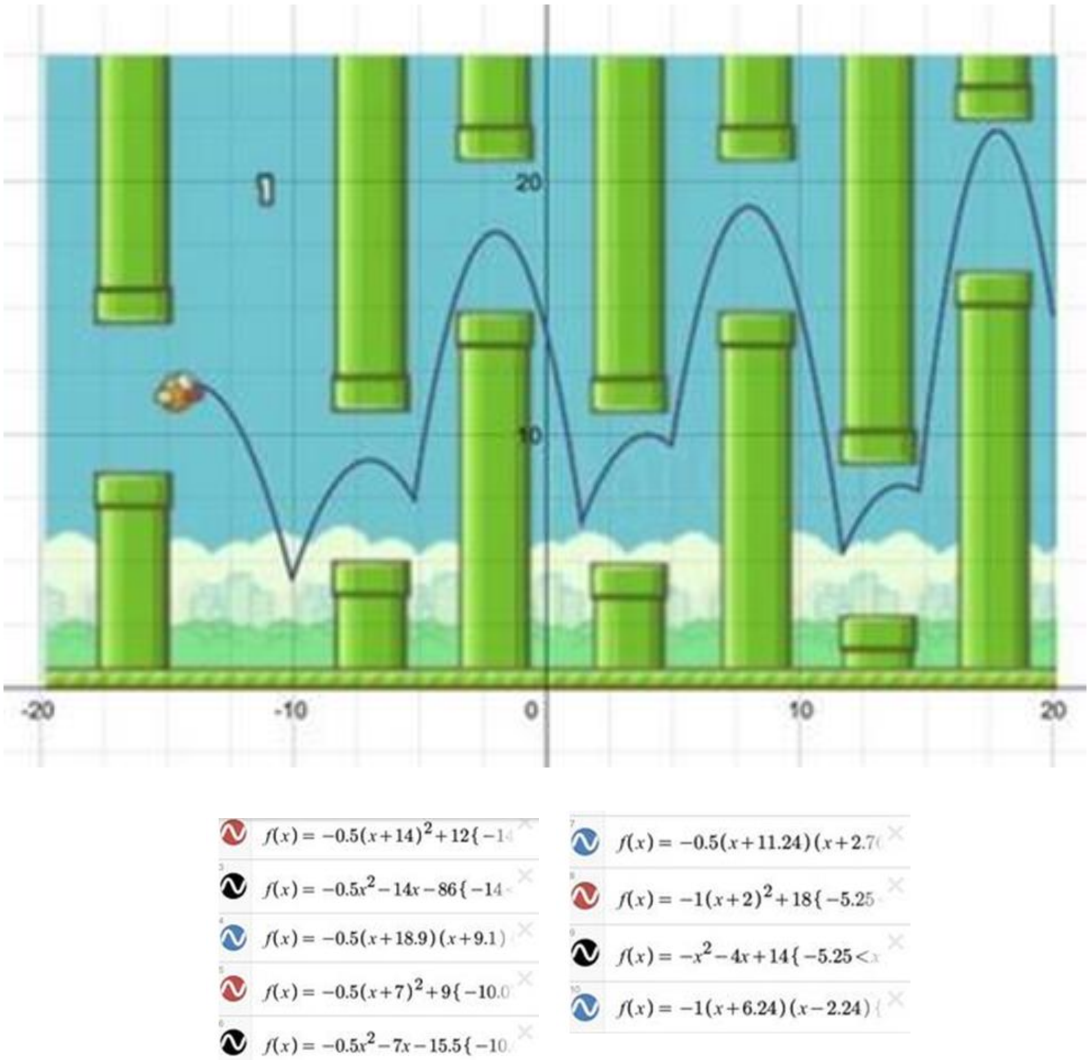
Honors Algebra 2



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Advice for students:

1. This is an honors class. We will typically cover new material fairly quickly. Once we have gone over the key fundamentals with fairly rote problems, we will often apply the new material to more challenging non-routine problems. These can be difficult, and one of my main goals this semester is to help you develop important problem-solving skills. In the face of challenges, try to be persistent, patient, and creative. Be proactive about seeking help when you need it. We do not expect everyone to get all homework problems correct.

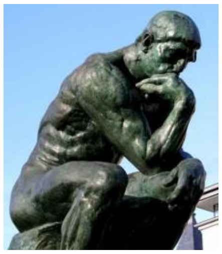
2. In sports, you have practices. In performing arts, you have rehearsals. In academics, you have homework. Homework in this class is designed to help you cement your understanding of the material by practicing straightforward problems and develop your problem-solving skills. Do your best to trying all the homework questions. If you have difficulty, come to the next class with specific questions that will help you advance your understanding.

3. Every problem set in this book comes with answers. They are at the end of each individual problem set. Checking your answers is essential. We recommend that you check answers after every few problems to make sure you are on the right track. We all make mistakes. Please let us know if you think you have found an error in the answers.

4. You will never need a formal textbook in class or for homework. All problems will be assigned from this book or supplemental handouts. You do not need to buy the green Algebra 2 book. Some of you may decide to buy it. Others can always use a library or math department copy if they want to look up the book's explanation. Still others will not use the book at all, and use Google or friends or parents when they need more explanation. For those using the book, we have tried to include textbook references where applicable in the title of each handout.

5. HELP!!! Get help when you need it. Some places (in no particular order):
- -Classmates
- -Parents
- -Your teacher
- -Internet (believe it or not, Google can help you find great explanations and practice problems)
- -Khan Academy videos: goto www.khanacademy.org -- the first section is Algebra.

6. Think more… memorize less!



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Unit 1 Warm-Up

Max and Daniel decide to head to Fresh Pond in Cambridge after school. They plan on hanging out there and having a picnic. Max skateboards east, through Monument Square, onto Concord Rd. and eventually onto the Minuteman Commuter Bikeway into Cambridge. He leaves CA at 2:45 pm and maintains a constant speed of 6.5 miles per hour on his skateboard. Daniel wants to go to Cumbies first to get a slushie, so he tells Max that he'll meet him in Cambridge. Daniel goes to Cumbies and then comes back to CA. He finally leaves CA at 3:05 pm on his bicycle and maintains a constant speed of 8.25 miles per hour.

The distance from CA to Cambridge is about 16.25 miles.

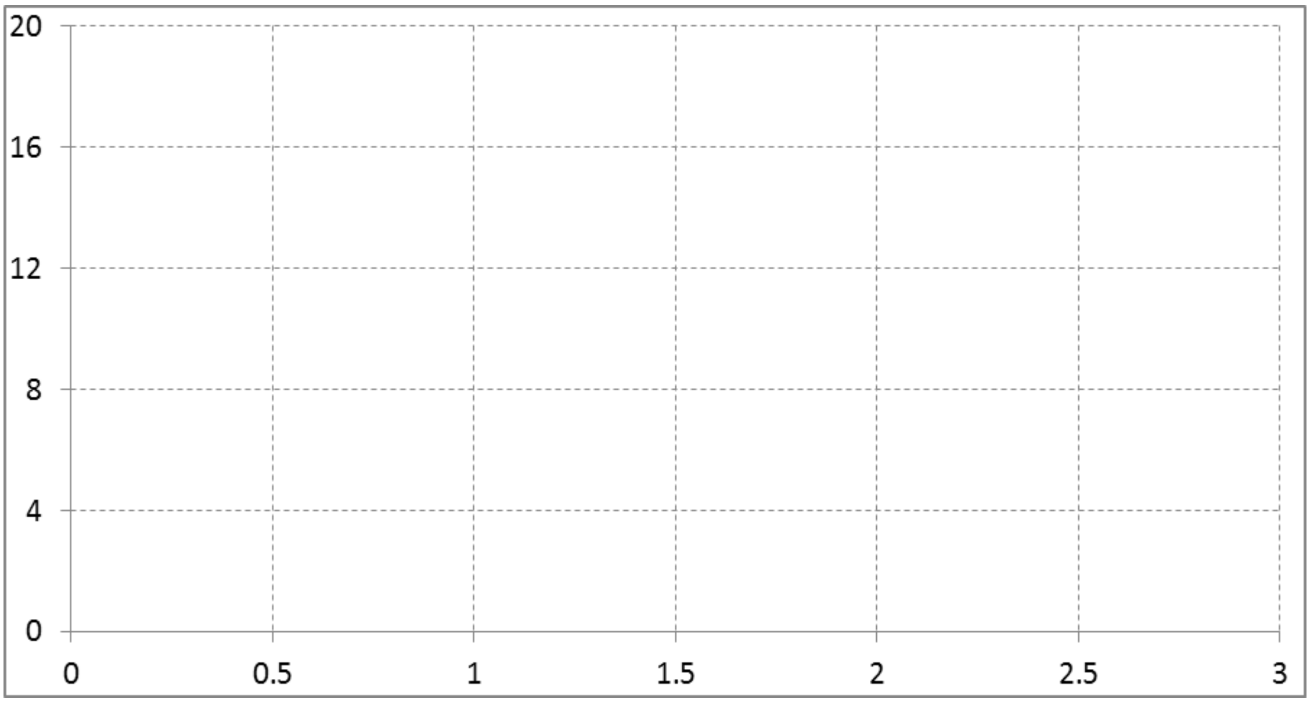
1. What time is it when Max arrives at Fresh Pond? Daniel?

2. Daniel catches up to Max along the way to Cambridge. How do you know?

3. When did Daniel catch up to Max? How far from CA were they at that point?

4. What is the equation that represents Max's distance from CA as a function of time elapsed? What about the equation for Daniel?

5. Graph the two functions you wrote in the prior question. What is the significance of the intersection?



Unit 1 Handout #1: Function Representations

1. A relation between variables *x* and *y* is defined by the table below. Use it to answer the questions below (note: these are the only inputs and outputs):

x	*y*
3	1
5	2
6	5
8	0
-2	5
-5	6

a. What are the domain and range of this relation?

