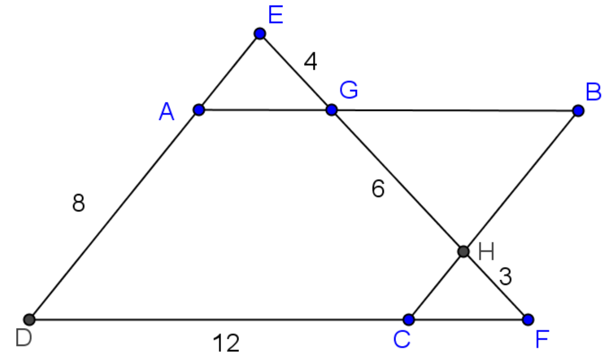
a. One segment in the diagram above has length equal to *m*. Which one and why? Think about the meaning of slope!

b. What are the lengths of segments $\overline{AC}$ *and* $\overline{BC}$, in terms of *x*, *y*, x_1, and y_1?

c. Based on similar triangles, you know that $\frac{BC}{AC} =$ what?

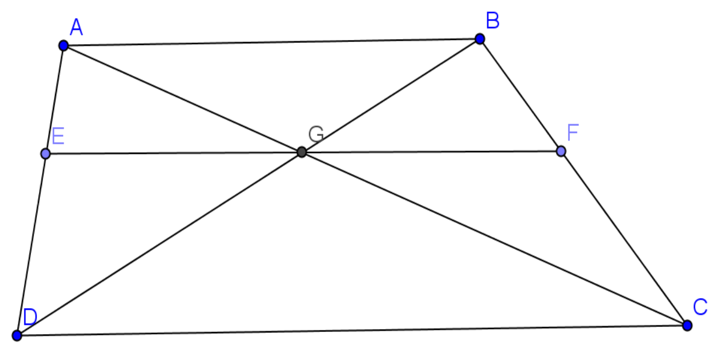
d. Show that the equation you wrote in part *c* above can be written as $y - y_1 = m(x - x_1)$, which is the point-slope form of the equation of a line.

20. In the diagram below ABCD is a parallelogram with sides 8 and 12. Two sides are extended to points E and F, which are connecting by $\overline{EF}$. $\overline{FH}$ is 3 units long, $\overline{GH}$ is 6 units long, and $\overline{EG}$ is 4 units long. Find the length of all other segments in the diagram. Hint: you may want to draw additional line segments.



21. In trapezoid ABCD below, $\overline{EF} \parallel \overline{AB}$ and contains point G, where the diagonals intersect.

a. Given that AB=20 and CD=26, find the length of $\overline{EF}$. And don't expect a nice, even number!



b. Show that, no matter what the lengths of $\overline{AB}$ *and* $\overline{CD}$ are, G is the midpoint of $\overline{EF}$. Let the lengths of $\overline{AB}$ *and* $\overline{CD}$ be *a* and *b.*

c. The harmonic mean of two numbers is the reciprocal of the average of their reciprocals. So the harmonic mean of *a* and *b* is $\dfrac{1}{0.5\left(\dfrac{1}{a}+\dfrac{1}{b}\right)}$. Show that then the length of $\overline{EF}$ is the harmonic mean of the lengths of the bases. [One application of harmonic mean: if you run one ***mile*** at 8 mph and one ***mile*** at 4 mph, your average speed is NOT 6 mph, but instead is the harmonic mean of 8 and 4 which is 5.33 mph. If you run one ***hour*** at each speed, then your average speed ***is*** 6 mph]

Answers

1a. both are OK; Angela's uses similarity b. 6 for both

2. one could cross multiply and cancel the ab terms. Or try this: $\frac{c+a}{a}=\frac{d+b}{b}$ → split the fractions on each side to get $\frac{c}{a}+\frac{a}{a}=\frac{d}{b}+\frac{b}{b}$ this is $\frac{c}{a}+1=\frac{d}{b}+1$ which is $\frac{c}{a}=\frac{d}{b}$.

3a. multiply both sides by DB/AE b. 28/3 c. yup! 4. 35/9 5. x=16/3; y=6

6a. 21/4 b. 6 (by factoring)

7. Steven is wrong. He cannot use the side-splitter theorem because BC is not one of the sides being split! Steven needs to use similar triangles, with a ratio $\frac{7}{21}=\frac{5}{x}$ to find that x is 15.

8. 12 and 18 9.draw a segment parallel to GH through point I. Its pieces will be equal in length to a and c since parallelograms are created… 10. 65/8 11. 3.5

12b. 5.4 c. isosceles since $\angle CBE \cong \angle BCD$ *and* $\angle BEC \cong \angle DCA$ *and* $\angle BCD \cong \angle DCA$ c. 5.4

13. same logic as in #12: ΔECA is isosceles so AC=a, then use side-splitter

14a. ΔCBF and ΔAEB b. 7.5 and 7.2 15. x=7.5, y=60/7 and z=30/7

16. the right triangle are similar so GI=2.8 and thus IK=9.8 so LK=3.92 and big square has side of 13.72

17a. 15 (avg of base lengths) b. 8 c. 6; 6; 3 d. 2 18f. 2/3

19a. DE since slope is rise over run is DE/AD and AD=1 b. AC=x-x1 and BC=y-y1

c. m/1, which is m d. $\frac{y-y_1}{x-x_1}=m$ *so* $y-y_1=m(x-x_1)$

20.draw segments through G parallel to AD and through H parallel of CD.
CH=8/3; BH=16/3; AG=24/5; BG=36/5; AE=32/9 and CF=18/5

21a. BG/GD = 20/26 so EG/AB=26/46 so EG=260/23
AG/CG=20/26 so AG/AC=20/46 so GF/CD=20/46 so FG=520/46=260/23 so EF=520/23

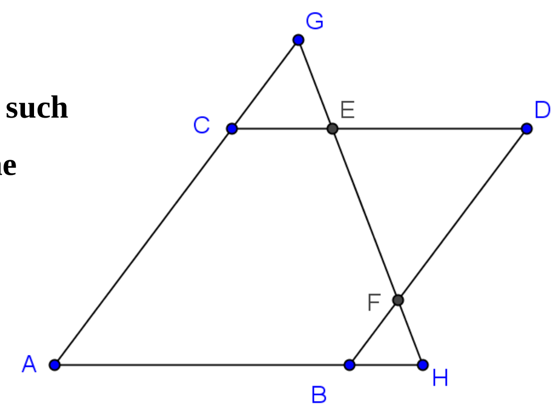
b. like part *a*, EG=$\frac{b}{a+b}\cdot a$ and GF=$\frac{a}{a+b}\cdot b$ so both are equal to $\frac{ab}{a+b}$ so G is the midpoint of EF

c. The harmonic mean of a and b is $\frac{1}{0.5\left(\frac{1}{a}+\frac{1}{b}\right)}$ which is $\frac{2ab}{a+b}$. This is the length of EF from part *b*.

Unit 5 Handout 5: Practice Similarity Problems

Some more challenging problems require the use of similar triangles several times.

Example #1: ABCD is a rhombus with side 10. Sides $\overline{AB}$ *and* $\overline{AC}$ are extended to points H and G respectively, such that BH=2 and CG=3. If $\overline{GH}$ is 11 units long, then find the perimeter of ΔFED.



Solution #1 ("the slow way"):

There are lots of similar triangles here!

Start by finding CE and BF; then we can subtract these from CD and BD to get DE and DF.

CE: ΔGCE~ΔGAH so $\frac{CG}{AG}=\frac{CE}{AH}$. Thus $\frac{3}{13}=\frac{CE}{12}$ and CE=$\frac{36}{13}$ so DE=$10-\frac{36}{13}=\frac{94}{13}$.

BF: ΔHBF~ΔHAG so $\frac{HB}{HA}=\frac{BF}{AG}$ thus $\frac{2}{12}=\frac{BF}{13}$ and BF=$\frac{26}{12}=\frac{13}{6}$ so DF=$10-\frac{13}{6}=\frac{47}{6}$.

EF is harder: one approach is to find GE and HF and subtract both of them from 11.

GE: ΔGCE~ΔGAH so $\frac{GE}{GH}=\frac{CG}{GA}$ thus $\frac{GE}{11}=\frac{3}{13}$ and GE=$\frac{33}{13}$.

HF: ΔHBF~ΔHAG so $\frac{HB}{HA}=\frac{HF}{HG}$ thus $\frac{2}{12}=\frac{HF}{11}$ and HF=$\frac{22}{12}=\frac{11}{6}$.

So EF=$11-\frac{33}{13}-\frac{11}{6}=\frac{858}{78}-\frac{198}{78}-\frac{143}{78}=\frac{517}{78}$

Therefore the perimeter of ΔFED=$\frac{517}{78}+\frac{94}{13}+\frac{47}{6}=\frac{517+564+611}{78}=\frac{1692}{78}=\frac{282}{13}$. Whew!

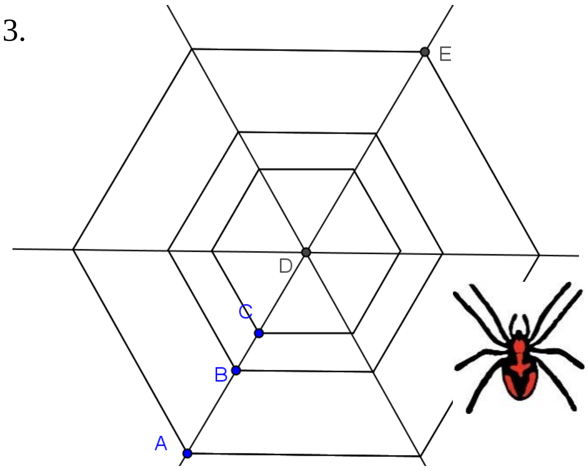
Solution #2 ("the fast way"):

Note that ΔDFE~ΔAGH. The perimeter of ΔAGH is 12+13+11=36.

We can find that DE is $\frac{94}{13}$ as in the first solution above. Then we can use the fact that

$\frac{DE}{AH}=\frac{perim\,\Delta DFE}{perim\,\Delta AGH}$. So $\frac{94/13}{12}=\frac{perim\,\Delta DFE}{36}$. So the perimeter of ΔDFE is $\frac{282}{13}$. Much easier!

1. Three regular hexagons are concentric (have the same center). The perimeter of the medium-sized hexagon is 72. The lengths $\overline{DC}$, $\overline{CB}$, *and* $\overline{BA}$ are in the ratio 2:1:3.

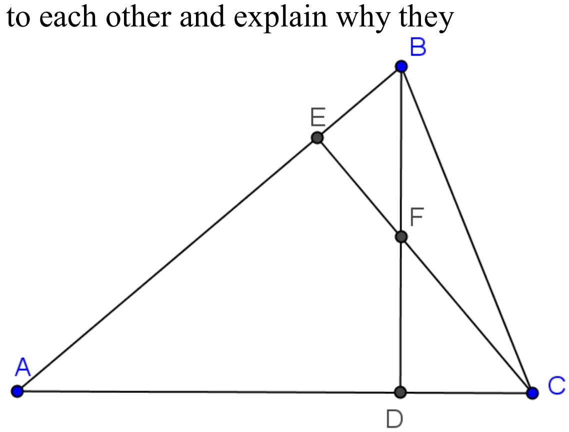


a. Find the perimeter of the smallest and largest hexagons.

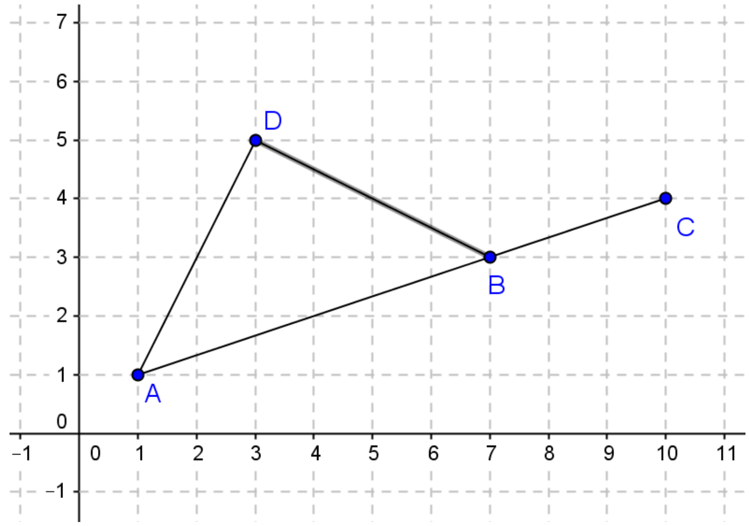
b. Find the length of $\overline{AE}$.

c. Find the perimeter of the smallest isosceles trapezoid in the diagram.

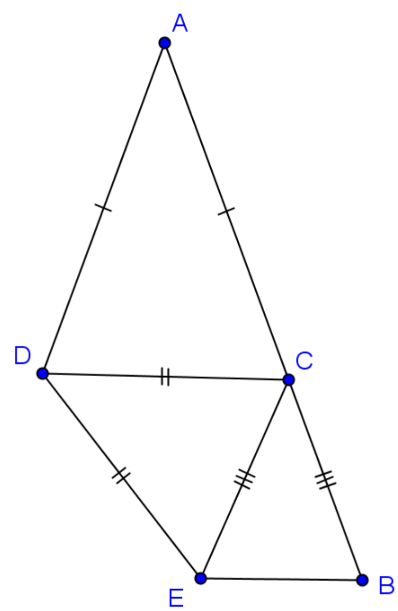
2. $\overline{CE}$ *and* $\overline{BD}$ are altitudes of ΔABC. Find four triangles similar to each other and explain why they must be similar.



3. Find one set of coordinates for point E if ΔABD~ΔACE. Then find a set of coordinates for F such that ΔABD~ΔCBF.



4. The diagram at the right is composed of three isosceles triangles. Points A, B, and C are collinear and $\overline{CD} \parallel \overline{BE}$. The lengths of $\overline{AD}$ and $\overline{CE}$ are 16 and 9 respectively.

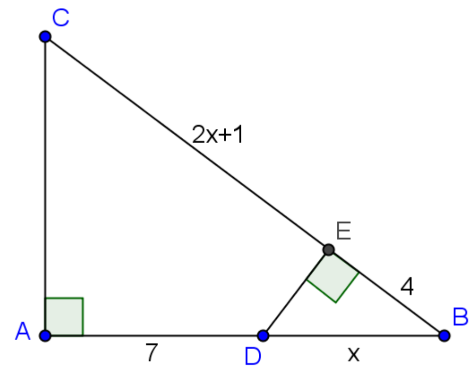


a. Show that the three triangles are similar. Hint: let the measure of angle A be 2*x* and find the measure of all angles in terms of *x*.