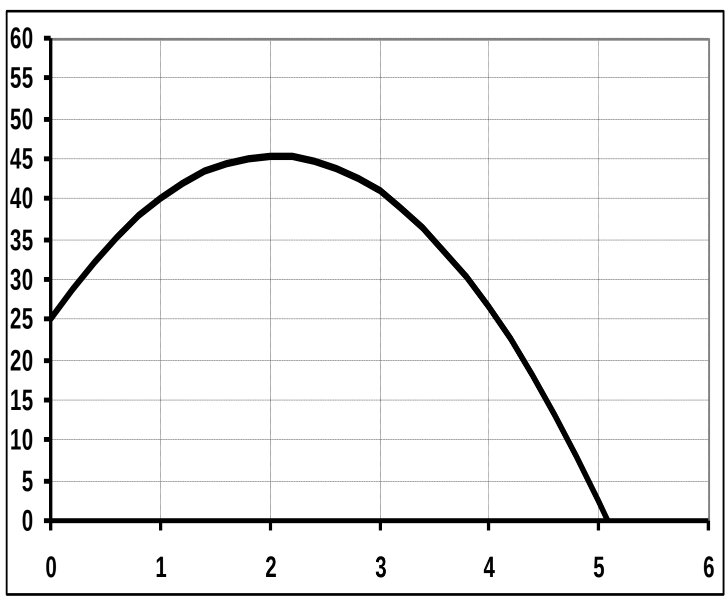
b. What is the output that corresponds to an input of 6? Note: this is $f(6)$.

c. What is the input or inputs that correspond to an output of 6? Note: this is $f(x) = 6$.

d. At what input values, if any, does the graph of the relation intersect the *x*-axis?

e. What is the difference between $f(5)$ and $f(x) = 5$? Answer numerically as well as in terms of input and output.

2. The graph below shows how the height of a ball (output variable *y*) relates to the time since it was thrown (input variable *x*).



a. What are the approximate domain and range of this relation?

b. What is the height 2 seconds after the ball is thrown?

c. When is the height equal to 20 meters?

d. When does the ball land?

e. Give any 2 times where the ball's height is at least 30 meters. For about how long is the ball's height at least 30 meters?

3. A relation is defined algebraically as $f(x) = 3x - 4$. The domain is all real numbers, positive and negative.

a. Give the coordinates of any three points on the graph of this relation:

b. What is $f(-2)$.

c. Solve the equation $f(x) = 17$.

d. Fill in the missing variable in the table below:

x	*f(x)*
0	
-5	
	5
	0
	4/3

e. The ***zero*** of a function is the input (or inputs) where the output is zero. It is similar to the *x*-intercepts (though there is a difference we will see when we get to imaginary numbers). What is the zero of $f(x)$?

4. Alexi is a babysitter. She gets paid \$8 per hour; she also gets a \$5 tip each night she works.

a. We want to look at the relationship between the number of hours she works and the amount she makes. What should the input (independent) and output (dependent) variables be?

b. Fill in the table below with the missing inputs and outputs

Input	*Output*
1	
3	
	21
	11
	17

c. What is a reasonable domain for this relation?

d. Write an equation that shows the algebraic relationship between the input and output variables.

e. Sketch a rough graph of this relation.

5. Given the function $f(x) = -2x^2 - 3x + 1$ do the following:

a. Give the coordinates of any two points on the graph of this function.

b. What $f(-3)$?

c. What are the coordinates of the *y*-intercept of the graph of this function?

d. Are there any numbers that cannot be input into this function? If so, what are they? What does this tell us that the domain of this function is?

6. Answer the following questions about $f(x) = -2x + 12$.

a. What is $f(11)$?

b. What is the difference between $f(3)$ *and* $f(x) = 3$?

c. What shape is the graph of $f(x)$? Why?

d. What are the coordinates of the *x*- and *y*-intercepts of $f(x)$?

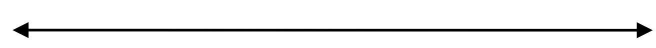
e. What are the coordinates where the graph of $f(x)$ intersects the graph of $g(x) = x - 5$? Hint: use *x*'s and *y*'s and solve this system of equations using either substitution or elimination (aka linear combination).

f. The output of $f(x)$ is at least 5 for what inputs? In other words, solve the inequality $f(x) \geq 5$, which is $-2x + 12 \geq 5$. Graph your solution on a number line.

7. Given the linear functions $f(x) = -x + 6 \;\; and \;\; g(x) = 3x - 4$, do the following:

a. Find the coordinates of the point of intersection between the graphs of $f(x)$ and $g(x)$.

b. For what x values is $f(x) > g(x)$? In other words, solve the inequality $-x + 6 > 3x - 4$. Give your answer symbolically (ie,,, x<___) and also graph it on a number line.



8. Write the equation of a linear function $f(x) = mx + b$ where $f(7) = 12 \;\; and \;\; f(11) = -4$.

9. The function $f(x)$ is a linear function whose y-intercept is 7. Its zero is 3, meaning $f(3) = 0$.

a. Find the function $f(x)$.

b. Where does the graph of $f(x)$ intersect that of $g(x) = -3x + 11$?

10. Find the missing parameters: (note: for a linear function $f(x) = mx + b$, the slope m and y-intercept b are parameters—they determine how a particular line looks)

a. If $f(x) = -2x + k$ and the graph of $f(x)$ goes through the point (-3,-8) then find k.

b. If $f(x) = mx + 5$ and $f(-2) = 8$ then find m.

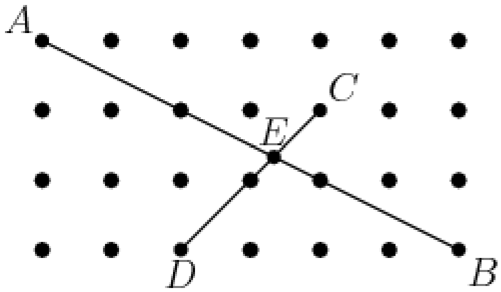
c. If $f(x) = -\frac{3}{5}x + k$ and $f\left(\frac{7}{9}\right) = \frac{11}{2}$, then find k.

d. If $f(x) = 2x + k$ and $g(x) = 5 - 2x$ and the graphs of $f(x)$ and $g(x)$ intersect when x=7, then find k.

e. For $f(x) = -x^2 + bx + 1$, you are told that $f(-2) = 7$. What is b?

f. For $f(x) = 2x^2 + bx + c$, you are told that $f(1) = 9$ *and* $f(-2) = 3$. What are b and c? Hint: this will involve solving a system of two equations with two variables.

11. The diagram shows 28 lattice points, each one unit from its nearest neighbors. Segment *AB* meets segment *CD* at point *E*. Find the length of segment *AE*. Leave your answer as a radical. Hint: you may want to put the coordinate axes on the diagram.



Answers

1a. domain {-5,-2,3,5,6,8} range {0,1,2,5,6} b. 5 c. -5 d. 8 e. $f(5)=2$, when input is 5 output is 2 the solutions; the solns to $f(x)=5$ are x=6 or -2; output is 5 when input is -2 or 6

2a. domain 0≤x≤5.1 (or so) range: 0≤y≤45 (or so) b. 45 c. about 4.3 sec d. about 5.1 sec
e. from 0.25 seconds until 3.75 seconds (roughly)—so for 3.5 seconds

3a. (-2,-10), (-1,-7), (0,-4), (1,-1), (2,2), (3,5), …. b. -10 c. 7
d. see table on left below e. x=4/3

x	*y*
0	-4
-5	-19
3	5
4/3	0
16/9	4/3

Input	*Output*
1	13
3	29
2	21
¾	11
1.5	17

4a. input is hours; output is $ b: see table above on right c. x>0 d. y=8x+5

5a. (0,1) (1,-4) (2,-13) and many more! b. -8 c. (0,1) d. no, domain is all numbers

6a. -10. The graph goes through (11, -10)
b. $f(3)=6$ → input of 3 and output of 6 (3,6) on the graph;
f(x)=3 means output=3 and input x=4.5 → on graph (4.5, 3)
c. a line since in form of y=mx+b → no squareds or other fancy stuff
d. x-int is y=0 which is x=6 so (6,0) y-int is when x=0 which is 12 so (0,12)
e. (17/3, 2/3) f. $x \le 3.5$

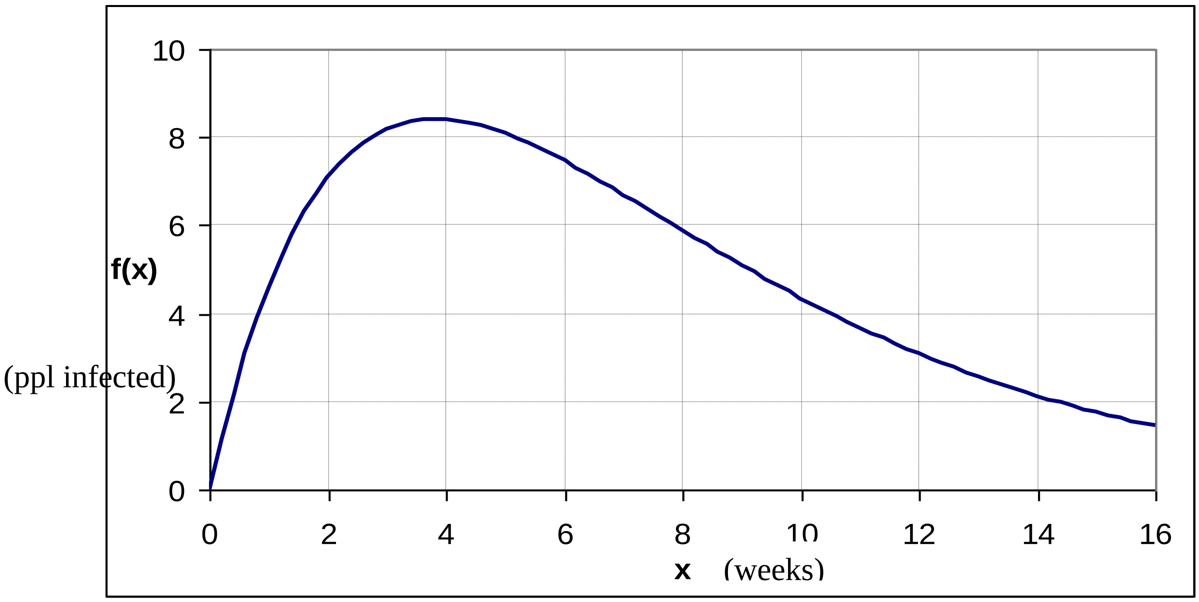
7a. (2.5,3.5) b. x<2.5 8. $f(x)=-4x+40$ 9a. $f(x)=-\frac{7}{3}x+7$ b. (6,-7)

10a. -14 b. -1.5 c. 179/30 d. -23 e. -5 f. b=4; c=3

11. if the lower left corner is the origin the lines are $y=-0.5x+3$ and y=x-2 so they meet at (10/3,4/3)
and AE is $\sqrt{\left(\frac{10}{3}\right)^2+\left(\frac{5}{3}\right)^2}=\sqrt{\frac{125}{9}}=\frac{5\sqrt{5}}{3}$

Unit 1 Handout #2: More Function Representations

1. The graph of the function $f(x)$, shown below, describes how a flu epidemic spread through a city. The function $f(x)$ shows the number of individuals infected (in thousands) x weeks after the epidemic began.



a. What is *f(2)* and what does it mean in terms of the epidemic?

b. About how many infected people were there at the height of the epidemic? When did that occur?

c. Solve $f(x) = 4.5$. How many solutions are there, and what do they represent?

d. Estimate the solution to the inequality $f(x) \geq 6$. Graph it on a number line and explain what it means, in terms of the epidemic.

2. Answer these questions about function $g(x) = \frac{3}{4}x + 1$. Give your answers as fractions.

a. What is $g(-2)$?

b. Evaluate $g(x)$ when $x = ½$.

c. A point on the graph of $g(x)$ has a y-coordinate of 10. What is its x-coordinate?

d. Find the solution to the equation $g(x) = \frac{1}{8}$.

e. Find the zero(s) of $g(x)$.

f. Find the solution to $-3 \leq g(x) \leq 5$

3. The function $f(x)$ shows the height of a ball (in meters) x seconds after it is thrown.

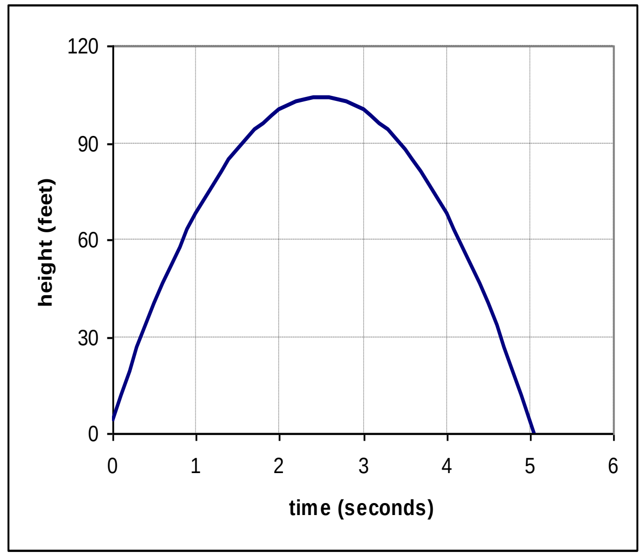
a. What is the meaning of the statement $f(2)=25$? Specify units.

b. What are the units and meaning of the expression $f(5)$?

c. In function notation, express the fact that the ball lands ten seconds after it is thrown. (What is its height when it lands?)

d. What is the difference between $f(3)$ and $f(x)=3$ (in the context of the problem)?

4. Johnny Damon hit a fastball solidly. The ball was four feet off the ground when he hit it. One second later it was 68 feet high. It continued to get higher, then it started to fall before it cleared the outfield fence in Yankee Stadium. The graph below shows *h(t)*, its height as a function of time. Use it to *approximate* answers to the following questions.



a. About how high was the ball after 3 seconds?

b. What is the value of *h(4)* and what does it mean?

c. What is the solution to $h(t) = 30$ and what does it mean?

d. Solve the equation *h(t)* = 0. What does this mean?

e. What was the maximum height of the ball and when was it attained?

f. At what input values (times) was the ball was more than 60 feet high.

g. Solve the inequality *h(t)* < 30. What does this mean?

h. What are the domain and range of *h(t)*?

i. The ball cleared the outfield fence 4.5 seconds after it was hit. The fence was 12 feet high. By about how much did it clear the fence?

5. The function $f(x)$ shows how much a backpack weighs (in pounds) when there are x textbooks in it. The equation is $f(x)=3x+2$.

a. What is the meaning of $f(8)$ and what is its numerical value? Specify units.

b. What is the solution to the equation $f(x)=11$? How can you interpret it?

c. What is the solution to $f(x) \geq 23$? What does this mean in the context of the problem?

d. What are the coordinates of the y-intercept on the graph of $f(x)$? What meaning do they have?

e. What are the zeros of $f(x)$? What meaning, if any, do they have?

f. What is a reasonable domain for the function $f(x)$? What is a reasonable range?

6. A linear function $g(x)$ is parallel to $f(x) = -\frac{1}{3}x + 7$ and $g(-2) = \frac{3}{5}$.

a. Write the equation of $g(x)$.

b. What is/are the zeros of $g(x)$?

c. What ate the coordinates of the point where $g(x)$ intersects the graph of $h(x) = \frac{3}{2}x + 1$?

7. A linear function $g(x)$ is perpendicular to $f(x) = -\frac{1}{3}x + 7$ and $g(-2) = \frac{3}{5}$.

a. Write the equation of $g(x)$.

b. Find the *x*-values corresponding to output values of at least 7, ie the solutions to $g(x) \geq 7$.

8. The following questions refer to a clock (an old-fashioned one, with minute hand and hour hand that both move continuously). They refer to the clock only between **12:01 pm one afternoon and midnight that night.**

a. Is the time of day a function of the position of the hour hand (I mean the exact angle/direction it is pointing)? Again, this means that the input is position and output is time. Explain.

b. Is the position of the hour hand a function of the time of day? Explain.

c. Is the time of day a function of the position of the minute hand? Explain.

d. Is the position of the minute hand a function of the time of day? Explain.

9. Determine whether each statement is true. For each one that is true, give the function.

a. The area of a circle is a function of its radius.

b. The radius of a circle is a function of its area.

c. The circumference of a circle is a function of its area.

d. The area of a rectangle is a function of its perimeter.

e. The area of a square is a function of its perimeter.

f. The perimeter of a 30°-60°-90° triangle is a function of the length of its hypotenuse.

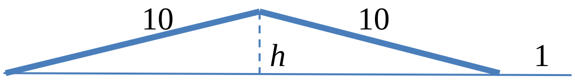
10. Two sticks 10 cm long are connected by a hinge. Initially they are lying flat. The left end remains fixed while the right end slides to the left at a rate of 1 cm per second.

a. What is the height of the highest part (the hinge) 1 second after the right end starts to move?

b. Write a function $h(t)$ that shows the height of the highest part t seconds after the right end starts to move.

Initially:

After one second



ANSWERS

1a. After 2 weeks there were about 7000 people infected b. About 8500 people; after about 4 weeks
c. x= about 1 or about 10; at these times there were 4500 people infected
d. about 1.3≤x≤8; more than 6000 people infected between weeks 1.3 and 8

2a. -1/2 b. 11/8 c. 12 d. -7/6 e. -4/3 f. $\frac{-16}{3} \le x \le \frac{16}{3}$

3. a. 2 seconds after it was thrown it is 25 m high b. meters; the height 5 seconds after it was throw
c. f(10)=0 d. f(3) is height after 3 seconds; f(x)=3 is when height is 3

4a. about 100 feet b. $h(4) \approx 68$; after 4 second the ball was 68 feet high
c. $t \approx 0.4,\ 4.6$ after 0.4 seconds and 4.6 seconds, the ball was 30 feet high
d. $t \approx 5.1$; the ball landed after about 5.1 seconds e. 105 feet after 2.5 seconds
f. approximately $0.8 < t < 4.2$ g. $t < 0.4$ *or* $t > 4.6$ before 0.4 seconds or after 4.6 seconds, the ball's height is less than 30 feet. (you could also say $0 \le t < 0.4$ *or* $4.6 < t \le 5.1$)
h. domain is about $0 \le t \le 5.1$; range is about $0 \le h(t) \le 105$ i. about 30 feet

5a. 26 pounds; weight with 8 books in it b. x=3; weighs 11 pounds when there are 3 books in it
c. x≥7; backpack weighs at least 23 pounds when there are at least 7 books in it
d. (0,2); backpack weighs 2 lb when no books are in it
f. -2/3 → no meaning; with -2/3 book in it, its weight would be zero!
g. domain is {0,1,2,3,4…} → non-negative integers; the range is {2,5,8,11,14,…}

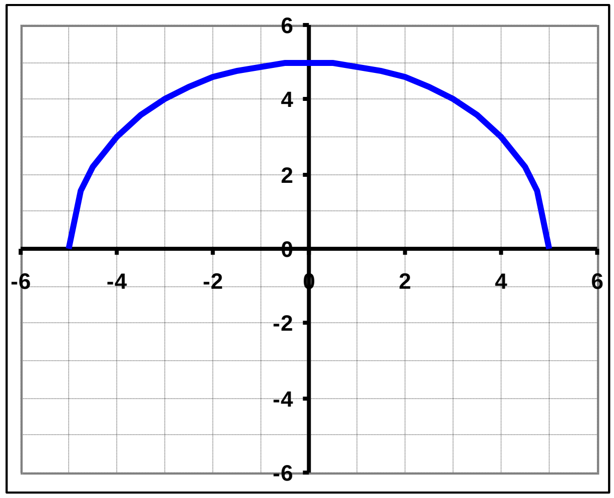
6a. $g(x) = -\frac{1}{3}x - \frac{1}{15}$ b. x= -1/5 c. $\left(-\frac{32}{55}, \frac{7}{55}\right)$ 7a. $g(x) = 3x + \frac{33}{5}$ b. $x \ge \frac{2}{15}$

8a. yes b. yes c. no d. yes 9. all are true except d a. $A(r) = \pi r^2$ b. $R(a) = \sqrt{a/\pi}$
c. $C(a) = 2\pi\sqrt{a/\pi} = 2\sqrt{\pi a}$ e. $A(p) = \left(\frac{p}{4}\right)^2$ f. $P(h) = 1.5h + \frac{h\sqrt{3}}{2}$

10a. $\sqrt{9.75} \approx 3.12$ cm b. $h(t) = \sqrt{10t - 0.25t^2}$

Unit 1 Handout #3: Functions: Solving Equations Graphically and Algebraically

1. Answer the questions below about the function *f(x)* that is graphed on the right.



a. What is *f(3)*?

b. Find all solutions to *f(x)* = 2.

c. What are the domain and range of *f(x)*?

d. What are the zeros of *f(x)*?