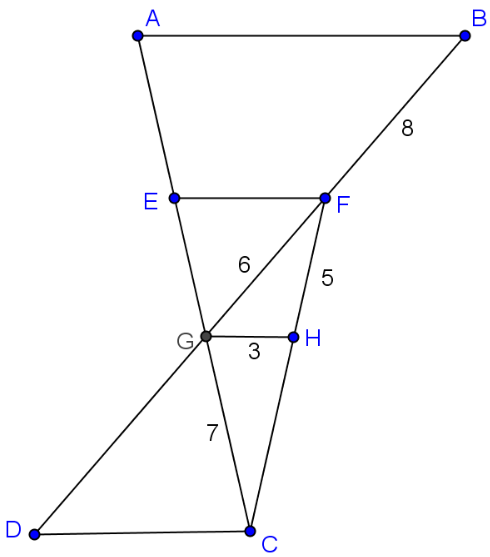
b. Must it be the case that $\overline{CE} \parallel \overline{AD}$?

c. Find the length of $\overline{CD}$.

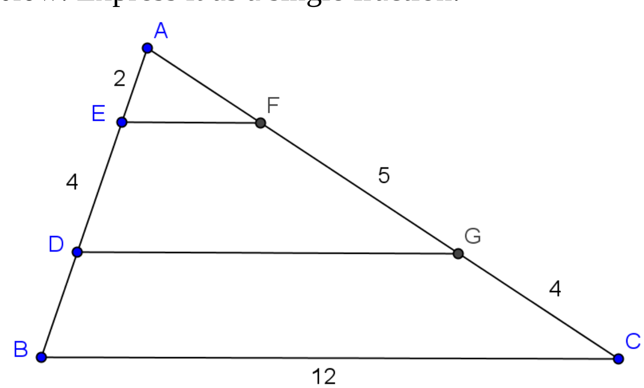
5. Find x in the diagram below.



6. In the diagram below, $\overline{AB} // \overline{EF} // \overline{GH} // \overline{DC}$ and ΔCHG is isosceles with base $\overline{GH}$. Find the lengths of all of the segments. Don't be afraid of fractions!

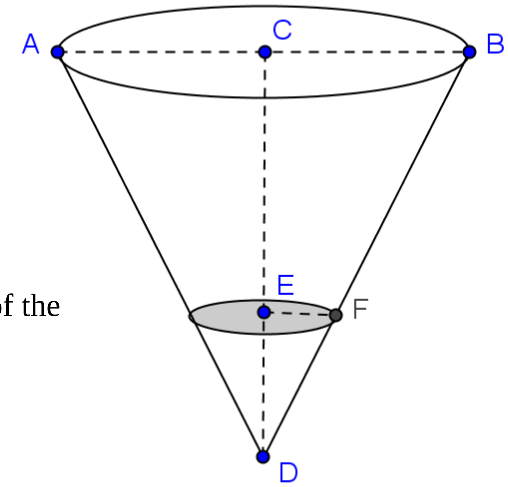


7. Given that $\overline{EF} // \overline{DG} // \overline{BC}$, find perimeter of ΔAEF below. Express it as a single fraction.



8. A cup is shaped like a cone, so each cross-section is a circle. The height of the cone ($\overline{CD}$) is 16 cm and the radius at its top ($\overline{BC}$) is 10 cm. The level of the water in the cone is the shaded circle.

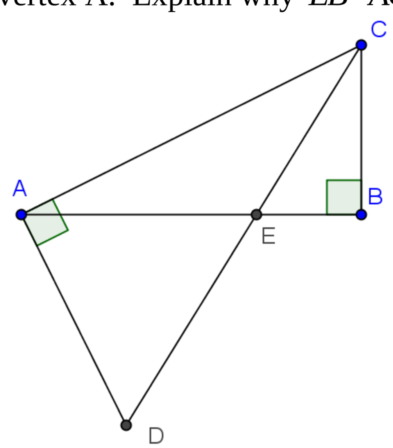
a. Why is ΔDCB~ΔDEF?



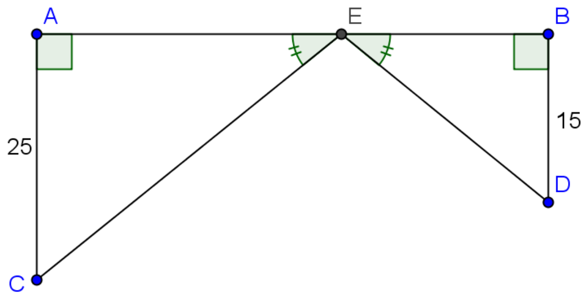
b. The water level is 7 cm from the bottom ($\overline{DE}$). What is the radius of the circle of water at the surface ($\overline{EF}$)?

c. More water is added to the cup. The radius of the circle of water at the surface is 7 cm. How far from the top of the cone is the water level?

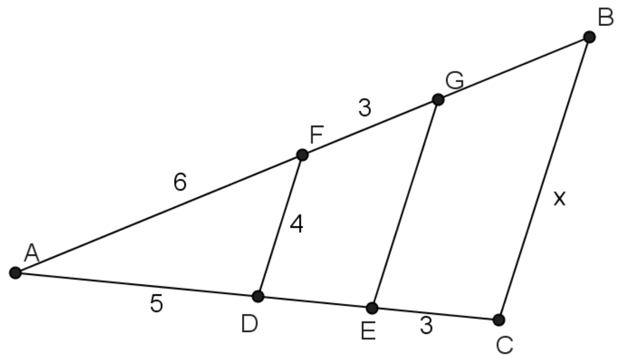
9. In ΔABC, $\overline{CD}$ is drawn to bisect angle ACB and it is extended until it meets $\overline{AD}$, a perpendicular from vertex A. Explain why $EB \cdot AC = BC \cdot AD$.



10. AB=50; find BE.

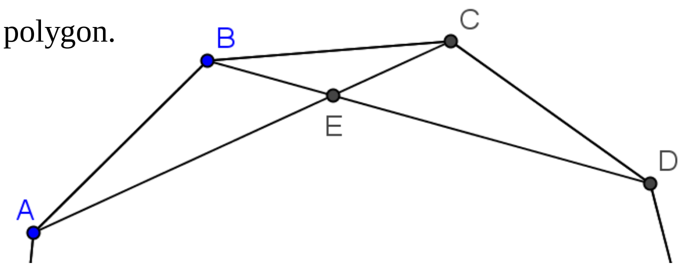


11. Given that $\overline{DF} \parallel \overline{GE} \parallel \overline{CB}$, find the value of *x*.



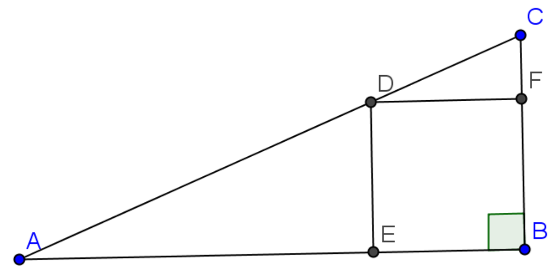
12. The diagram below shows part of a regular ten-sided polygon.

a. Must ΔABC~ΔBEC? Explain.

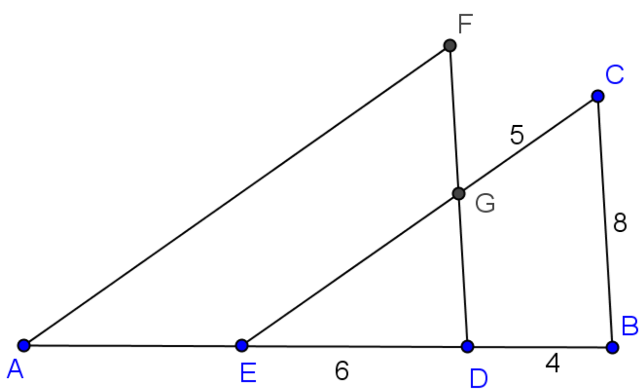


b. Instead of being part of a regular ten-sided polygon, imagine it was part of a regular *n*-sided polygon. Must it still be the case that ΔABC~ΔBEC? Explain. Hint: write some angles in terms of *n*.

13. The area of square BEDF is 45 square cm and AE exceeds CF by 12 cm. Find AB.

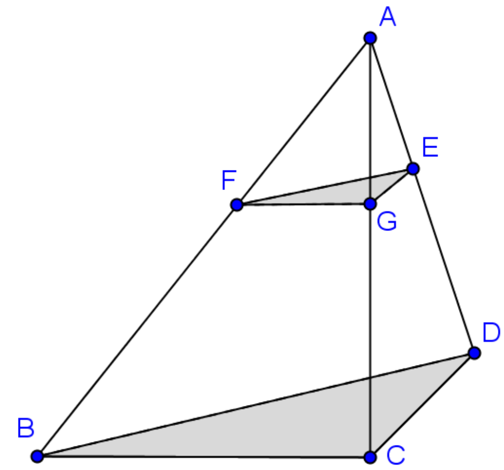


14. In the diagram below, $\overline{FD} \parallel \overline{BC}$ *and* $\overline{AF} \parallel \overline{EC}$. Some lengths are given on the diagram. Additionally, DF=10. Find the lengths of $\overline{FG}$ *and* $\overline{AF}$. Be prepared for fractions (or decimals)!



15. ABCD is a triangular prism: base BCD is a triangle and edges $\overline{AB}, \overline{AC},$ *and* $\overline{AD}$ create three triangular faces. Triangle FGE is in a plane parallel to ΔBCD, so $\overline{FG} \parallel \overline{BC}, \overline{CD} \parallel \overline{GE},$ *and* $\overline{FE} \parallel \overline{DB}$.

a. Why must ΔAGE~ΔACD?



b. Why must $\frac{EG}{CD} = \frac{FG}{BC} = \frac{EF}{BD}$?

Given that GE=4, BC=10, EF=7, AC=20, AF=8, and CD=12, find the lengths of:

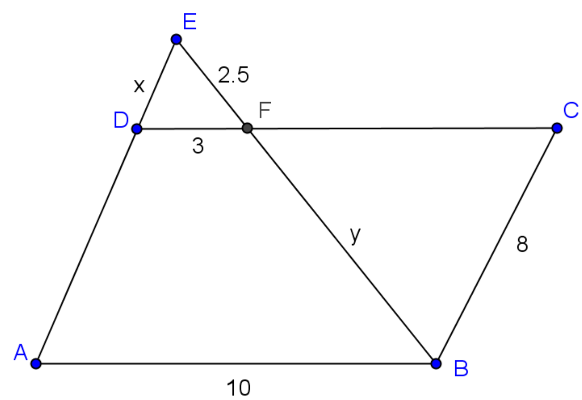
c. $\overline{FG}$

d. $\overline{BD}$

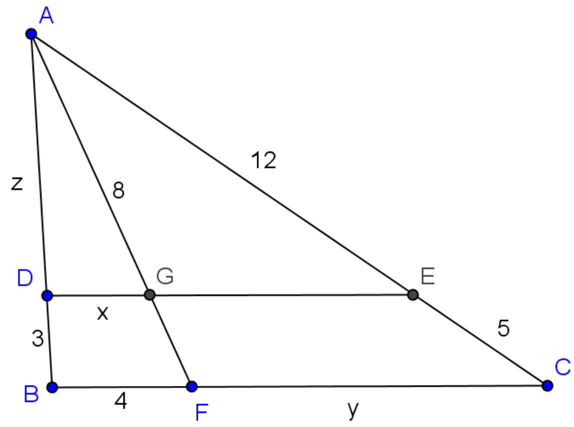
e. $\overline{BF}$

f. $\overline{CG}$

16. Side $\overline{AD}$ of parallelogram ABCD is extended to point E, which is joined to B by a line segment. Find the values of *x* and *y*.

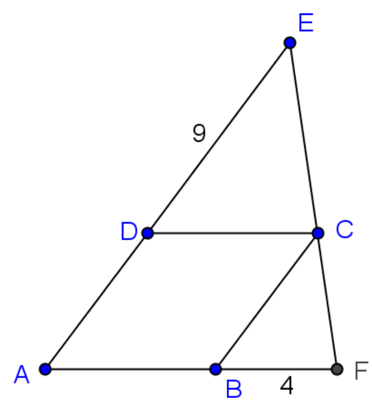


17. In the diagram below, $\overline{DE} // \overline{BC}$ and $\overline{DE}$ measures 11 units. Find *x*, *y*, and *z*.



18. Rhombus ABCD has sides $\overline{AB}$ *and* $\overline{AD}$ extended to F and E so that $\overline{EF}$ contains point C. $\overline{EF}$ is 12 units long.

a. Find $\overline{AB}$.

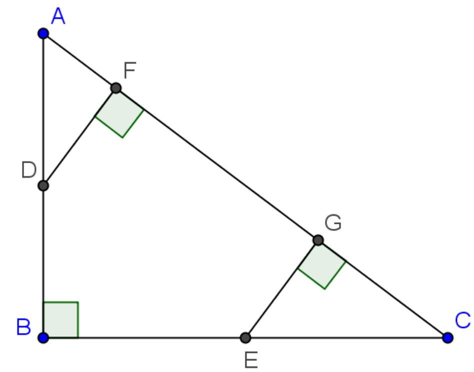


b. Find $\overline{CF}$.

c. Rank the angles A, E, and F from small to large.

19. D and E are midpoints of the legs of right triangle ABC. AB=12 and BC=16.

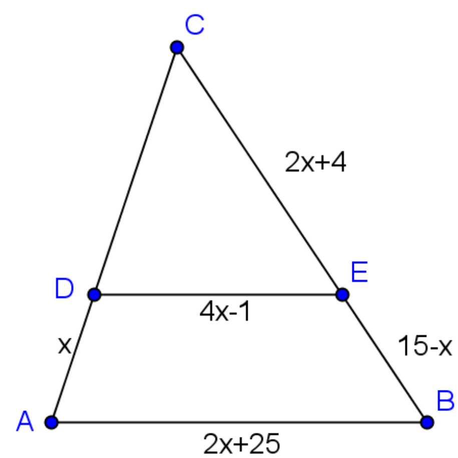
a. Use the Pythagorean Theorem to find AC.



b. Find DF.

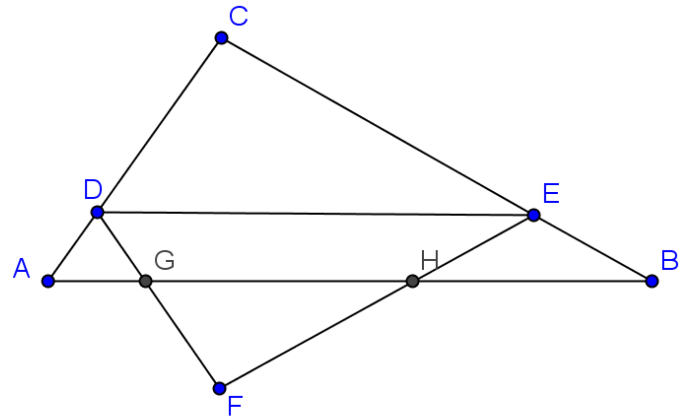
c. Find CG.