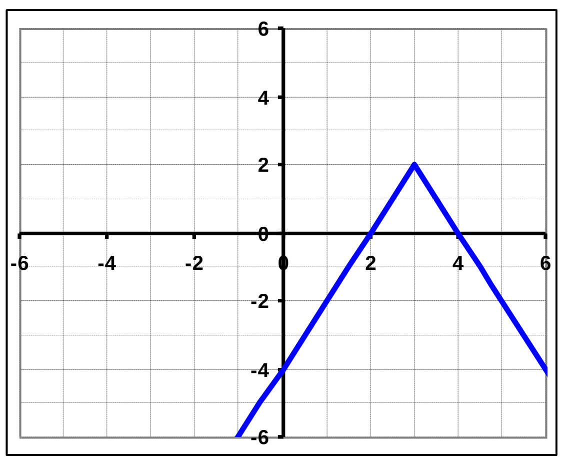
e. Fill in the following table of values:

x	*f(x)*
4	
-3	
	5

f. What is the solution to the inequality $f(x) \geq 3$?

2. Answer the following questions about *w(t)*, graphed below. **It continues forever in both directions.**

a. What is $w(1)$?



b. Solve the equation *w(t)*=1.

c. Find all solutions to $w(t) = 4$.

d. What are the domain and range of *w(t)*?

e. What are the zeros of *w(t)*?

f. Graph the solution to the inequality *w(t)* < -2 on a number line and then write an inequality showing the values of *t* that solve it.

3. Answer the following questions about $f(x) = -2x + 2$ **<u>algebraically</u>**.

a. Evaluate $f(4)$.

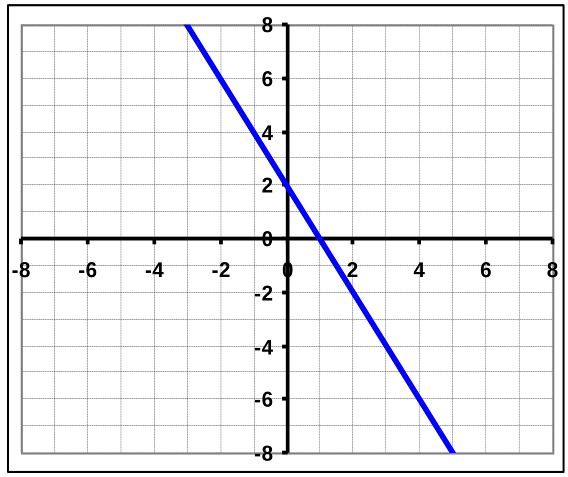
b. Solve the equation $f(x) = -3$.

c. Find the zero of $f(x)$.

d. Solve the inequality $f(x) < 4$

e. Where does the graph of $f(x)$ intersect the graph of $g(x) = 0.5x - 1$?

4. The graph below is that of $f(x) = -2x + 2$. **Use the graph** to answer the following questions.



a. What is $f(4)$?

b. Solve the equation $f(x) = -3$.

c. Is this the same as solving the equation $-2x + 2 = -3$? Why or why not?

d. Find the zero of $f(x)$.

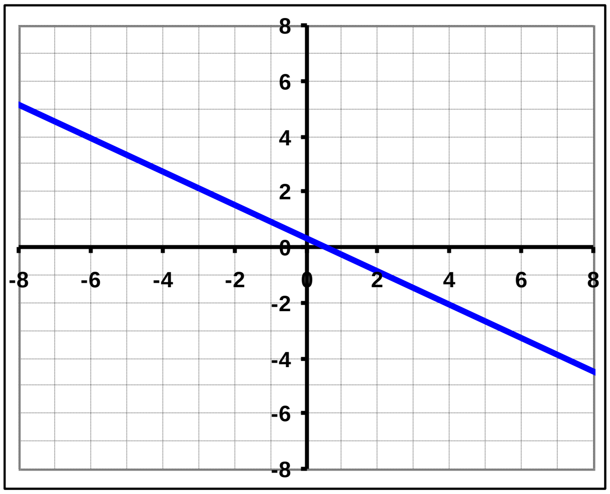
e. Solve the inequality $f(x) < 4$

f. On the same set of axes, graph $g(x) = 0.5x - 1$ and estimate the coordinates of the intersection of $f(x)$ *and* $g(x)$.

5. Note that the function $f(x) = -2x + 2$ was the same in both questions 3 and 4 above. In question 3, you answered questions **algebraically**. In question 4, you answered the same questions **graphically**. Check to confirm that your answers were the same.

6. The function $f(x) = -\frac{3}{5}x + \frac{1}{3}$ is graphed below.

a. Use to it evaluate $f(8)$. Is it exact or approximate?



b. Find $f(8)$ algebraically.

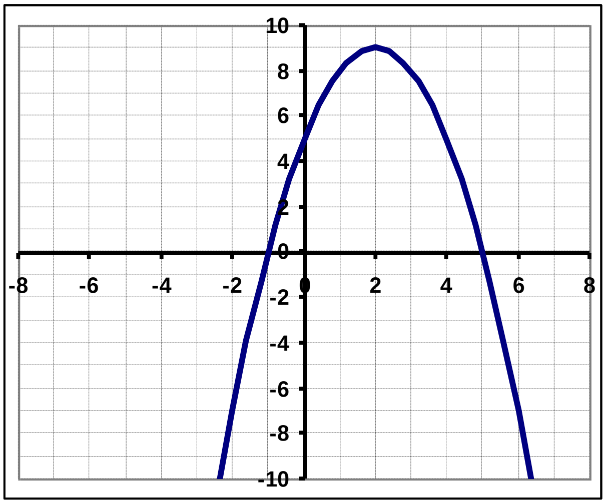
c. Use the graph to estimate the solution to $-\frac{3}{5}x + \frac{1}{3} = -2$. Then find it algebraically. How close were the two?

d. Use the graph to estimate the solution to $-\frac{3}{5}x + \frac{1}{3} < -3$. Then find it algebraically. How close were the two?

e. Either graphically or algebraically, approximate the zero of $f(x)$. Why did you choose the method that you did?

7. The function $f(x) = -x^2 + 4x + 5$ is graphed below.

a. Use the graph to approximate the zeros of $f(x)$.



b. Evaluate $f(-2)$ algebraically and graphically. Confirm that your answers are the same.

c. Are there any solutions to the equation $-x^2 + 4x + 5 = 10$? How do you know? [On the graph, this is $f(x) = 10$]

d. **Use the graph** to solve the equation $-x^2 + 4x + 5 = -7$. Then solve it algebraically (by factoring) if you remember how to.

e. **Use the graph** to find the solution to $-x^2 + 4x + 5 \le 0$ (in other words $f(x) \le 0$).

8. The function v(t) measures the vocabulary (in number of words) of a typical child t months old.

a. Describe the meaning of the equation v(23)=112. Be sure to specify units.

b. The typical vocabulary of an 18-month old is 45 words. Use function notation to express this.

9. Sketch the graph of a function with a domain of all real numbers, a range of $f(x) \geq -4$, and zeros of $x = -2$ and $x = 2$ only.

10. Sketch the graph of any function with zeros of 3 and 6 and a *y*-intercept of -4.

11. Sketch the graph of any function a domain of $0 \leq x \leq 8$ and a range of $-5 \leq y \leq 4$.

12. Sketch the graph of any function $f(x)$ where the following are all true: $f(0) = 7$, $f(4) = 0$, $f(-1) = -3$, and $f(6) = 3$.

13. Sketch the graph of any function $f(x)$ with zeros of -3 and 2, and where the solutions to $f(x) = 3$ are x=-2 and x=1.

14. Sketch the graph of any function such that the solution to $f(x) > 0$ is $x > 3$ *or* $x < -4$ and the range is $f(x) \geq -5$.

15. Write the equation of a linear function $f(x) = mx + b$ that has a zero of 7 and a y-intercept of -3.

16. Write the equation of a linear function where the solution to $f(x) \leq -2$ *is* $x \geq 4$ and the solution to $f(x) \geq 5$ *is* $x \leq 0$.

ANSWERS

1a. 4 b. ≈4.6 & -4.6 c. domain is -5≤x≤5; range is 0≤*f(x)*≤5 d. -5, 5 e. 3, 4, 0 f. -4≤x≤4
2a. -2 b. 2.5, 3.5 c. none d. domain: all numbers ; range is *w(t)*≤2 e. 2, 4 f. t<1 or t>5
3a. -6 b. 2.5 c. 1 d. x>-1 e. (6/5, -2/5)
4a. -6 b. 2.5 c. yes, since f(x) = -2x+2 d. 1 e. x>-1
5. they should all be the same
6a. about -4.5 b. -67/15 c. x≈4 on graph; algebraically it is 35/9 (3+8/9)
d. on graph it is x>5.5 (or so); algebraically it is x>50/9
e. graph about 0.5; fast but inexact; algebraically is exact but more work; I get 5/9
7a. -1 and 5 b. -7 c. no, graph never gets as high as 10 d. -2 and 6 e. x≤-1 or x≥5
8a. a 23-month old knows 112 words b. $v(18) = 45$
9-14. many possible answers
15. $f(x) = \frac{3}{7}x - 3$ 16. line must go thru (0,5) and (4,-2) so the equation is $f(x) = -\frac{7}{4}x + 5$

Unit 2 Handout #1: Linear Modeling

1. The value (in dollars) of a Honda Accord automobile t years old is given by the equation $V(t) = -1200t + 24000$.

a. Is the graph of $V(t)$ a line? How do you know, just by looking at the equation?

b. What is the slope and what does it mean?

c. Algebraically find the x- and y-intercepts of the graph of $V(t)$. What meaning do they have (watch the units).

d. Give the coordinates of any two other points on the graph of $V(t)$.

e. What is a reasonable domain for the function $V(t)$?