20. If $\overline{AB} \parallel \overline{DE}$, then what is the length of $\overline{CD}$?



21. In ΔABC below, $\overline{DE} \parallel \overline{AB}$. Triangle CDE is then reflected over $\overline{DE}$ to create ΔFDE.
AB=24, DE=18, AC=16 and BE=5

a. Why are ΔADG and ΔHEB isosceles?

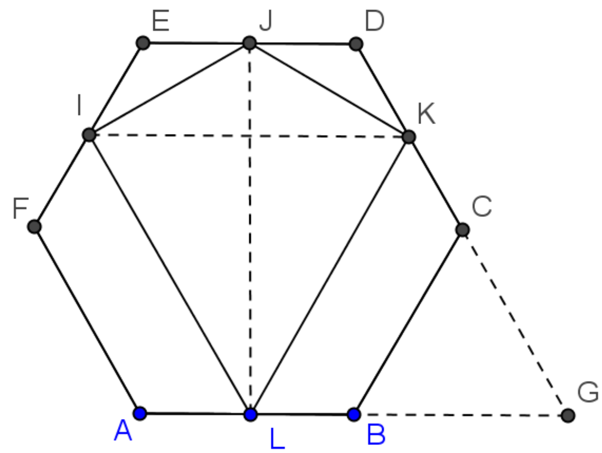


b. Find the lengths of $\overline{AD}, \overline{CE}, \overline{FG}$, *and* $\overline{GH}$.

c. Drop a line segment from C perpendicular to $\overline{DE}$ that hits $\overline{DE}$ at point I. If DI=27/4 then find AG.

22. In the diagram below, ABCDEF is a regular hexagon with side length of 6. Sides $\overline{AB}$ *and* $\overline{DC}$ are extended and meet at point G. Points I, J, K, and L are the midpoints of four of the hexagon's sides.

a. Must triangles GBC and GLK be similar? Explain why.



b. Must $\overline{KL} \parallel \overline{BC}$?

c. What is the length of $\overline{KL}$?

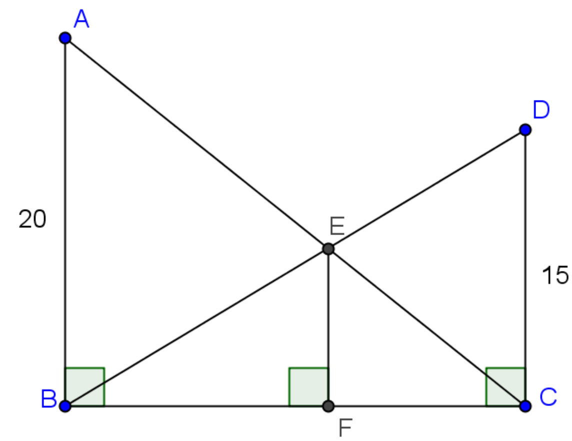
d. Why must JDBL be a rectangle?

e. Let the length of $\overline{JL}$ be *x*. Use simliar triangles to find the length of $\overline{JK}$ in terms of *x*.

f. Explain why ΔJKL must be a right triangle; then use the Pythagoran Theorem to find the value of *x*.

23. In the diagram below, segments $\overline{AB}$, $\overline{CD}$, *and* $\overline{EF}$ are all perpendicular to $\overline{BC}$.

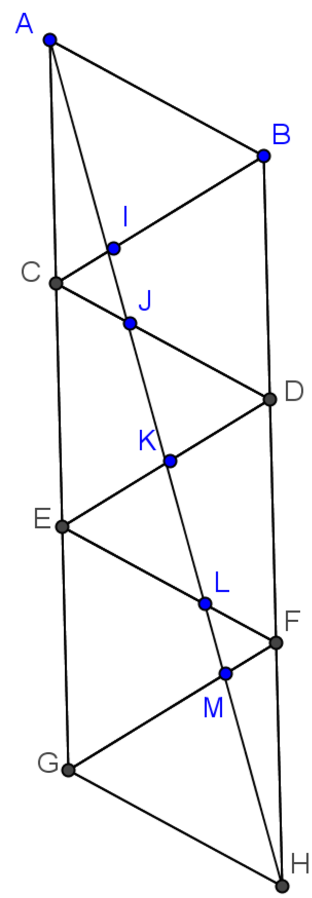
a. If the length of $\overline{BC}$ is 24, then find *EF*. Hint: define two variables and write two equations involving them.



b. Instead, if the legnth of $\overline{BC}$ is 30, find the legnth of $\overline{EF}$.

c. In general, show that if CD=a and AB=b then EF=$\frac{ab}{a+b}$, whatever then length of $\overline{BC}$ is. Then algebraically show that this means that $\frac{1}{a}+\frac{1}{b}=\frac{1}{EF}$

24. The diagram below consists of six equilateral triangles with side length of one. Points A, C, E, and G are collinear, as are points B, D, F, and H. Segment $\overline{AH}$ is also drawn; its length is $\sqrt{13}$ (adapted from *Exeter Math 2*)



a. Name all five triangles similar to ΔFMH.

b. Must $\overline{AG} \parallel \overline{BH}$? Must $\overline{AB} \parallel \overline{CD}$? Explain.

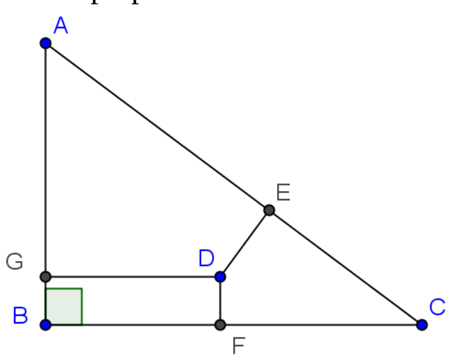
c. Why must K be the midpoint of $\overline{DE}$?

d. Find the lengths of $\overline{DK}$ *and* $\overline{FM}$.

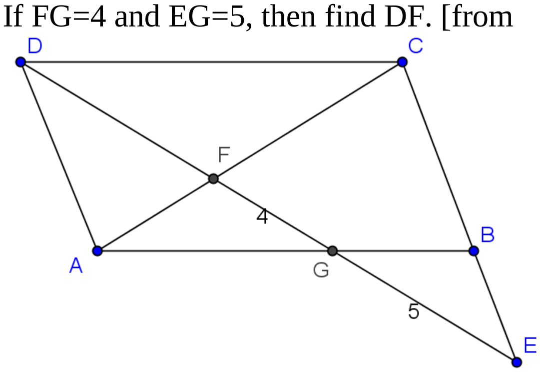
e. Name five triangles similar to ΔGMH.

f. Find the lengths of $\overline{FL}$, $\overline{AI}$, $\overline{IJ}$, *and* $\overline{JK}$.

25. In ΔABC, AB=6 BC=8 and AC=10. From point D in the interior, the distance along perpendicular $\overline{DE}$ to $\overline{AC}$ is two. And the distance along perpendicular $\overline{DF}$ to $\overline{BC}$ is one. What is the length of $\overline{DG}$, the perpendicular to $\overline{AB}$?



26. Given parallelogram ABCD with side $\overline{CB}$ extended to E. If FG=4 and EG=5, then find DF. [from *Geometry For Enjoyment and Challenge*]



Answers

1a. small=48; big=144 b. 48 c. 28

2. ΔBDA~ΔBEF (same angle B and right angle)… which are similar to ΔDCF (vertical angles and right angles) which are similar to ΔACE (same angle C and right angle)

3. E could be at (4,7) and F at (9,2) (where line DB meets line through C parallel to AD)

Since ΔABD is isosceles E can also be at (7,-2)→ draw CE//AD and 50% longer ; F can then be (8,5)

4a. A=2x; D and DCA=90-x so EBC=90-x so BEC=90-x and BCE=2x; so DCE=90-x and DEC does too, so CDE=2x & each Δ has a 2x and two 90-x angles b. yes $\angle ECD \cong \angle CDA$, which are alt int c. 12

5. 5 6. HC=7; EG=5; EF=36/7; AE=20/3; AB=12; GD=42/5; CD=36/5 7. 327/46

8a. both are right triangles and they share angle CDB. b. 35/8=4.375 cm c. 4.8 cm

9. ΔDAC~ΔEBC then use proportions of sides 10. 18.75 11. 8.4

12a. $\angle ABC = 144°$ so $\angle BAC = \angle BCA = 18°$; in ΔBCD, same logic shows $\angle CBD$ to be18° so BEC=144°.

b. instead of 144, we have $180 - \frac{360}{n}$ and instead of 18 we have $\frac{180}{n}$ and the same equalities hold

13. $15 + 3\sqrt{5}$ because $\frac{AE}{3\sqrt{5}} = \frac{3\sqrt{5}}{AE - 12}$ and AE=15.. 14. FG=5.2 and AF=15.625

15a. since GE/CD, angle AGE cong ACD so AA~

b. EG/CD = AG/AC by similar trianlges AGE and ACD; in similar triangles AFG and ABC, FG/BC = AG/AC… so by transitivity EG/CD=FG/BC. You can do the same thing with AF/AB, since it is shared by triangles ABC and ADB. C. 10/3 d. 21 e. 16 f. 40/3

16. x=24/7 and y=35/6 17. x=48/17, y=139/12, z=36/5 18a. 6 b. 4.8 c. $\angle E < \angle A < \angle F$

19a. 20 b. 24/5 c. 32/5 20. x=7 so CD=63/4

21a. $\angle A \cong \angle CDE$ and because of the reflection $\angle CDE \cong \angle GDE$ and $\angle GDE \cong \angle AGD$ because they are alternate interior angles. Thus, using transitivity a few times, $\angle A \cong \angle AGD$ and the sides of a triangle opposite congruent angles are congruent… b. AD=4, CE=15, FG=8, and GH=12 c. 4.5

22a. yes, both are equilateral b. yes since both are equilateral LKG=BCG=60° c. 9

d. angles LBD and BDJ are 90° and JD=LB; two segments congruent and parallel form parallelogram

e. BD=x & ΔDJK~ΔCDB so JK=0.5x f. $\angle DKJ = 30^\circ; \angle LKC = 60^\circ$; $(0.5x)^2 + 9^2 = x^2$ so $x = 6\sqrt{3}$

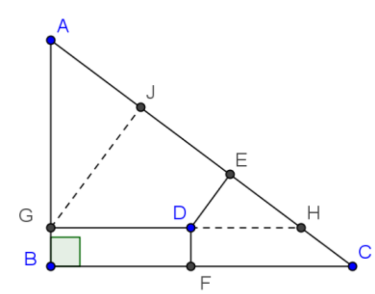
23a. 60/7 let EF=x & BF=y: $\frac{y}{x} = \frac{24}{15}$ *and* $\frac{24-y}{x} = \frac{24}{20}$ add them & get $\frac{24}{x} = \frac{14}{5}$ b. also 60/7 (cool!)

24a. ΔDKH, ΔBIH, ΔGMA, ΔEKA, ΔCIA

b. yes by congruent alt int angles: since Δ's are equilateral, $\angle DBC = \angle BCA = 60$ and $\angle ABC = \angle BCD = 60$

c. $\Delta DKH \cong \Delta EKA$ by ASA, then using CPCTC d. ½, ¼ e. ΔEKL, ΔCIJ, ΔFML, ΔDKJ, ΔBIA

f. FL=1/3; AI= $\sqrt{13}/4$; IJ= $\sqrt{13}/12$; JK= $\sqrt{13}/6$

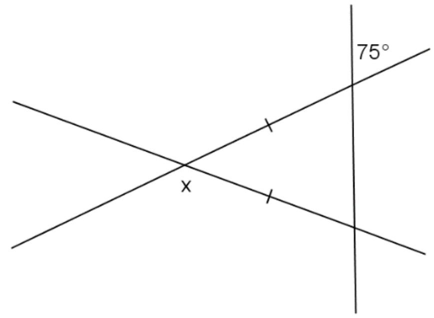


25. continue GD until it meets AC at H; also draw perpendicular from G meeting AC at point J..
GH=(5/6)(8) = 20/3 JAG~BAC so JG=4 so DG=10/3 looking at HJG

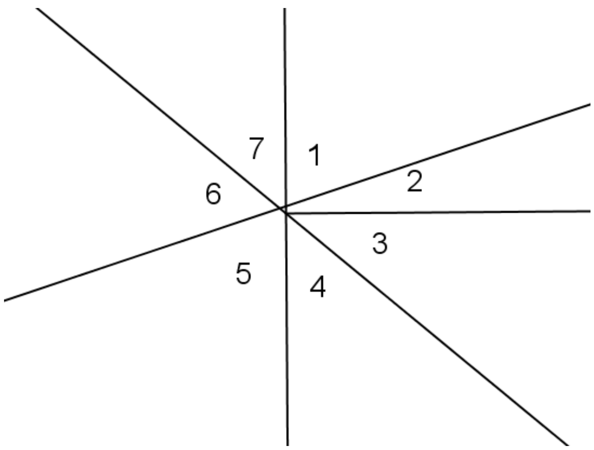
26. 6 → Let DF=x and DC=y then AG=4y/x and BG=4-4y/x and so (9+x)/y = 5 / (y)(1-4/x))

Geometry Final Exam Practice Problems: Part I

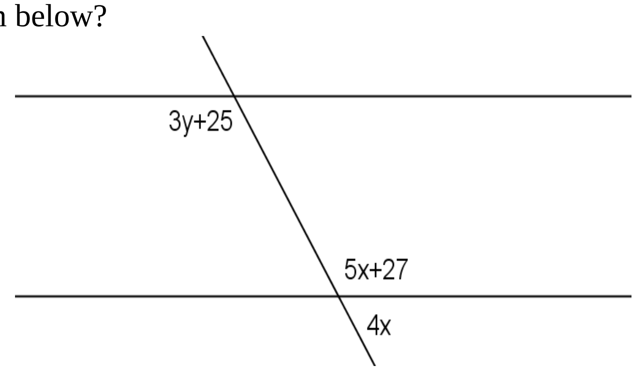
1. Find the measure of angle *x* in the diagram below.



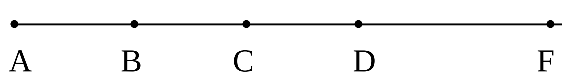
2. In the diagram below, four line segments intersect in a central point. Angles 3 and 7 are equal and complementary and angle 2 measures 25°. Find the measures of angles 1, 3, 4, 5, and 6.