2. When Brandon uses his Ipad while it is charging, it takes 90 minutes to charge (from empty). When he does not use his Ipad while it is charging, it takes 70 minutes to charge. How long can he use his full-charged Ipad without it being plugging in before it runs out of power? Assume that it charges at a constant rate and using it drains the battery at a (different) constant rate.

3. A bicycle rider is riding down hill. Five minutes after she starts her elevation is 1000 feet. Eight minutes after she starts her elevation is 700 feet.

a. Write the equation for the linear function *H(m)*, showing her elevation as a function of the number of minutes since she started riding.

b. Sketch a graph the function *H(m)*.

c. What is the value of *H(13)*? What meaning does it have in this problem?

d. What is the slope of the line representing *H(m)*? What is its meaning (units!)?

e. Solve the inequality $H(m) < 400$. What meaning does it have in this problem?

f. What is the *y*-intercept of *H(m)*? What meaning does it have in this problem?

g. What is the *x*-intercept of *H(m)*? What meaning does it have in this problem?

4. Julie's new phone card is worth \$40. Each minute she speaks reduces the value of the card by \$0.15.

a. Write the equation for the function $V(m)$, showing the card's value as a function of the number of minutes she has spoken. Sketch a rough graph.

b. What is the value of the card after she speaks for 105 minutes? Write this using function notation.

c. What is the slope of the line representing $V(m)$? What is its meaning?

d. Solve the inequality $V(m)>25$. What meaning does it have in this problem?

e. What is the y-intercept of $V(m)$? What meaning does it have in this problem?

f. Write an equation using function notation showing when the card becomes worthless. Solve it.

5. Lindsay has a one-quart (32 fluid ounces) water bottle. It is almost empty so she stops at the drinking fountain to fill it. Five seconds after the she starts there are 12 fluid ounces in it. Eleven seconds after she starts filling it there are 25 fluid ounces in it. Write the equation of *f(x)*, a **linear function** describing the number of fluid ounces in it as a function of seconds after she starts filling it.

a. How full is it after 4 seconds?

b. When does it fill up?

c. What is the solution to the inequality $f(x) < 9$ and what does it mean.

d. How much was in it when she started to fill it?

6. Thirty seconds after Sam turns a partially-full hourglass over, there are 1000 grains of sand in the top part. Fifty seconds after she turned it over there are 400 grains of sand still in the top part.

a. Define a linear function *S(t)* showing the number of grains of sand left in the top part of the hour glass for any number of seconds after it was turned over. Graph it.

b. How many grains are left 60 seconds after she turns it over?

c. How many grains were in there when she first flipped it? Where on the graph do you see this?

d. What is the zero of the function you wrote? What is its meaning in this problem?

7. A train is going from Boston to Chicago, a distance of 900 miles. It is traveling at a constant speed of 70 miles per hour. At 4pm it is 120 miles from Boston.

a. Write a function showing distance from Boston as a function of the hours after noon. Graph it.

b. When will it arrive in Chicago?

c. What is the zero of this function and what meaning does it have (in context)?

d. What is the domain of this function?

8. Three companies offer cell-phone service and have different pricing schemes. Atlantic Company charges $10 per month plus 10 cents per minute. Bert's charges $15 per month plus 5 cents per minute. Continental charges $18 per month with all minutes free.

a. For what range of minutes per month used (if any) is Bert's the least expensive?

b. For what range of minutes per month used (if any) is Atlantic the cheapest and Bert's the most expensive? Hint: it may help to think of this as two separate inequalities joined by an "and".

9. Henry and Annie are running a race. Henry gives Annie a 10-second head start for the 800 meter race and still beats her by 40 meters (she has 40 meters to go when he finishes). Annie runs at a speed of 4 meters per second.

a. What is Henry's speed?

b. How many seconds after Henry started running did he pass Annie? Hint: if you are having trouble setting this up, make a table. Where are Henry and Annie 0 seconds after Henry starts? One second? Two seconds? x seconds?

10. A train track leads directly from New York towards Atlanta. At 1 pm, a train is 200 miles from New York, and at 5 pm, it is 440 miles from New York. Assuming it is traveling at a constant speed, Zora says one way to write the distance from New York (d) as a function of the hours since noon (t) is $d-200=60(t-1)$.

a. Is she correct? Explain.

b. Yanice disagrees; she thinks the correct equation is $d-440=60(t-5)$. They argue for a while. Help them resolve their disagreement.

c. Xavier looks at the situation and determines that the relationship can be described as $d-d^*=60(t-2.5)$. What is the numerical value of d^* and what does it represent in the context of the problem?

11. Three rental car companies have different pricing schemes. Each has a daily base rate (a certain number of dollars) plus some additional charge per mile driven. Hertz charges $30 per day plus 40 cents per mile. Budget charges $40 per day plus 25 cents per mile. Avis charges $50 per day plus 15 cents per mile. Angela wants to rent a car for one day.

a. If the total cost of renting a car for one day from Avis is more than the total cost of renting a car for one day from Hertz, then how many miles might Angela have driven? Your answer should be an inequality, perhaps like $20<m<30$ or $m>100$.

b. For what range of miles driven, if any, if Budget the cheapest of the three rental companies?

c. Is it ever the case where Hertz is the cheapest, following by Avis, with Budget the most expensive? If so, how many miles were driven?

12. In triangle ABC, angle A measures $x°$ and angle C measures $(3x\text{-}40)°$. The lengths of the sides satisfy the inequality AB > BC > AC. Write an inequality describing all possible values of x. Hints: try a few numbers for x (perhaps 30, 50, and 65). And there's something from geometry about which side is opposite which angle…!

13. Annie and Susan are running along a road from west (left) to east. They start together. Annie runs a mile every 6 minutes (fast!) and Susan runs a mile every 8 minutes. Rob starts along the same road far away from the women and runs towards them at a rate of 7 minutes per mile. Twenty five minutes after Annie and Susan start running, Rob and Annie pass by each other. How minutes ensue from then until the time that Rob and Susan pass by each other? Leave your answer as a fraction.

Annie → ← Rob
Susan

ANSWERS

1a. yes; it is in slope-intercept form b. the value falls by $1200 per year
c. (0,24000) when the car is new it is worth $24000 (20,0) the car is worthless in 20 years
d. many possible answers e. $0 \le t \le 20$ (negative time and negative values make no sense)
2. 315 minutes 3a. *H(m)*=-100m+1500 c. H(13)=200; after 13 mins her elev is 200 ft
d. -100; elev decreases 100 feet every minute
e. $-100m+1500 < 400$ so $m>11$; anytime after 11 mins her elev is < 400 ft
f. (0,1500) after 0 mins (starting), her elev is 1500 feet g. (15,0) after 15 mins her elevation is zero
4a. *V(m)*=-0.15m+40 b. V(105)=$24.25
c. -0.15, meaning each minute reduces value by 0.15 dollars (15 cents)
d. -0.15m+40>25 so m<100; if Julia uses it less than 100 minutes then its value is over $25
e. (0,40), after 0 minutes (new card), the value is $40 f. *V(m)*=0 so m=266.67 minutes
5. $f(x)=\frac{13}{6}x+\frac{7}{6}$ a. $f(4)=\frac{59}{6}$ *or* 9.83 *fl oz* b. $f(x)=32$ *so* $x=185/13$ c. $x<\frac{47}{13}$ means that there are fewer than 9 oz in the bottle for the 1st 47/13 (almost 4) seconds d. $f(0)=7/6$ fl oz
6a. *S(t)*=-30t+1900 b. S(60)=100 c. 1900; when t=0 S(t)=1900
d. S(t)=0 when t=63.33 seconds → it hourglass empties
7a. $d(t)=70t-160$ b. 900=70t-160 so t=15.14 hours so a little after 3 am
c. d(t)=0 when t=16/7 so a little after 2:15 pm 8a. never b. between 60 and 80 minutes
9a. 40/9 m per sec (800 m in 180 sec) b. after 90 sec (he gains 4/9 m/s and needs to gain 40 meters)
10a. yes; it goes 60 miles per hour so the distance beyond 200 miles is 60 times the number of hours after 1 b. both are correct; the distance beyond 440 miles is 60 times the hours since 5 pm
c. 290; the train’s distance at 2:30 pm
11a. $m<80$ b. $66.67<m<100$ c. never: need $50+0.15m>30+0.4m$ and $40+0.25m>50+0.15m$ so $m<80$ and $m>100$ and these never both occur.
12. $44<x<55$ since $x>0$, $3x-40>0$, $220-4x>0$ and $3x-40>x>220-4x$
So $x>40/3$ and $x<55$ and $x>20$ and $x>44$ 13. 35/9 minutes

Unit 2 Handout #2: Piecewise Linear Functions

1. Wilson's father is a baker, and one morning Wilson helps him by decorating the cupcakes. His father offers to pay him 10 cents per cupcake for the first 50, 15 cents per cupcake for the next 30, and 20 cents per cupcake for all the others. Let the function $W(c)$ represent the total number of cents Wilson earns if he decorates c cupcakes.

a. Is $W(c)$ a linear function? Explain.

b. Sketch a rough graph of $W(c)$. Don't plot any points… just think about the shape of it.

c. What is $W(60)$ and what meaning does it have?

d. Wilson is tired and about to stop. He realizes that if he decorates 6 more cupcakes then he'll make another $1.10. How many has he decorated thus far?

e. Solve the equation $W(c) = 2810$.

f. Challenge: if Wilson makes an average of 14.8 cents per cupcake he decorated, then how many did he complete?

2. The federal income tax is a major source of revenue for our revenue-starved federal government. Here is how it works. I have adjusted the numbers to simplify the problem, but it is close to reality. A taxpayer pays 10% of her first $8000 of earnings plus 15% of everything between $8000 and $35,000,

Plus 25% of earnings between $35,000 and $80,000
Plus 28% of earnings between $80,000 and $170,000
Plus 35% of earnings above $170,000

(Note: economists call this "progressive taxation" because high earners not only pay more dollars of tax, but also a higher percentage of their earnings in taxes.)

Let the function $T(s)$ represent the dollars of taxes someone pays if her income is $*s*.

a. What is $T(50{,}000)$?

b. What is $T(210{,}000)$?

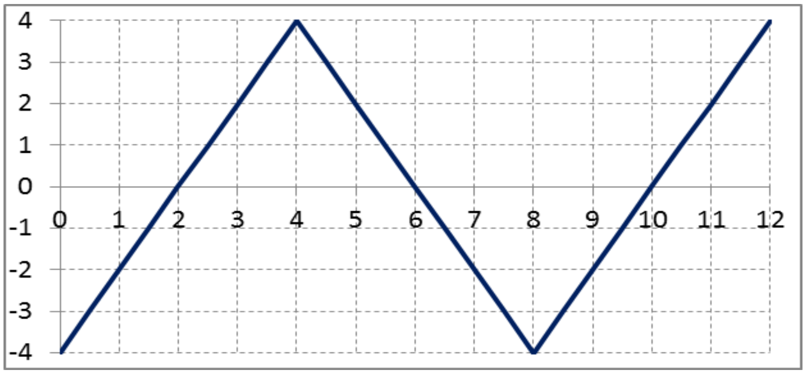
c. Solve the equation $T(s) = 8000$

d. Solve the equation $T(s) = 40{,}000$.

e. Linda gets a huge (but well-deserved) $30,000 raise in her annual salary and calculates that she will pay $9,100 of that in taxes. What was her salary before she got the raise?

f. (challenge) If someone's average tax rate (taxes paid divided by dollars earned) is 32%, then how much did he or she earn?

3. The function $f(x)$ has a domain of all real numbers and a range of $-4 \le f(x) \le 4$. A partial graph is shown below. The zig-zag pattern continues outside of the window, so when x is between 12 and 16. $f(x)$ falls from 4 to -4 before bouncing up again.



a. What is $f(81)$? $f(214.6)$?

b. Find all solutions to $f(x) = -2$ when x is between 12 and 20.

c. For all x's between 96 and 104, what are the solutions to $f(x) = 1$?

d. Put the following in order from low to high: 0, $f(41)$, $f(43)$, $f(46.5)$

e. On the domain $-80 \le x \le -76$, the graph of $f(x)$ is a line. What is the equation of that line?

e'. Find the smallest positive solution to $f(x) = -1.5$. What are the next three higher solutions?

f. If a is some number between 0 and 4 and $f(a) = k$, then what are the next three higher solutions to the equation $f(x) = k$? (your answer will be in terms of a.) Try numbers for a if you need to—it may help you understand the question!

g. Find the two lowest and one highest x values at which the graph of $f(x)$ intersects the graph of $y = 0.01x - 4$?

h. How many times does the graph of $f(x)$ intersect the graph of $y = 0.01x - 4$?

i. On the interval $0 \le x \le 100$, someone solves the inequality $f(x) > 2.5$ and graphs her solution on a number line. What percentage of the number line (between 0 and 100) is shaded?

j. Instead, if the number line representing the solution to $f(x) < k$ on the interval $0 \le x \le 100$ is shaded 30% of the time, then what is *k*?

4. Given that $f(x)$ is a linear function and $f(a) = b$ and $f(c) = d$ find the following:

a. $f(e)$ b. The y-intercept of the graph of $f(x)$. c. The zero of $f(x)$.

Answers

1a. No. For it to be a line each time you go one to the right (one more cupcake frosted) you need to go up the same amount (constant number of cents earned)

c. 650 ; makes $6.50 if he frosts 60 cupcakes d. 78 e. 173 f. $\frac{950+20x}{x+80} = 14.8$ so x=45 so 125

2a. 8,600 b. 55,300 c. 47,600 d. 165,357 e. 150,000

f. 606,667 because $\frac{41300+.35(x-170000)}{x} = 0.32$

3a.-2; -1.2 b.15, 17 c.98.5, 101.5 d. $f(41)$, $f(46.5)$, 0, $f(43)$ e. $y = 2x + 156$ e'.1.25, 6.75, 9.25,14.75

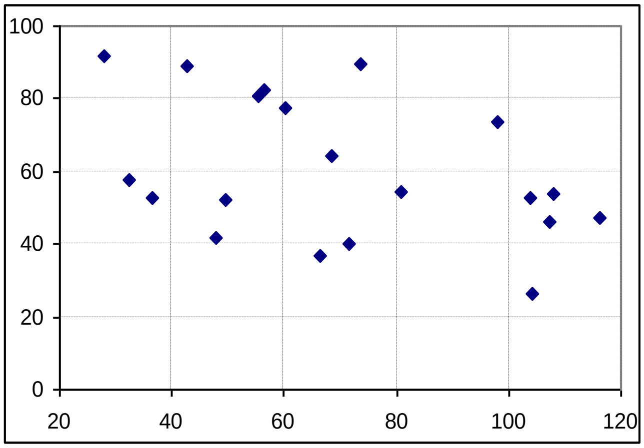
f. 8-a, 8+a, 16-a g. low is x=0, then x=16/2.01 ; high is x=1600/2.01 h. 200 i. 18.75% j. -1.6

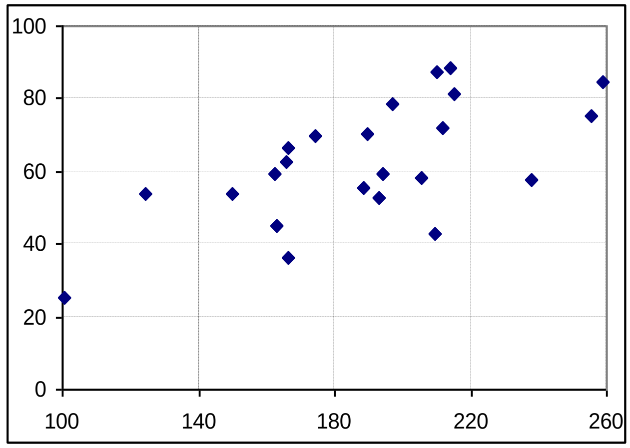
4a. a few ways: $b + \frac{d-b}{c-a}(e-a)$ *or* $d + \frac{d-b}{c-a}(e-c)$ (others too!)

b. one form is $b + \frac{d-b}{c-a}(-a)$ c. one form is $a - b \cdot \frac{c-a}{d-b}$

Unit 2 Handout #3: Best-Fit Lines

1. For each scatter-plot below, first determine if the correlation is positive, negative, or near-zero. Then draw in a reasonable best-fit line and write its equation. Be careful of the scale. Note that the **x-axis does not start at zero**, so what you think the y-intercept is may not actually be the y-intercept!





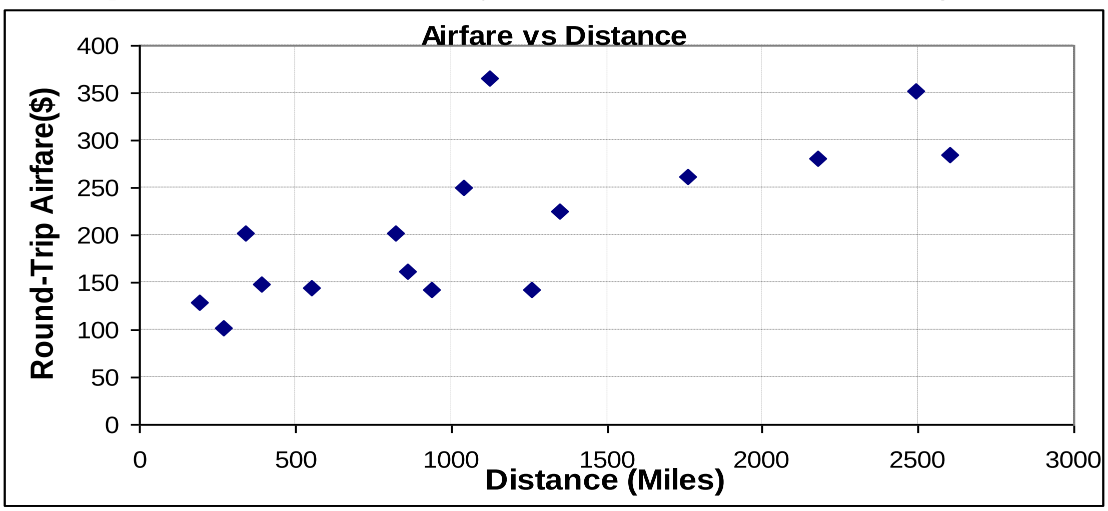
Correlation:

Equation of best-fit line:
(Approximate)

Correlation:

Equation of best-fit line:
(Approximate)

2. Here is some data showing how airfare relates to distance for a set of sixteen cities. Airfare is the round-trip fare from Boston to that city (dollars) and distance is the mileage from Boston to that city.



a. What is an approximate equation of the best-fit line?

b. What is the meaning of the slope of the best-fit line (include units!)

c. Use the **equation** of the best-fit line to predict the airfare to St. Louis (1177 miles). Confirm your answer by using the graph.

3. According to one textbook, the typical income for a family practitioner doctor each year is given below. Analyze the relationship between the **years since 1980** and the income (measured in 1000's).

1982	71
1985	79
1986	80
1987	91
1988	95
1989	96
1990	103
1991	112
1992	112

a. Load the data into your calculator and do a scatter plot. Estimate the correlation from the graph.

b. Have your calculator calculate the exact correlation.

c. Have your calculator compute the equation of the best-fit line. Write it down. What do the parameters (slope and y-intercept) mean in the context of the problem?

d. Use your best-fit line to predict income in 1993.

4. The data set below shows how age relates to recovery time for nine injured weight-lifters. Each had arthroscopic surgery on his shoulder. The experimenters then measured how many days elapsed before they were able to return to their sport.

	A	B	C	D	E	F	G	H	I
Age (x)	33	31	32	28	33	34	32	28	27
Days (y)	6	4	4	1	3	4	2	3	2

a. Calculate the equation of the best-fit line. Describe its meaning.

b. Person F has a longer-than-average recovery. Use the equation of the best-fit line to predict his recovery time. What do these data points tell us?

c. What would you predict the recovery time for a 45-year old to be? How good a forecast do you think this is?