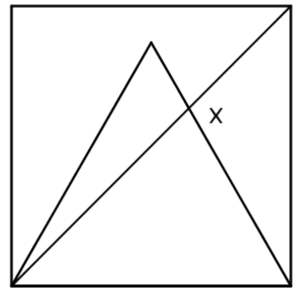
3. For what value of *y* are lines *a* and *b* parallel in the diagram below?



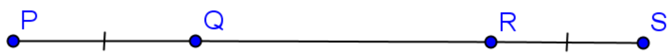
4. In line segment AF below, B and C trisect segment AD and D bisects BF. If AB=6, find AF.



5. In the diagram below, the square and the equilateral triangle share a side. What is the measure of the angle *x*?



6. In the line segment below. PR=5x+5, QS=2x+17, and QR=6x-7 find RS.



7. Determine which of the following statements are true:

a. Two distinct planes cannot meet in a point.

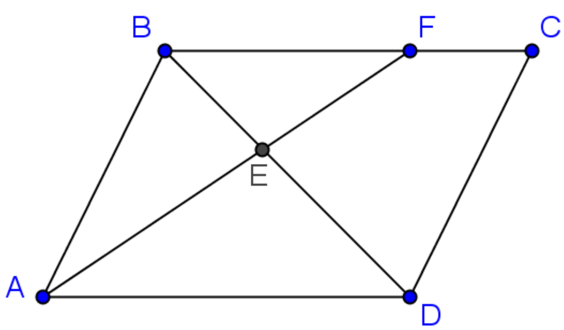
b. Skew lines are parallel.

c. Two lines in space that do not intersect must be parallel.

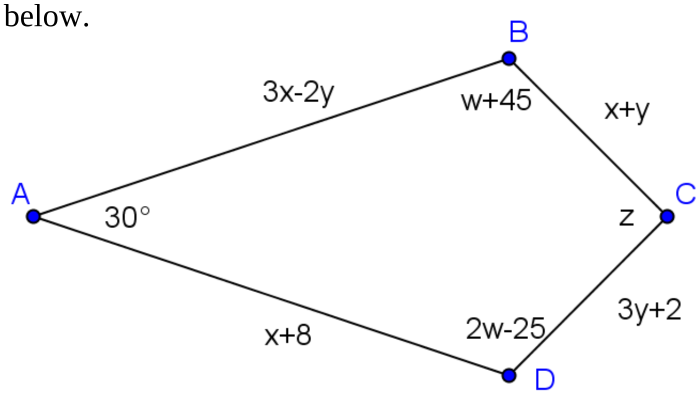
d. If two distinct coplanar lines are not parallel, then they must intersect in a point.

e. Two planes may intersect in a line.

8. In parallelogram ABCD, BD=5, BF=4, and CF=2. Find DE.



9. Find the values of w, x, y, and z in the kite below.



10. Answer the following questions about the statement "If A is a tennis player then A likes Andy Roddick"

a. What is the contrapositive of the statement?

b. Assuming the original statement is true, which of the following must also be true (circle all that apply)?

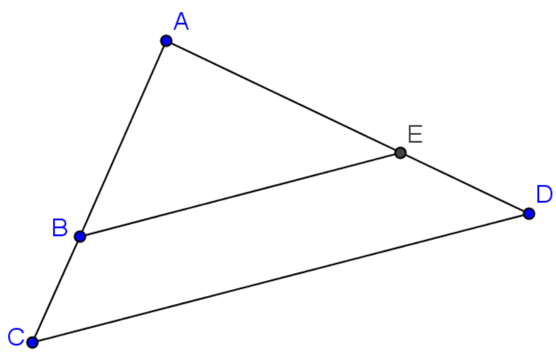
i. Jenny likes Andy Roddick so she plays tennis.

ii. Bill plays tennis so he likes Andy Roddick.

iii. Sally does not like Andy Roddick so she does not play tennis.

iv. Jesse does not play tennis so she does not like Andy Roddick.

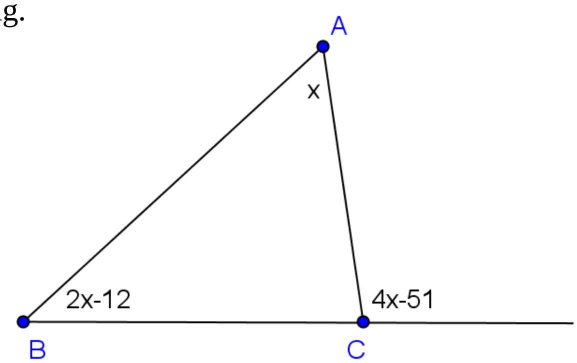
11. Given triangle ACD and BE || CD, which of the following must be true. Circle all that apply.



a. $\frac{AB}{BC} = \frac{AE}{ED}$ b. $\frac{AB}{AC} = \frac{AC}{AD}$ c. $\frac{AB}{AE} = \frac{AC}{AD}$ d. $\frac{AB}{ED} = \frac{AE}{BC}$ e. $\frac{AB}{BE} = \frac{AC}{CD}$

12. Given the statement, "if a man is from Crete, then he is a liar," write the inverse, converse, and contrapositive. Then determine which must be true.

13. Find the measure of angle ACB in the diagram below. Then rank the sides of ΔABC from short to long.

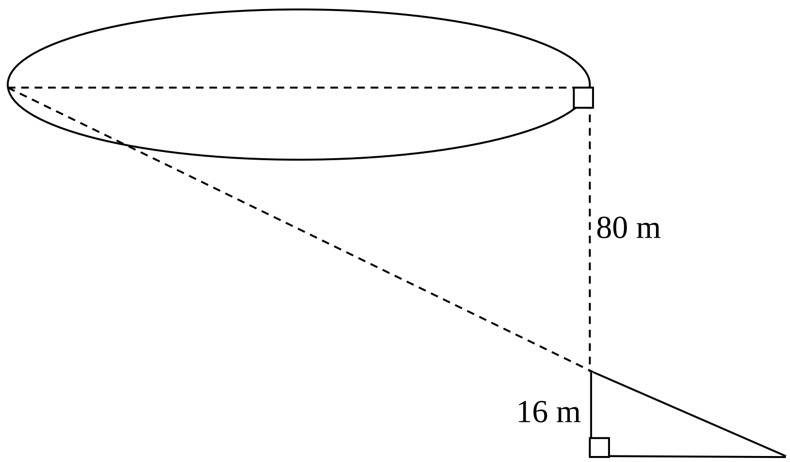


14. Two sides of a triangle are 7 and 11. The perimeter must be between what two numbers?

15. A triangle's perimeter is 80 and its sides are in the ratio 3:4:5. What is the longest side?

16. The angles in triangle ABC are in the ratio of 1:3:5. What is the triangle's largest angle, or is the triangle impossible?

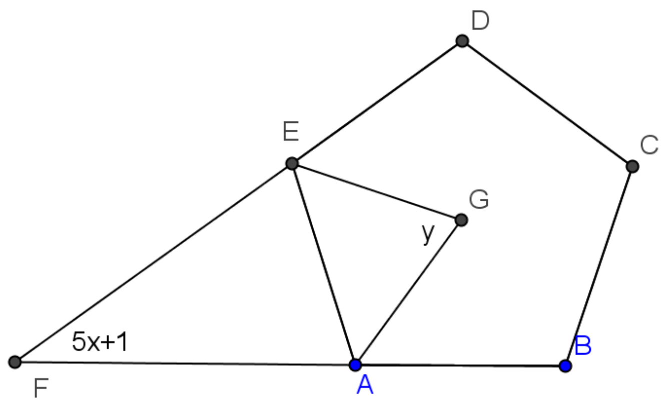
17. A forest ranger wants to determine the width of the lake so she sets up the two triangles below. How wide is the lake?



18. The exterior angles in triangle ABC are in the ratio of 1:3:5. What is the triangle's largest angle, or is the triangle impossible?

19. G is the center of regular pentagon ABCDE.

a. Find the values of *x* and y.



b. Is AGEF a kite?

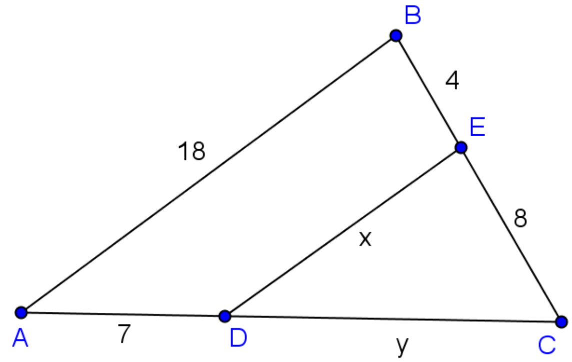
c. Is $\overline{AE}$ longer than $\overline{AG}$?

20. In the triangle below, $\overline{DE} // \overline{AB}$.

a. Find *x*.

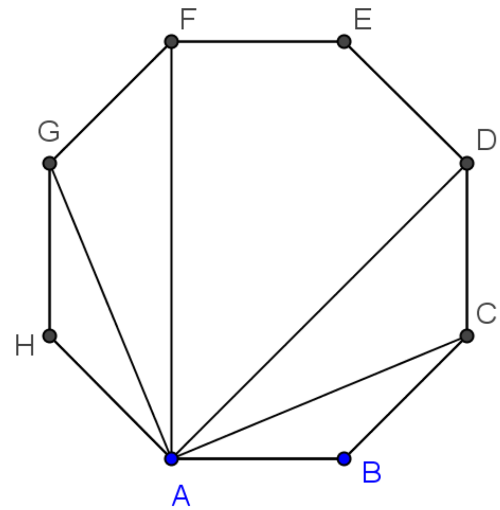
b. Find *y*.

c. Is angle E a right angle?



21. ABCDEFGH on the right above is a regular octagon.

a. What is the measure of each internal angle?

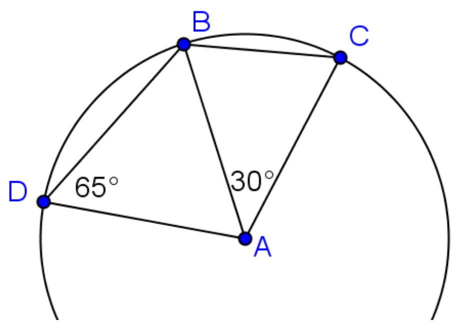


b. How many diagonals does it have?

c. Prove that $\overline{AD} \cong \overline{AF}$

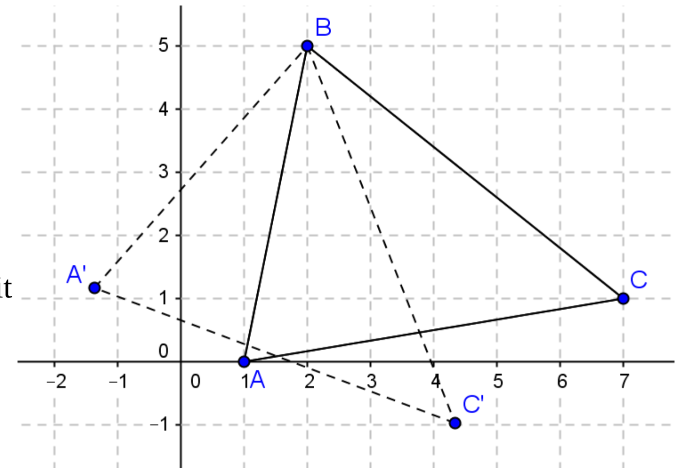
22. True or false: the diagonals of a trapezoid divide the trapezoid into four triangles and the "upper" and "lower" triangles of these four are always similar. Explain.

23. Point A is the center of a circle below. Find the measure of angle DBC.



24. Given triangle ABC below, where A=70° and C=50°, do the following:

a. If C is rotated 90° counter-clockwise around the A, where will it end up (note: this is not shown on the diagram)?



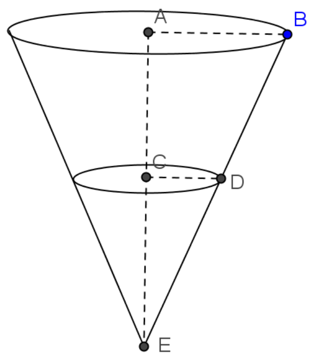
b. If A is rotated 90° clockwise around B, where will it end up (not shown on the diagram)?

c. Triangle ABC is rotated 30° clockwise around point B.

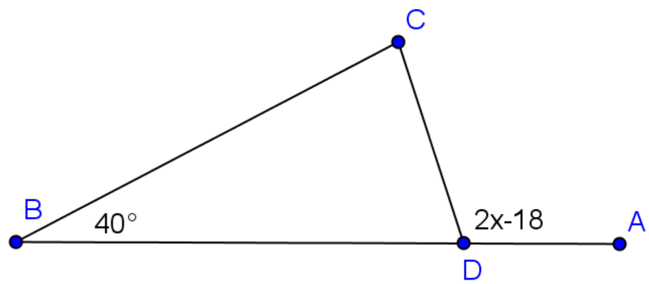
i. At what acute angle does $\overline{C'B}$ intersects $\overline{AC}$?

ii. If $\overline{AC}$ makes a 10° angle with the *x*-axis, then at what acute angle does $\overline{A'B}$ intersect the *y*-axis?

25. In the cone-shaped cup below, the height $\overline{AE}$ is 15 cm and the radius of the circular top is 7 cm. The cup is partially filled—the level of the water is 8 cm from the bottom (represented by length $\overline{EC}$). What is the radius of the water surface ($\overline{CD}$)? Hint: use similar triangles.



26. In triangle BCD below, side $\overline{BD}$ is the longest side and $\overline{CD}$ is the shortest. Find angle C in terms of *x* and then determine the restrictions on *x*.



27. The supplement of an angle is 20 degrees more than twice its complement. What is the supplement?

28. Polygon Properties: sometimes; always, or never

a. The diagonals of a kite are perpendicular to each other.

b. An equiangular polygon is also equilateral.

c. The diagonals of a parallelogram bisect the angles.

d. Adjacent angles of a parallelogram are supplementary.

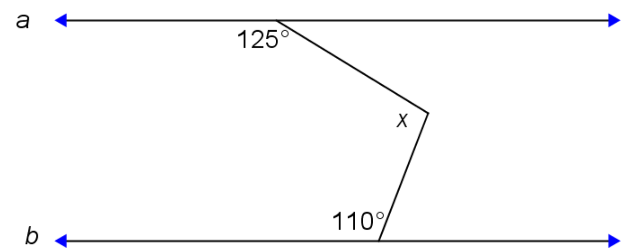
e. A parallelogram may have exactly one right angle.

f. A right triangle is equilateral.