Unit 2 Handout #4: Problem Solving with Lines

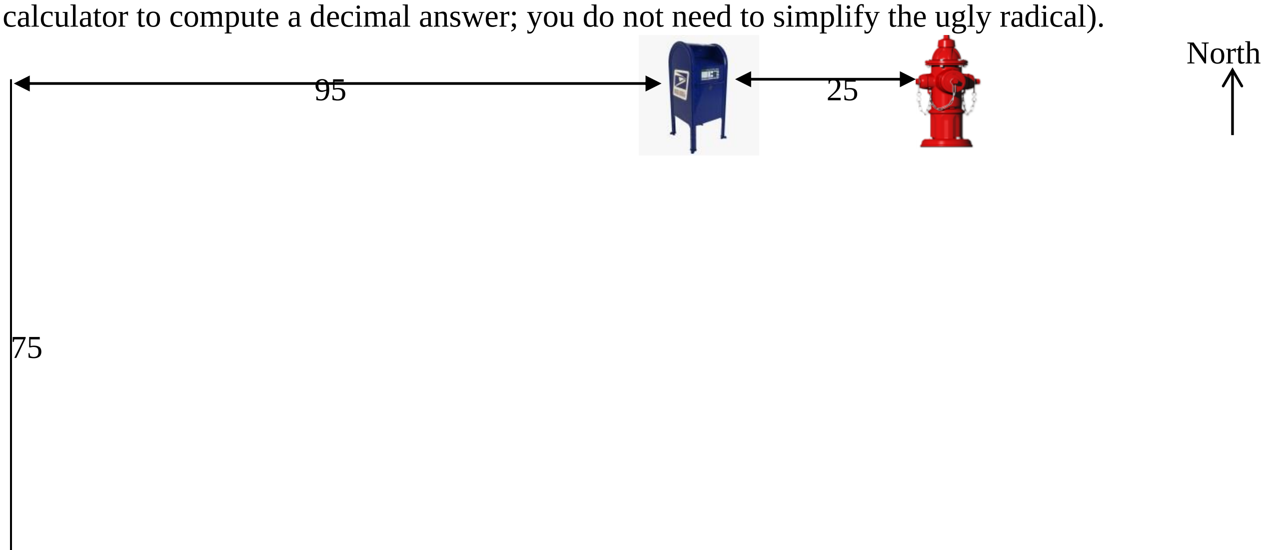
1. Diophantus’ riddle was written in verse almost 2000 years ago. Diophantus is sometimes considered the father of algebra; he lived in Alexandria Egypt in the 3rd century CE. This is a rough translation:

“Here lies Diophantus. The wonder behold-
Through art algebraic, the stone tells how old:
God gave him his boyhood one-sixth of his life,
One-twelfth more as whiskers grew rife;
Then yet one-seventh their marriage begun;
In five years there came a bouncing new son.
Alas, dear child of master and sage
Met fate at just half his dad’s final age.
Four years yet his studies gave solace from grief;
Then leaving scenes earthly he, too, found relief.”

How long did Diophantus live?

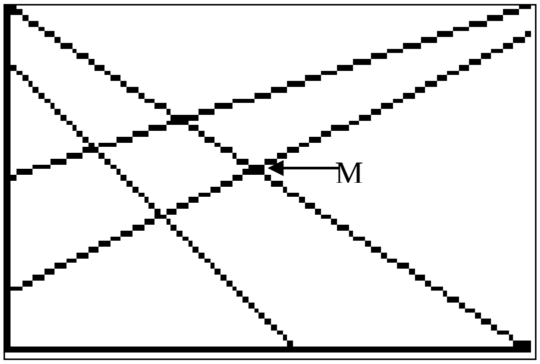
2. Alex was hired to unpack and clean 576 very small items of glassware. He got paid 5 cents for each one successfully unpacked but had to pay $1.98 for each item broken during unpacking/cleaning. He ended up with $22.71. How many items did he break?

3. Michael and Bob are standing in a park when they see a rabbit come out of its hole in the distance. They watch it and wonder how far away it is. Bob walks 5 meters east and sees the rabbit go back into its hole directly in line with the mail box. Michael walks 5 meters north and sees the rabbit go back into its hole directly in line with the fire hydrant. Use the map below to determine how far the rabbit hole is from the spot where Michael and Bob were standing when they first saw it (you may use a calculator to compute a decimal answer; you do not need to simplify the ugly radical).



* initial location of Michael and Bob

4. The calculator screen shows the graphs of the four equations listed below. The lower left-hand corner of the screen is the origin (0,0). What are the exact coordinates of point M?



$y = -x + 6$
$3x + 2y = 10$
$2y – x = 6$
$4y - 3x = 4$

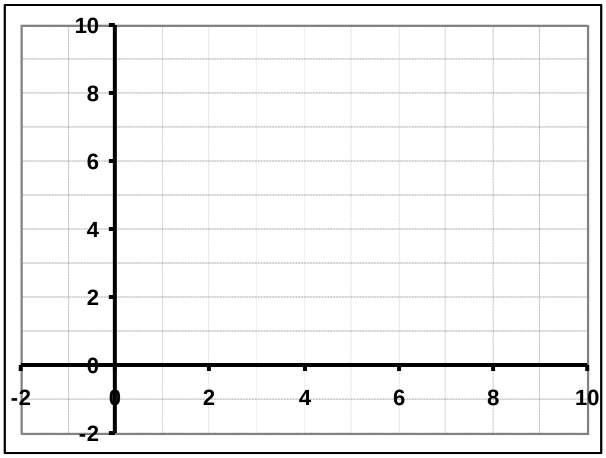
5. Conventional wisdom holds that dogs age/mature seven human years each year, so a two-year old dog is roughly the maturity of a fourteen year-old person. Mark is 43 and his dog Charleston is 10.5 years old. How long ago were they at the same level of maturity?

6. The top scale balances. Which way does the second scale tip, and why?

ⓅⓅⓅ
¤ ¤ ¤ ¤ ⓅⓅⓅ ¤ ¤ⓅⓅⓅⓅ

ⓅⓅ
¤ ⓅⓅ ⓅⓅⓅ

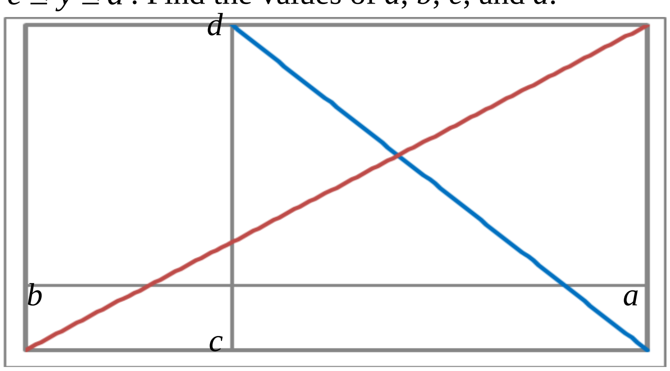
7. Sketch the graph of the line $3x+4y=24$ by finding the *x*- and *y*-intercepts.



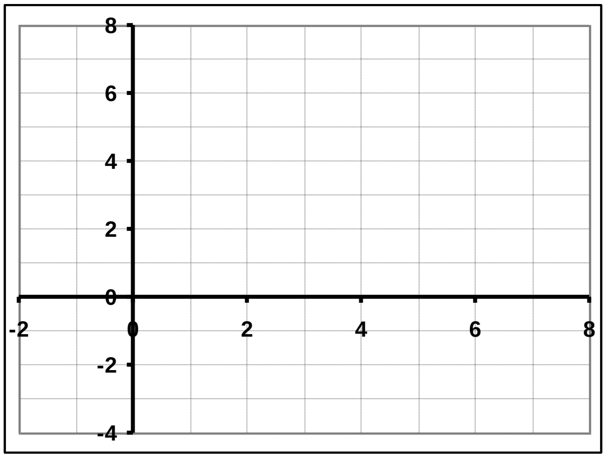
a. What are the area and perimeter of the triangle in the first quadrant whose sides are the *x*-axis, the *y*-axis, and this line?

b. A rectangle has one corner at the origin and the opposite corner on the graph of the line, at the point (2, 4.5). Its perimeter is 13. Is there another rectangle with one corner at the origin and the opposite corner on the graph of the line (in the first quadrant) whose perimeter is 15? How about 10?

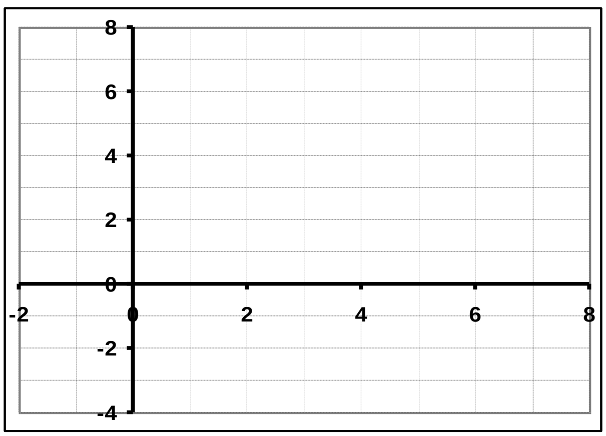
8. The lines $5x+4y=32$ *and* $-5x+6y=8$ are graphed below. The window is $b \le x \le a$ and $c \le y \le d$. Find the values of *a*, *b*, *c*, and *d*.



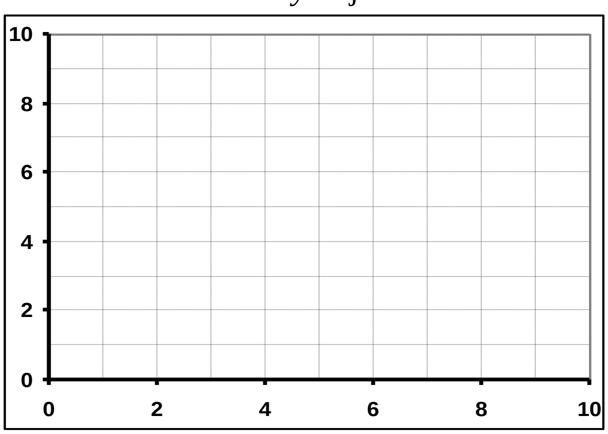
9. Draw the lines $y = x - 4$ *and* $y = 14 - 2x$ below. There is a triangle (in the first quadrant) whose sides are these two lines and the *x*-axis (the *x*-axis is the bottom). What are the area and perimeter of this triangle?