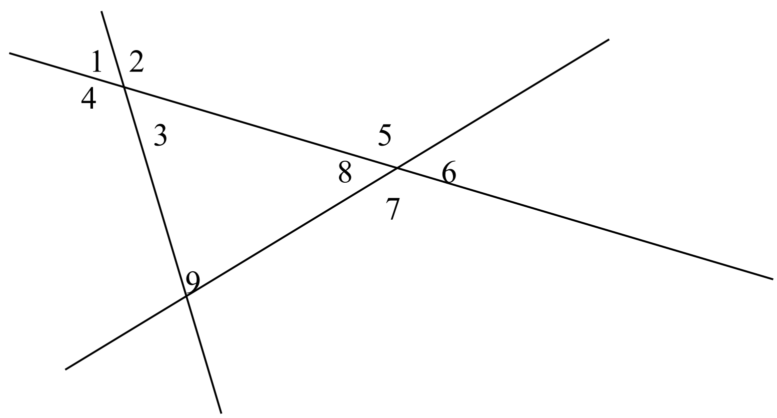
g. An equilateral triangle is isosceles.

h. An obtuse triangle has two acute angles.

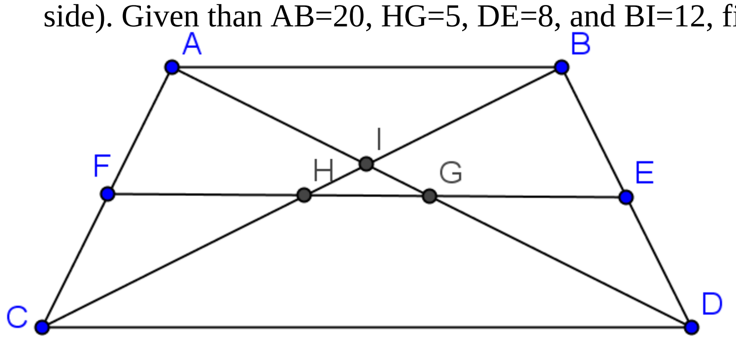
29. Lines *a* and *b* are parallel. Find *x*.



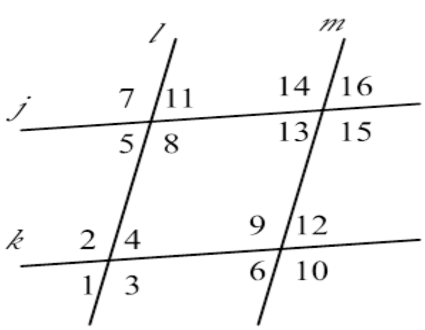
30. In the diagram below, angle 1 measures 80° and angle 2 is 40° more than angle 6. Find the measure of angle 9.



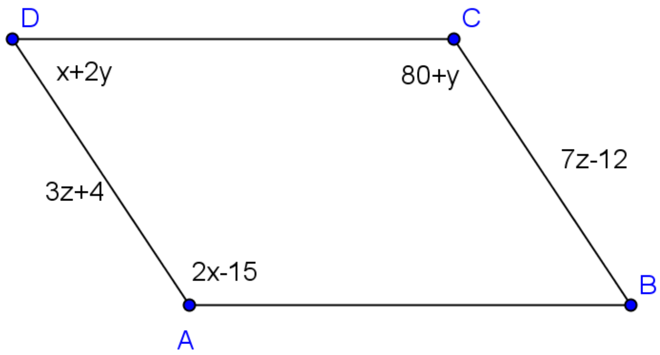
31. $\overline{EF}$ is the midsegment of trapezoid ABDC below (meaning it joins the midpoints of the opposite side). Given than AB=20, HG=5, DE=8, and BI=12, find FH, CD, HI, and CH.



32. In the diagram below, angles 4 and 12 are congruent. Angle 2 must be congruent to which other angles?



33. If ABCD is a parallelogram, find the values of *x*, *y*, and *z*.



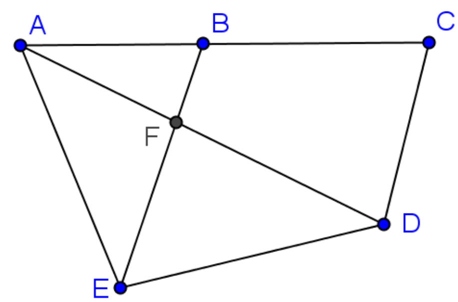
34. Find the following based on the diagram below:

a. $\overline{AD} \cap \overline{FA}$

b. $\overline{AC} \cap \overline{FE}$

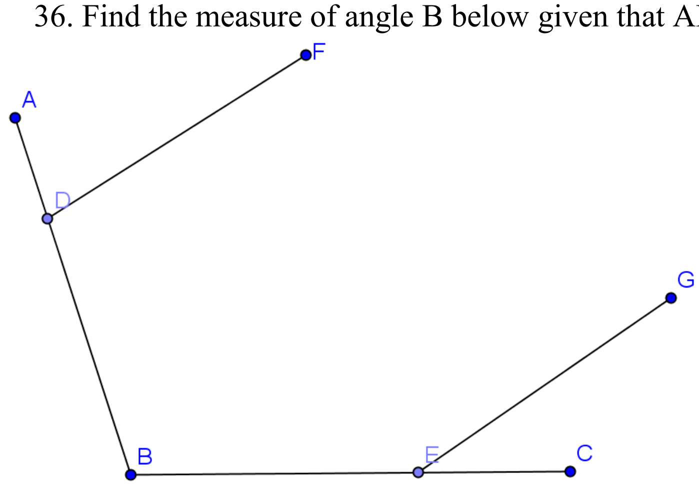
c. $\overline{BC} \cup \overline{BA}$

d. $\overline{FB} \cup \overline{FD}$

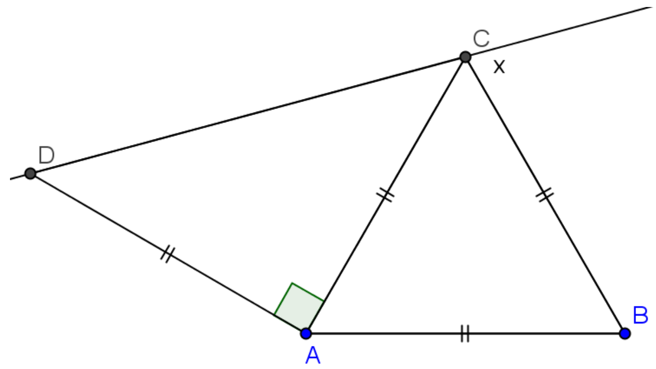


35. Given three points in a plane, describe how you could find the center of the circle containing them.

36. Find the measure of angle B below given that ADF=80°, CEG=35°, and $\overline{GE} // \overline{DF}$.



37. Find the value of *x* in the diagram below.



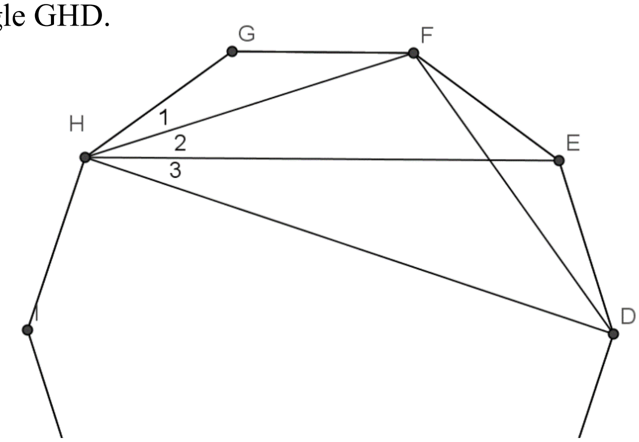
38. One angle of a ***right*** triangle is 5 times as large as another angle of that same triangle. Give all possble values for the smallest angle in the triangle.

39. ΔABC~ΔDEF.

a. B is 30° more than D and F is two thirds the measure of A. What is the measure of E?

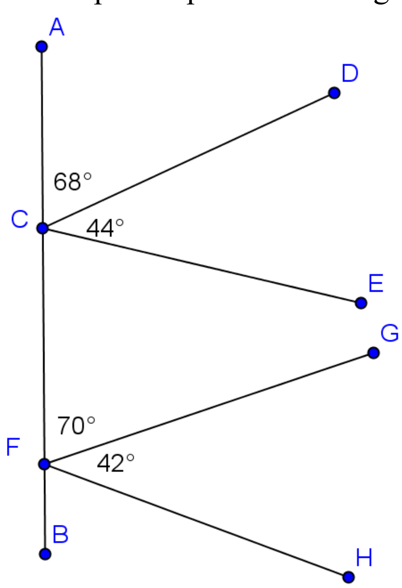
b. BC is three units longer than AB. DE is five units longer than AB. EF is six units longer than DE. Find AB.

40. DEFGH below are vertices of a regular decagon. It can be shown that $\overline{FG} // \overline{EH}$ (you don't need to prove it in this problem). Show that HE and HF trisect angle GHD.

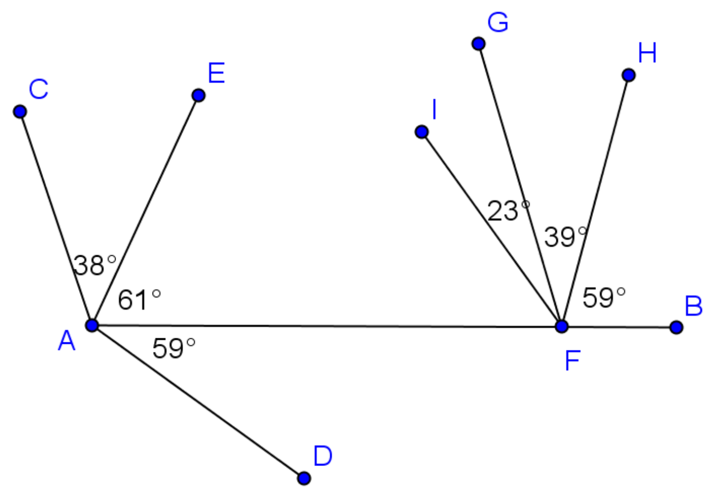


41. Find a pair of parallel line segments in the diagrams below and explain why they must be parallel

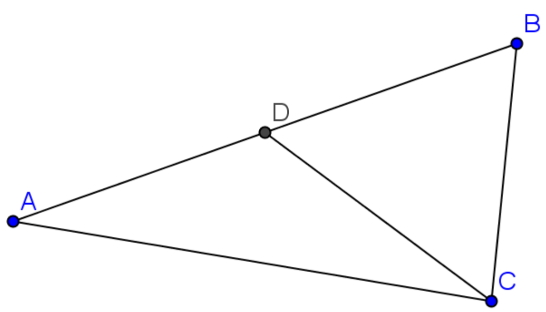
a.



b.

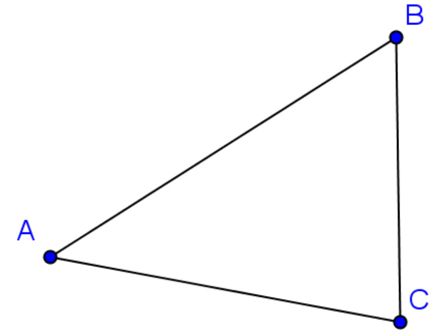


42. In triangle ABC, median $\overline{CD}$ is one half of the length side $\overline{AB}$. Find the measure of angle ACB. Hint: let angle B measure x degrees and find the measure of all angles in terms of x.



43. In triangle ABC, angle A measures x° and angle B measures $2x$-30°.

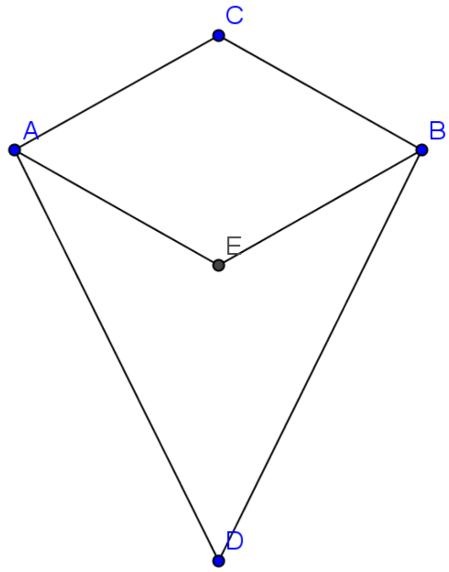
a. Find the measure of angle C in terms of x.



b. Assume that AC>AB>BC. What range of x's are possible?

44. ACBD below is a kite. Angle C measures 120° and CAD measures 105°. Point E is located in a way that makes ACBE a rhombus.

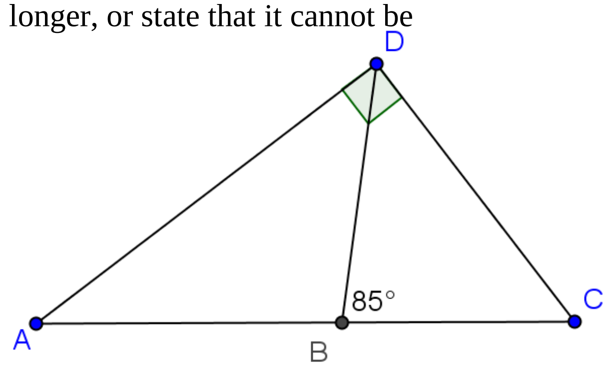
a. Find the measure of $\angle EBD$.



b. Which is longer, $\overline{BC}$ *or* $\overline{DE}$? Explain.

c. Explain why C, E, and D are collinear.

45. $\overline{DB}$ bisects angle D. Determine which segment in each pair is longer, or state that it cannot be determined.



a. $\overline{AB}$ *or* $\overline{BD}$

b. $\overline{AB}$ *or* $\overline{BC}$

c. $\overline{AD}$ *or* $\overline{CD}$

d. $\overline{AC}$ *or* $\overline{AD}$

46. ABC is an equilateral triangle; squares are then constructed on each side.

a. Is hexagon DEHIFG equiangular?

b. Is hexagon DEHIFG equilateral?

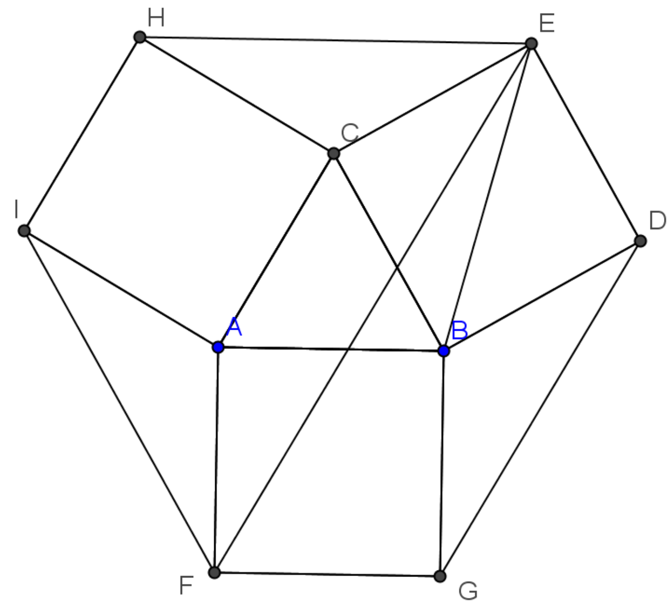
c. Give the name of any equilateral pentagon in the diagram (without drawing additional line segments). Is it equiangular as well?

d. Can H, C, B, and G be consecutive vertices of a regular polygon? If so, how many sides does it have?

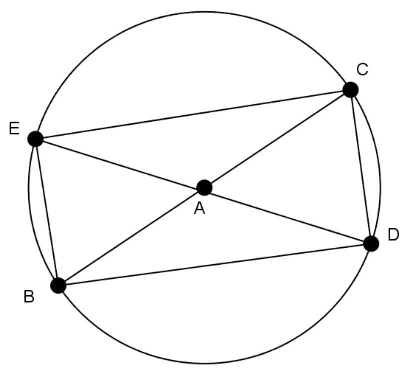
e. Can G, B, and E be consecutive vertices of a regular polygon? If so, how many sides does it have?

f. Find the measure of angles GBE and BEF.

g. Explain why ACEF is an isosceles trapezoid.



47. In circle A below, diameters $\overline{BC}$ and $\overline{DE}$ are drawn. Explain why BDCE must be a rectangle. You may use quadrilateral properties.



Answers

1. 150° 2. 65°, 45°, 45°, 65°, 70° 3. 29 4. 30 5. 105° 6. 8

7a. true b. false c. false d. true e. true 8. 3 9. w=70, x=6, y=2, z=100

10. "If A does not like Andy Roddick then A is not a tennis player" b. ii, iii 11. a, c, e

12. inverse: "if a man is not from Crete then he is not a liar"
Converse: "if a man is a liar then he is from Crete"
Contrapositive: "if a man is a not liar then he is not from Crete" last one is only that must be true

13 ACB=75° (x=39°) ; BC<AC<AB 14. 22<p<36 15. 100/3

16. 100° 17. 120m 18. Impossible; an exterior angle would need to be 200°

19. x=7; y=72; yes, opposite larger angle 20a. 12 b. 14 c. no; the Pythagorean Thm does not hold

21. 135° b. 20 c. yes, by using SAS twice $\Delta ABC \cong \Delta AHG$ *and* $\Delta ACD \cong \Delta AGF$

22. True: they are ~ b/c one pair of angles are vertical and the other two pairs are both alt interior angles.

23. 140° 24a. (0,6) b. (-3,6) c.i. 80° ii. 40° 25. 56/15 cm

26. C=2x-58 2x-58>198-2x>40 so 64<x<79 27. 160° 28. A, S, S, A, N, N, A, A

29. 125° 30. 40° 31. FH=10; CD=30; HI=3; CH=15 32. 9, 10, and 3 33. x=55; y=15; z=4

34a. AF b. Nothing c. AC d. Angle DFB

35. draw perp bisectors thru 2 pairs and see where they meet 36. 115° 37. 75° 38. 15° or 18°

39a. 86.25° b. 5 40. G=144 and HGF is isosceles so 1=18; since HE//FG (1+2)=180-144=36 so 2=18 also FDH is isosceles and F=108 so (2+3)=36 and 3=18

41a. CE and FH b. AD and IF 42. 90° 43. 210-3x b. 48<x<52.5

44a. 45° b. CB=BE and DE>BE so DE>BC c. angle CED=180° by adding CEB and DEB

45a. AB b. AB c. AD (look at triangle ACD) d = AC

46a. yes b. no c. ACEDB; no d. yes; 12 sides e. no since BG is not equal to BC
f. 165° and 15° (FBC is isosceles)
g. ACE=150° and CEF=30°…so bases are parallel and sides are congruent and not parallel

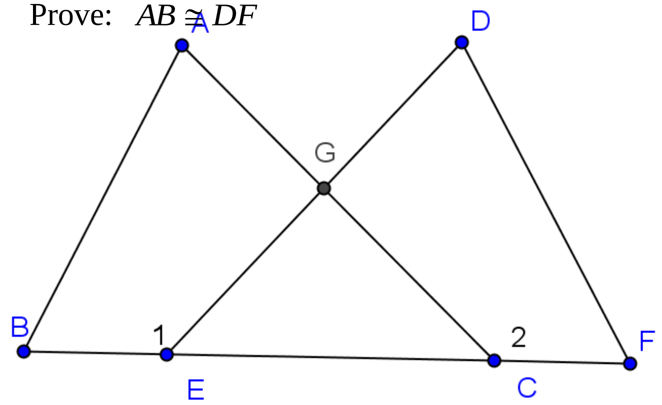
47. triangles BAE and CAD are congruent by SAS (radii and vertical angles) so angles ABE and ACD are congruent, making BE and CD parallel. Since they are also congruent (by CPCTC), a quadrilateral with sides parallel and congruent is a parallelogram. Triangle EBD is congruent to triangle CDB by SSS (diameters are same, EB=CD and DB=DB) so angles EBD and CDB are congruent. They are also supplements because EB and CD are parallel. Since they are congruent and supplementary, they must be right angles. Thus BDCE is a parallelogram with a right angle, so it is a rectangle.

Geometry 1 Final Exam Review Part 2: Proofs and Coordinate Geometry

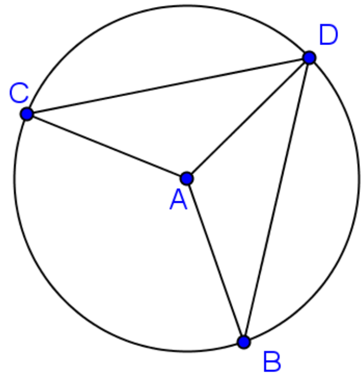
Proofs: Use a two-column or paragraph framework (be sure you can do some in a two-column way)

1. Given: $\angle 1 \cong \angle 2$, $\overline{BE} \cong \overline{CF}$, $\overline{AC} \cong \overline{DE}$

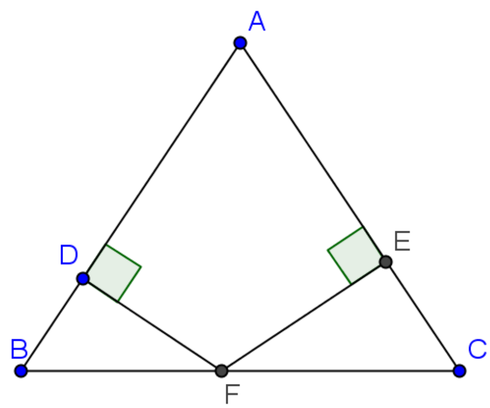
Prove: $\overline{AB} \cong \overline{DF}$



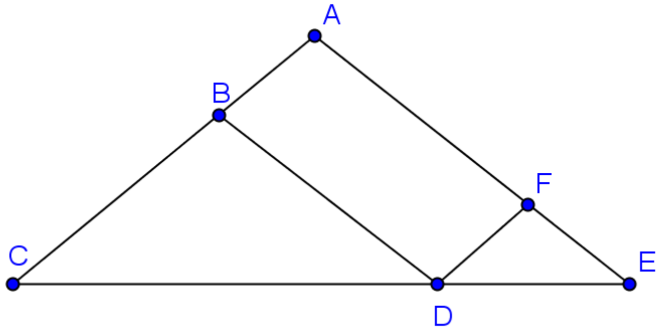
2. Given circle A and $\overline{CD} \cong \overline{BD}$, prove $\angle B \cong \angle C$



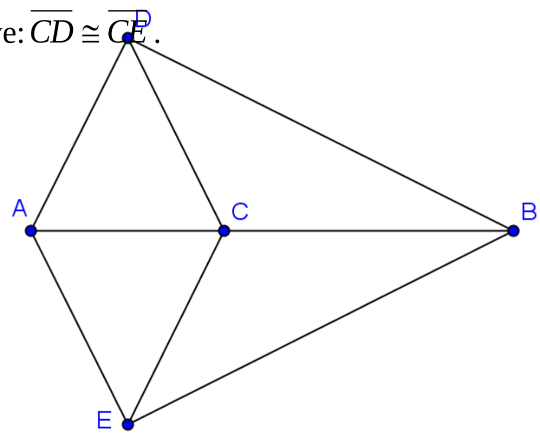
3. Given: $\overline{AB} \cong \overline{AC}$, $\overline{DF} \perp \overline{AB}$, $\overline{EF} \perp \overline{AC}$ Prove: $BD \cdot EF = CE \cdot DF$



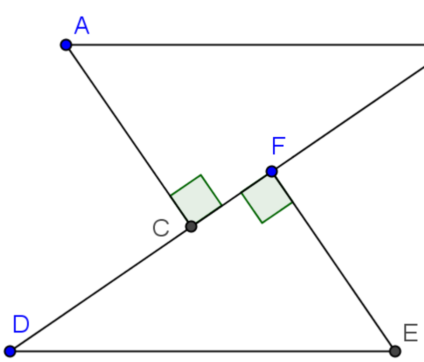
4. Given: $\overline{AC} \cong \overline{AE}$ $\angle CBD \cong \angle EFD$
Prove: BC/FE = CD/ED



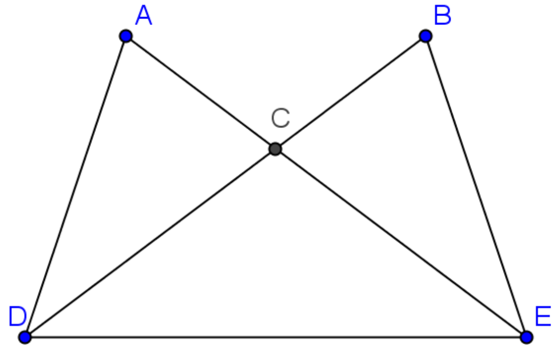
5. Given: A, B, and C are collinear, $\overline{AD} \cong \overline{AE}$, and $\overline{BD} \cong \overline{BE}$
Prove: $\overline{CD} \cong \overline{CE}$.



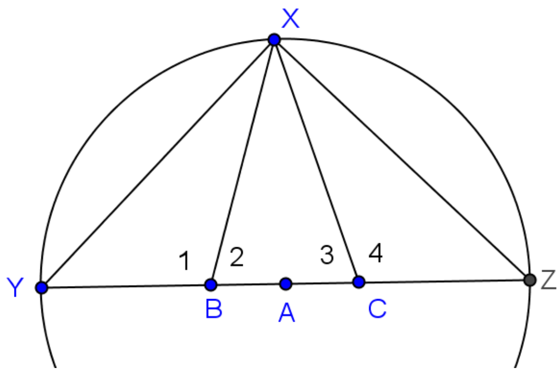
6. Given: right angles C and F; $\overline{AC} \cong \overline{EF}$, $\overline{DC} \cong \overline{BF}$ Prove $\overline{AB} \cong \overline{ED}$ and $\overline{AB} \parallel \overline{ED}$



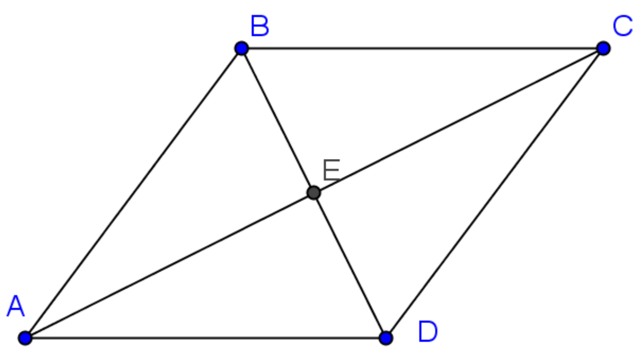
7. Given: $\overline{AC} \cong \overline{CB}$, $\angle A \cong \angle B$ Prove: ΔDCE is isosceles



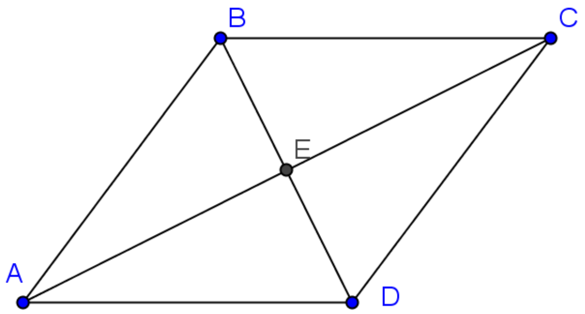
8. Given circle A where $\angle Y \cong \angle Z$, $\overline{BA} \cong \overline{CA}$ Prove: $\angle 2 \cong \angle 3$



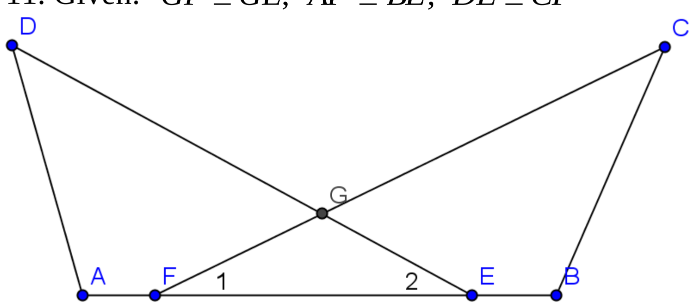
9. Given: $\overline{BC} // \overline{AD}$, $\overline{BC} \cong \overline{AD}$ Prove: $\overline{AB} // \overline{CD}$
(note: no using properties or definitions of quadrilaterals in this proof!)



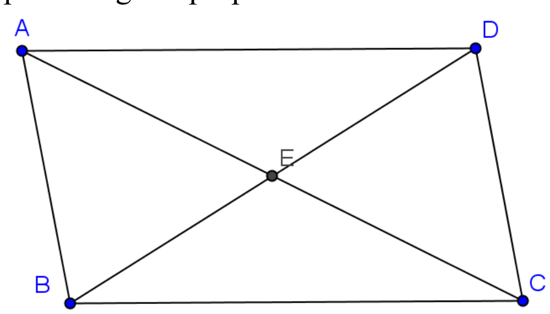
10. Prove that if the diagonals of a quadrilateral bisect each other that opposite sides are congruent.



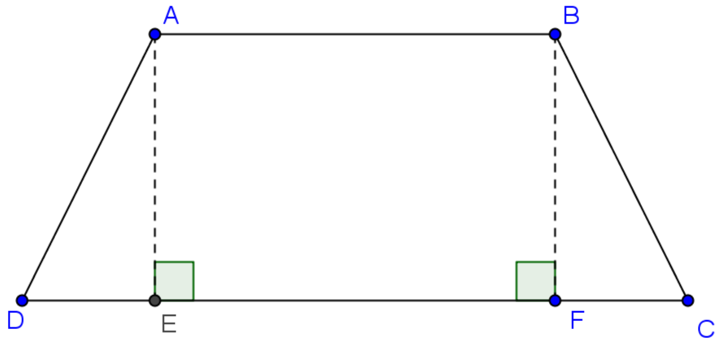
11. Given: $\overline{GF} \cong \overline{GE}, \ \overline{AF} \cong \overline{BE}, \ \overline{DE} \cong \overline{CF}$ Prove: $\angle C \cong \angle D$



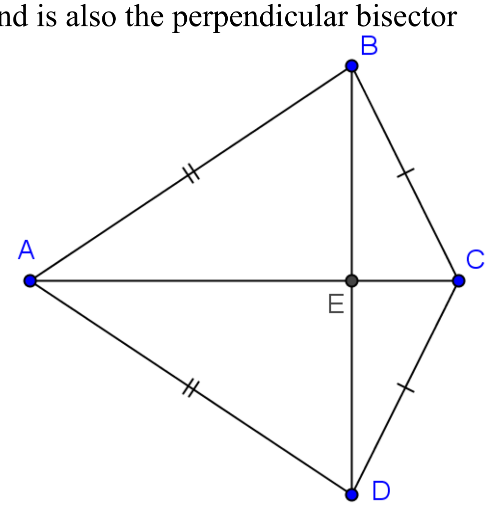
12. Prove that if the diagonals of a parallelogram are congruent that it must be a rectangle. No using parallelogram properties!



13. In the quadrilateral below, angles B and A are congruent, as are angles C and D. Prove it must an isosceles trapezoid. (paragraph proof is ok)



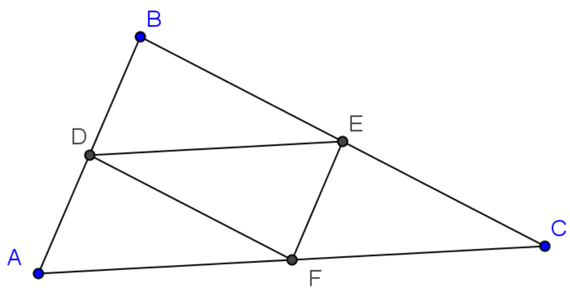
14. In the kite below, prove that $\overline{AC}$ bisects vertex angles A and C, and is also the perpendicular bisector of diagonal $\overline{BD}$.



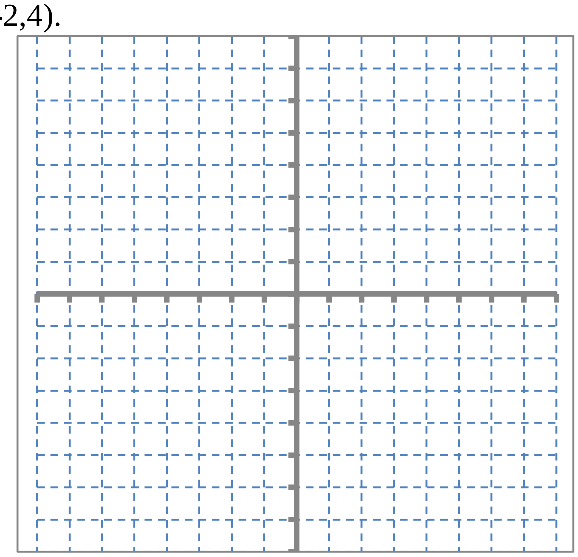
15 .A quadrilateral's sides are all congruent. Prove that the opposite sides are parallel (no using properties)

16. Prove that the diagonals of a rhombus bisect the angles.

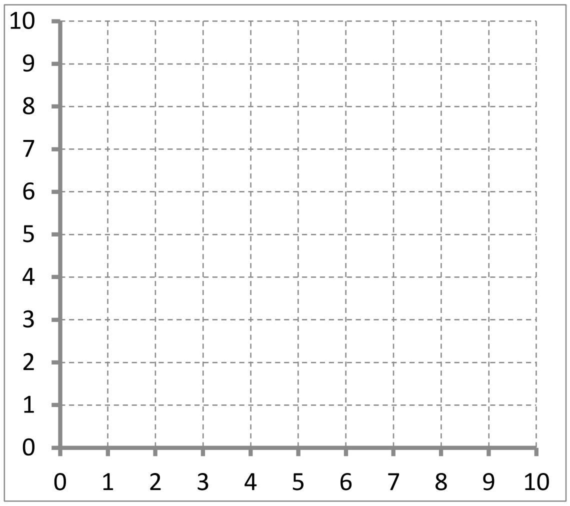
17. Given that D, E, and F are the midpoints of the sides of triangle ABC, prove that the four smaller triangles are all congruent.



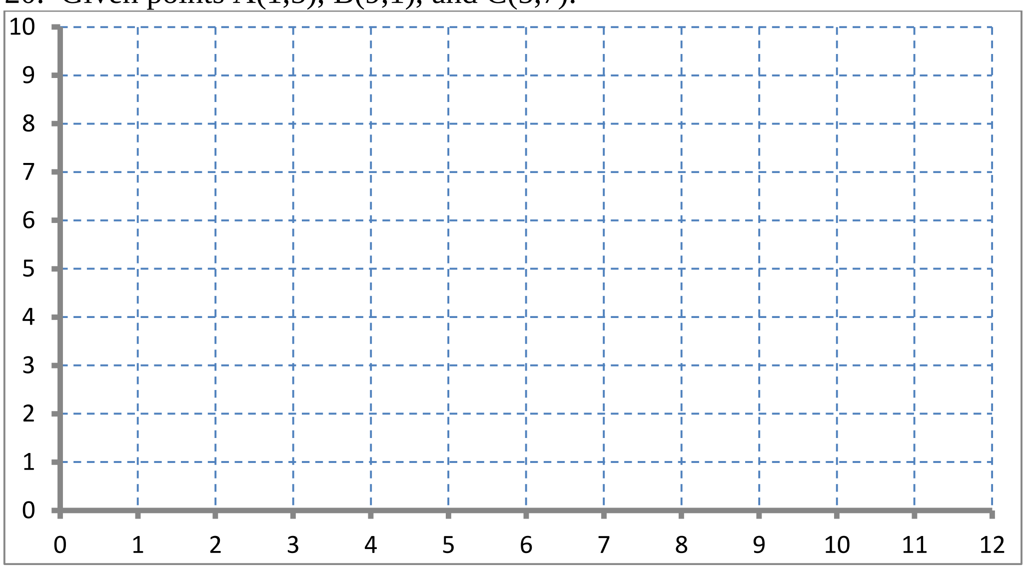
18. Find the equation of all points equidistant from A(6,0) and B(-2,4).



19. AB is the base of isosceles triangle ABC, where A's coordinates are (2,3) and B's are (6,9). Point C is located on the *y*-axis. What are its coordinates?



20. Given points A(1,5), B(9,1), and C(5,7):



a. Which angle of ΔABC is largest, and why?

b. If $\overline{AD}$ is a median of triangle ABC, then find the coordinates point D.

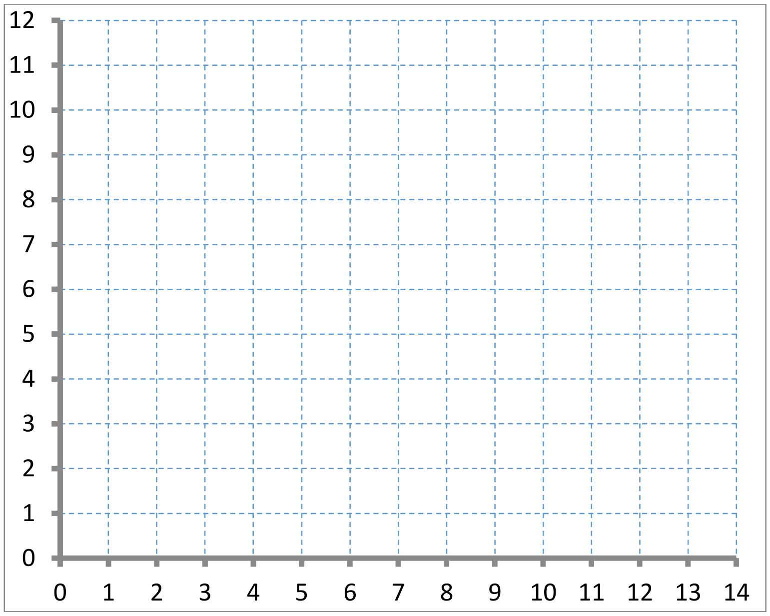
c. Let point E be on side $\overline{AB}$ such that $\overline{CE}$ is an altitude. Find the equation of line $\overline{CE}$.

d. Find the coordinates of point E.

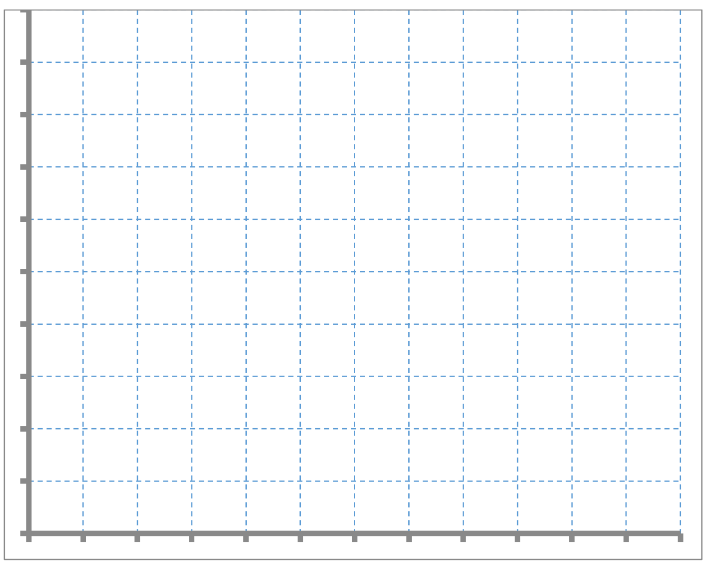
e. Where does the altitude $\overline{CE}$ intersect median $\overline{AD}$? Watch for fractions!

f. Point F has an *x*-coordinate of 10 and $\angle ABC \cong \angle BCF$. What is F's *y*-coordinate? There are actually two possibilities, but one is quite difficult to find (definitely extra credit potential!).

21. Given that three vertices of a parallelogram ABCD are A(3,1), B(5,8), and C(13,11). Find the coordinates of D then show that the diagonals $\overline{AC}$ *and* $\overline{DB}$ bisect each other.



22. Do the same as you did in the previous problem with the more general parallelogram ABCD where A(0,0), B(b,c), D(a,0) where a, b, and c are all positive.

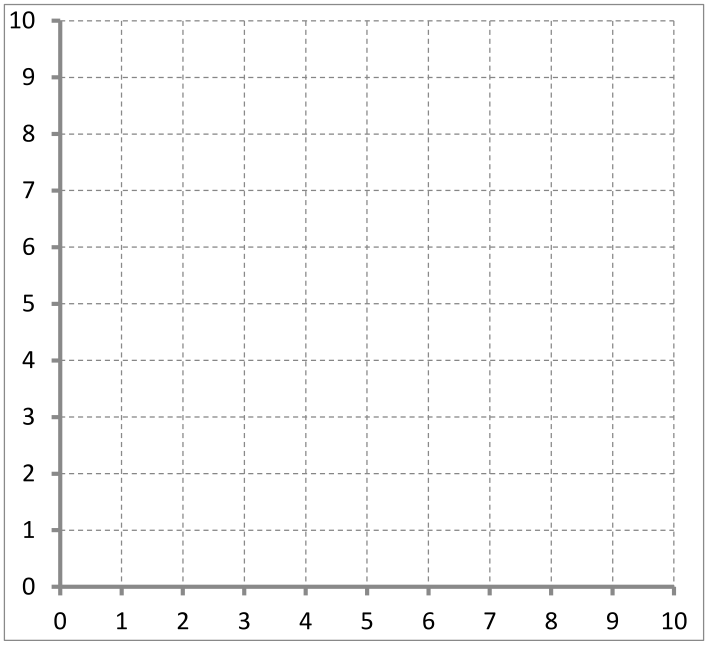


23. Given triangle ABC whose vertices have coordinates A(0,0), B(2,8) and C(10,4). Answer the following questions:

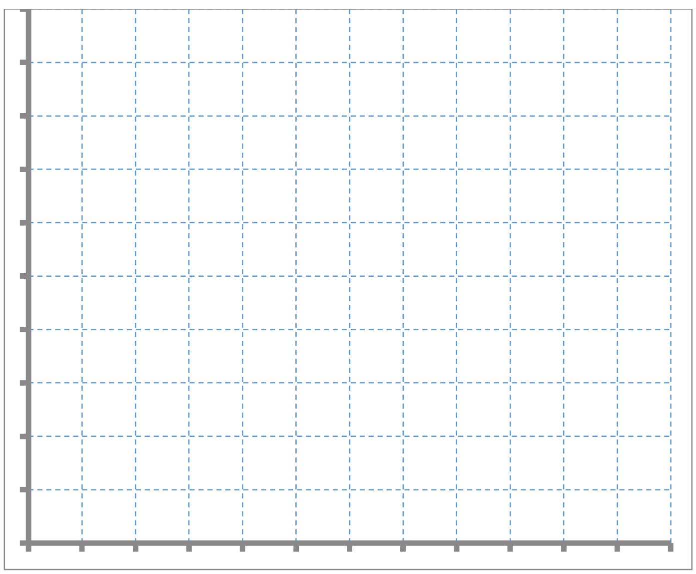
a. What are the exact coordinates where the median from B intersects side $\overline{AC}$?

b. What are the exact coordinates where the altitude from B intersects side $\overline{AC}$?

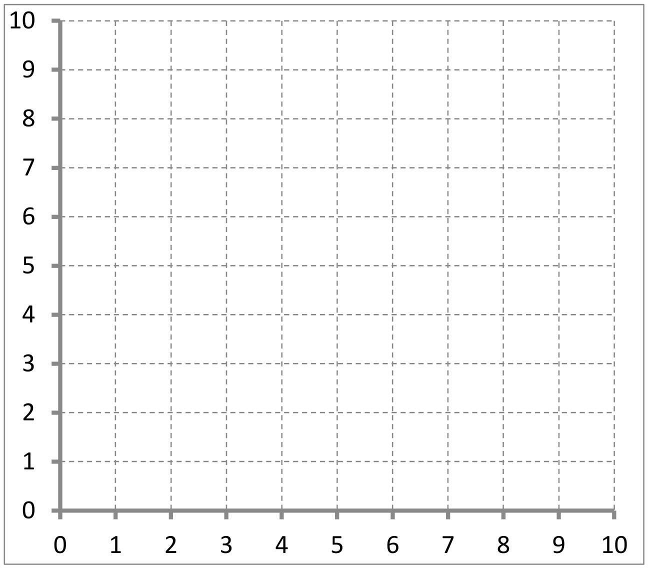
c. Triangle ABD is isosceles with $\overline{AD} \cong \overline{BD}$. Give any three possible coordinates for point D.



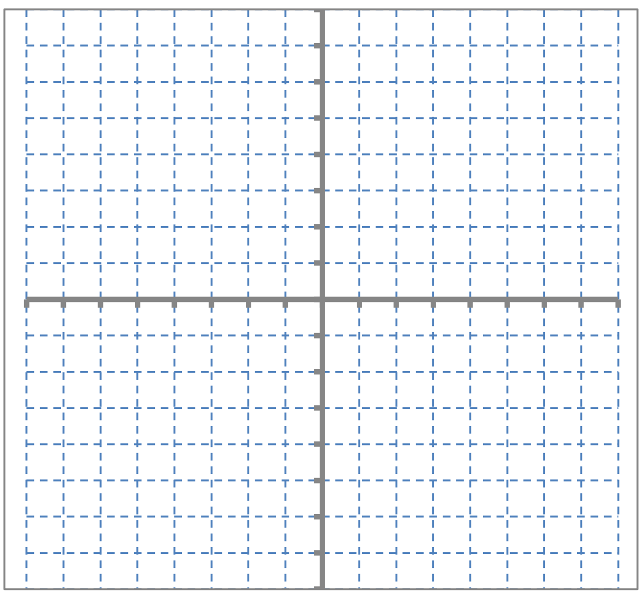
24. Prove that the segment connecting midpoints of two sides of a triangle is parallel to the third side and half of its length.



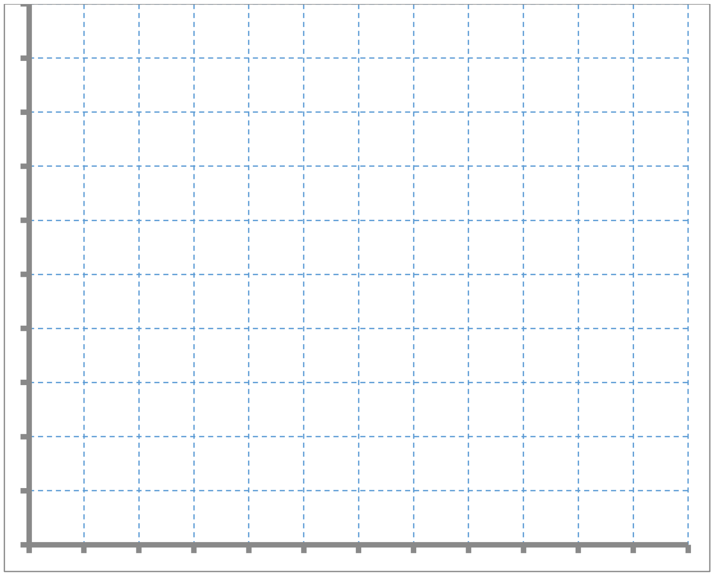
25. Give possible coordinates of a rhombus where no sides are parallel to the *x* or *y* axis.



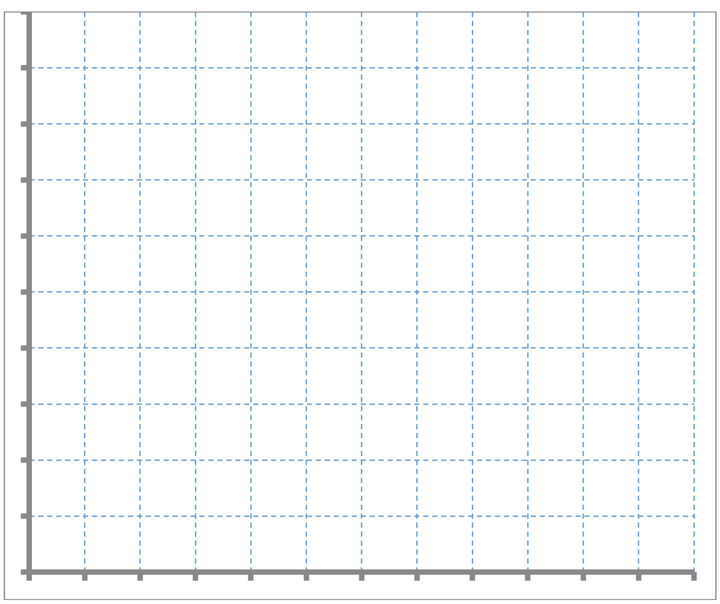
26. Given points A(0,3), B(4,9), C(5,4), and D (6,-1) prove that $\overline{AC}$ bisects angle BAD.



27. Given that three vertices of a parallelogram are (0,*a*), (*b*,*c*), and (*d*,*e*), find possible coordinates for the fourth. Assume all constants are positive.



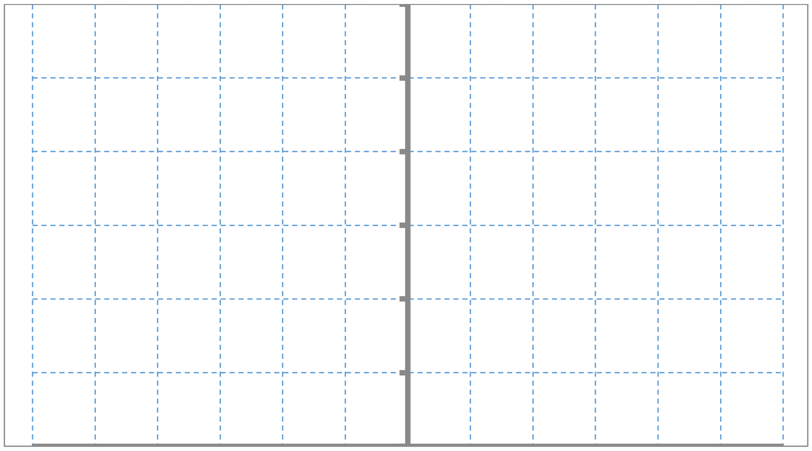
28. Use coordinate geometry to show that the midpoints of the sides of a rectangle form a rhombus.



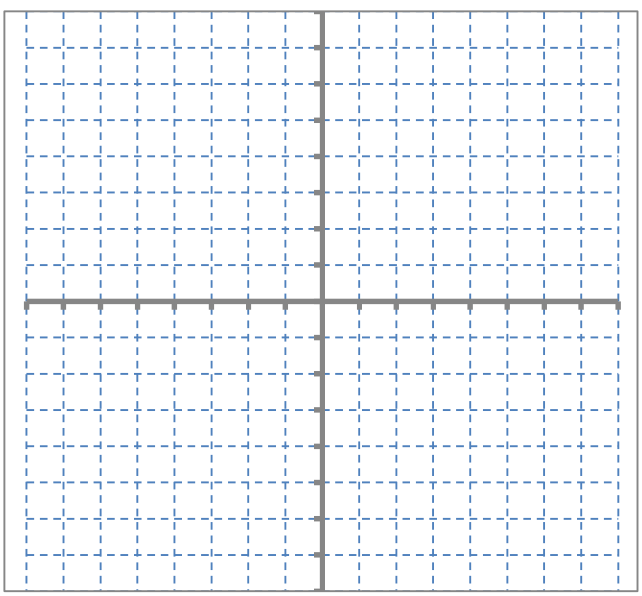
29. Put a "generic" isosceles trapezoid on the coordinate plane and use it to prove

a. Its diagonals are congruent.

b. Connecting the midpoints of its adjacent sides forms a rhombus.



30. Given points A(-4,-8), B(8,8), C(1,7), and D (-5,-1). Verify that ABCD is an isosceles trapezoid. Remember: there are **<u>three things</u>** that must be true for ABCD to be an isosceles trapezoid!



Solutions / Answers

1.

1) $\angle 1 \cong \angle 2$	1) Given
2) $\overline{BE} \cong \overline{CF}$	2) given
3) $\overline{AC} \cong \overline{DE}$	3) given
4) BC = FE	4) 2; addition property (adding EC to both)
5) < DEF = <ACB	5) 1; supplements of equal angles are equal
6) ΔDEF=ΔACB	6) 3, 4, 5; SAS
7) AB = DF	7) 6; CPCTC

2.

1) circle A and $\overline{CD} \cong \overline{BD}$	1) Given
2) AC=AD=AB	2) 1; all radii are congruent
3) ΔACD = ΔABD	3) 1, 2; SSS
4) $\angle B \cong \angle C$	4) 3; CPCTC

3.

1) $\overline{AB} \cong \overline{AC}$	1) given
2) angle B=angle C	2) 1; in Δ = sides → = opposite angles
3) <FDB=<FEC	3) given; all right angles are =
4) ΔBDF~ΔCEF	4) 2, 3; AA
5) BD/EC=DF/EF	5) 4; def of similarity
6) (BD)(EF)=(EC)(DF)	6) 5; multiplication principle

4.

1) $\overline{AC} \cong \overline{AE}$	1) Given
2) $\angle CBD \cong \angle EFD$	2) Given
3) <C = < E	3) 1; Δ = sides → = opposite angles
4) ΔCBD ~ΔEFD	4) 2, 3, AA
5) BC/FE=CD/DE	5) 4, Definition of Similarity

5. use SSS to get big triangles congruent; then CPCTC to get congruent angles; then SAS in smaller triangle.

6. DF=BC so right triangles are = by SAS; AB=DE by CPCTC and // because <D=<B by CPCTC and = alternative interior angles implies parallel lines

7. ΔACD=ΔBCF by ASA (using ACD and BCF as vertical angles); Therefore CD=CE by CPCTC..

8. XY=XZ b/c isosceles triangle; BY=CZ since radii are = and we subtract ='s from them…so triangles BYX and CZX are congruent, making angles 1 & 4 =. Then 2&3 are equal since they have = supplements.

9. angles BDA and DBC are = since alt interior; angles BEC and AED are vertical; so triangles BEC and AED are = by AAS; then use CPCTC to get triangles DEC and BEA =; then CPCTC and alt interior..

10. triangles BEC and DEA are = by SAS (using vertical angles) so AD=BC; do same thing with triangles ABE and CDE.

11. angles 1 & 2 are = b/c FGE is isosceles; so triangles DAE and CBF are = by SAS; so C=D by CPCTC

12. AD//BC and CD//AB by def of parallelogram; angles DBC and BDA are =, as are CDB and ABD. So triangles ABC and CDB are congruent and BC=AD and AB=CD. Then triangles DCB and ABC are = by SSS so angles ABC and DCB are = by CPCTC. Since lines are parallel, these two angles must be supplements. If they are = and supplements then they are right angles. And a parallelogram with at least one right angle is a rectangle.
13. since A+B+C+D=360 and B=C and A=D we know B+C=180 so AB//CD. AE//BF since adjacent inter angles are supplements. Thus ABFE is a rectangle and AE=BF. Then triangles AED and BFC are = by AAS so BC=AD by CPCTC….
14. triangles ABC and ADC are = by SSS so using CPCTC, AC bisects BAD and BCD. Then using SAS (EC, ECB, BC), triangles BEC and DEC are =. So ED=BE (thus bisects) and angles BEC and DEC are =. And since they are equal and supplementary, they are right angles.
15. draw a diagonal; two triangles are = by SSS. So CPCTC gives you some = angles which establish parallel lines
16. Draw a diagonal; triangles are = by SSS. Get a pair of = alternate interior angles; but both triangles are isosceles. So transitivity shows diagonal bisects vertex angle…
17. because midlines are // to sides, we know angles BDE and DAF are =, as are DBE and ADF.. so by ASA we get triangles BDE and DAF =… same with EFC.. and can get any of those = to middle triangle using SAS
18. perp bisector is perp bisector is y=2x-2 19. (0, 26/3)
20a. angle C is opposite the longest side b. (7,4) c. y=2x-3 d. (17/5, 19/5) e. (49/13, 59/13)
f. "the easier one": make CF//AB so CF's equation is y= -0.5x+9.5 and when x=10, y=4.5
"the harder one": make BCG an isosceles triangle by placing G at the intersection of the perp bisector of BC and side AD, which is (37/7, 20/7). The equation of CG is y = -14.5x + 79.5 when x=10 y=-65.5
21. D=(11,4); hard way: AC's eqn is y=x-2 BD's is y=(-2/3)x+34/3 meet at (8,6) and can use distance formula a bunch of times easy way: midpoint of AC=(8,6) and midpoint of BD=(8,6) → therefore they must bisect each other
22. C is (b+a,c).. same midpoint: 23a. (5,2) b. (130/29, 52/29) c. anywhere on the line y=(-1/4)x + 17/4
24. let triangle be A(0,0), B(2a,0) and C(2b,2a): one midsegment connects (b,c) to (a+b,c); slope is 0 so parallel to AB and length is a, so half the length of AB
25. many possible—easiest if diagonals are parallel to coord axes
26. using SSS, triangles ACD and ACB are congruent, so by CPCTC
27. hard! (d-b, e+a-c) , (d+b, e-a+c) , (b-d, a-e+c)
28. rectangle is (0,0), (2a,0), (0,2b), and (2a,2b) – midpoints are (a,0), (2a,b), (0,b) and (a,2b) and using distance formula, adjacent ones are all $\sqrt{a^2+b^2}$ apart.
29. (2a,0), (-2a,0), (2b,2c), and (-2b,2c)

a. diagonals lengths are $\sqrt{(2a+2b)^2+(2c)^2}$ *and* $\sqrt{(2b+2a)^2+(2c)^2}$ which are equal

b. midpoints are (0,0), (a+b,c), (0,2c), and (-a-b,c) and adjacent ones are $\sqrt{(a+b)^2+c^2}$ apart

30. AB//CD since slopes are both 4/3; BC=AD= $\sqrt{50}=5\sqrt{2}$; and thirdly, the legs cannot be parallel, else it is a parallelogram—and BC is not parallel to AD since slopes are 1/7 and -7 (it is a coincidence that they are perpendicular—it means that this particular isos trapezoid can be thought of as part of an isosceles right triangle—a generic isos trap is just part of any isos triangle)