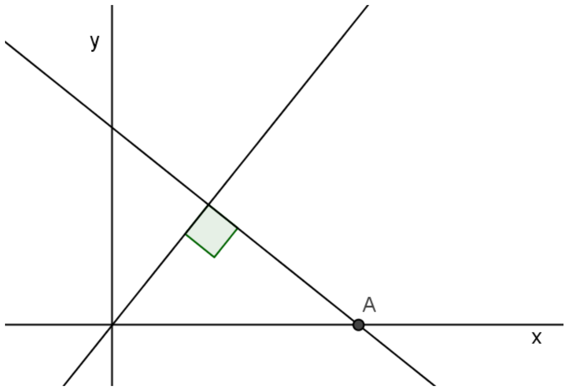
10. Graph the inequality $y < x + 3$ (recall that you may want to graph the line $y = x + 3$ and then shade the region on one side). Then graph $y < -0.5x + 6$ *and* $y > 2$. What is the area of the region where all three inequalities are true?



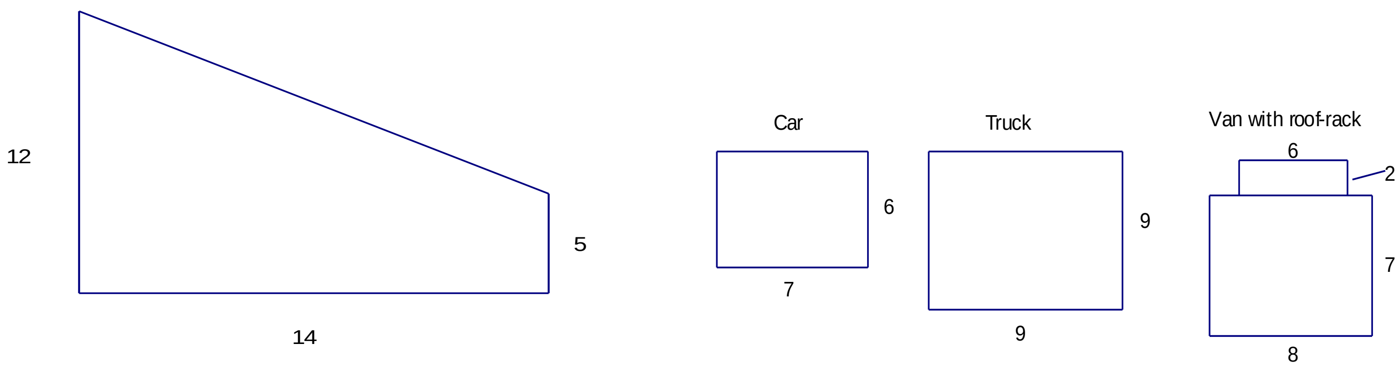
11. A triangle has vertices (2,3), (8,3) and (8,6). Draw the triangle. Now write a system of three linear inequalities that describes the points inside the triangle (including its sides). The inequalities may contain both and *x* and *y* or just one of them.



12. The lines below meet at (4,5). Point A has a *y*-coordinate of zero. What is its *x*-coordinate?



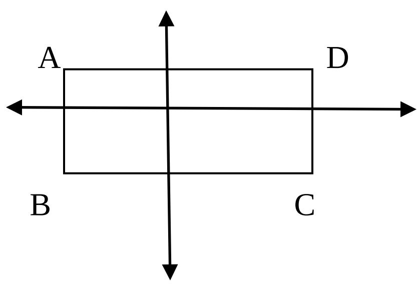
13. There is a tunnel whose mouth is the shape and dimensions below. Some vehicles with the dimensions below (all dimensions are in feet) try to drive through the tunnel, staying one foot from the



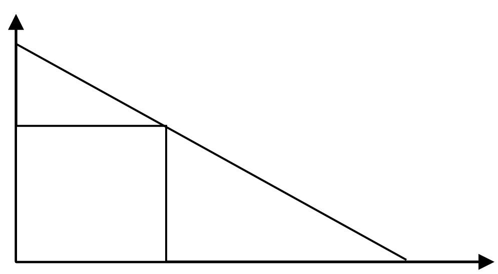
left wall. Which of the vehicles can fit through the tunnel? By how much do they make or miss fitting?

14. A line intersects the x-axis at (a,0) and the y-axis at (0,b). At what x-value does it intersect the line $y = x$? Your answer should be in terms of a and b.

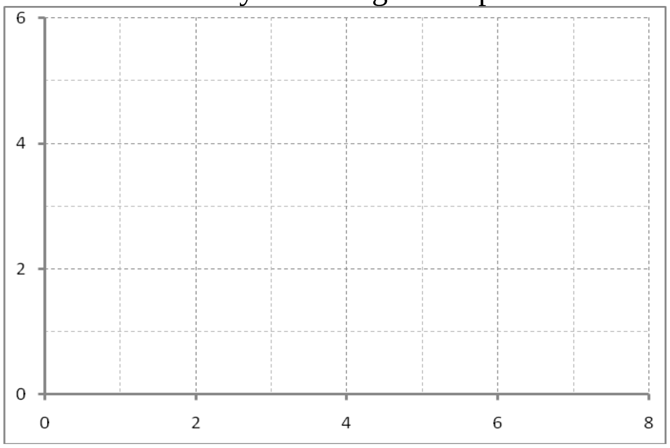
15. The diagram below shows a rectangle whose sides are parallel to the coordinate axes. Its width is three times its height and perimeter is 56. Point D is (9,2). Find the coordinates of the other three corners.



16. The graph below is the line $4x + 6y = 24$. The square has one corner at the origin and the opposite corner on the line. What is the area of the square?



17. A triangle's vertices are A (1,3)   B (2,1) and C(6,4). Draw the triangle below. Then you will calculate its area by following the steps below.



a. The altitude from A to BC must be perpendicular to BC. Write the equation of the line it lies on.

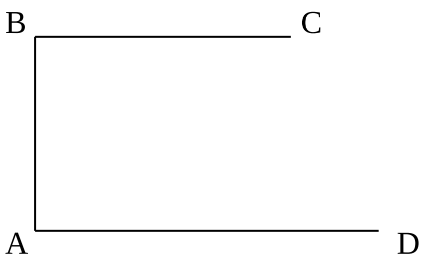
b. Find the coordinates of point D, where this altitude intersects side BC.

c. Using BC as the base and AD as the altitude, find the area.

d. Was there an easier way to find the area? Describe it, if you can think of one.

18. A triangle's coordinates are (2,2), (12,2) and (6,8). The line $y = x - 4$ divides this triangle into two pieces—a triangle and a quadrilateral. What is the ratio of the area of the triangle to that of the quadrilateral?

19. In the diagram below, angles A and B are right angles. AB=4, BC=6, and AD=8. Given that AC and BD meet at point E, find the area of triangle ABE. Hint: it may help to put it on the coordinate plane!



20. A line goes through the points (a,b), (a+3,b-2), and (a+9,3b+8). Find b.

21. When mathematicians refer to the distance from a point to a line, they mean the shortest possible distance. It turns out that this occurs when the segment connecting the point to the line is perpendicular to the line. What point on the line $y = 0.5x - 6?$ is closest to the point (0,5), and what is the distance? If need be you can write the distance as a decimal approximation instead of working with the radical.

22. Point A's coordinates are (-1,5). Point B's coordinates are (5,3). Find the *y*-intercept of the line through points A and B. Now assume Point B's coordinates are (w,3) where w>0. Find the *y*-intercept of the line through A and B in terms of w.

23. A square's sides are parallel to the coordinate axes and opposite corners are (1,3) and (7,9). A line with slope 1/3 cuts the square into two congruent trapezoids. What is the *y*-intercept of this line?

24. A triangle's vertices are (0,2), (14,2) and (4,8). A line through the point (10,2) cuts the triangle into two regions of equal area. What is the slope of this line?

25. A ***lattice point*** is defined as a point whose coordinates are all integers. How many lattice points in the **first quadrant** does the line $3x + 7y = 2015$ go through?

26. A line segment connects the points (10,8) and (640,200). How many lattice points does it go through (including the endpoints)?

Answers

1. 84 2. 3 3. $\sqrt{\left(\frac{110}{3}\right)^2+\left(\frac{475}{18}\right)^2} \approx 45.2\,\mathrm{m}$

4. the lines that meet at M are y=-x+6 and 4y-3x=4; by substitution they meet at (20/7, 22/7)
5. 61/12 years ago 6. ¤=2℗ so the right side is heavier
7.the ints are (0,6) and (8,0) a. the area is ½*b*h=24 and perimeter is 6 + 8 + $\sqrt{6^2+8^2}$ or 24 too!
b. at(6,1.5) perim is 15; can't make perimeter 10 8. a=8, b=-4, c=-2, d=8
9. the lines meet at (6,2) so the triangle's vertices are (6,2), (4,0) , and (7,0). The area is ½*3*2=3 and the perimeter is $3+\sqrt{8}+\sqrt{5}$ 10. 13.5 11. $y \geq 3$ *and* $x \leq 8$ *and* $y \leq \frac{1}{2}x+2$ 12. 10.25
13. car makes it by 2 feet ; truck misses by 2 feet; van with roof rack misses by 1 foot at roof rack
14. $x=\frac{b}{1+b/a}=\frac{ab}{a+b}$ 15. A is (-12,2) B is (-12,-5) and C is (9,-5)
16. 144/25 17a. $y-3=-\frac{4}{3}(x-1)$ *or* $y=\frac{-4}{3}x+\frac{13}{3}$ b. $\left(\frac{58}{25},\frac{31}{25}\right)$ c. area = 5.5 18. 3/7
19. 48/7 20. -7 21.point is $\left(\frac{22}{5},-\frac{19}{5}\right)$; dis is $\frac{22}{\sqrt{5}} \approx 9.84$ 22. 14/3; $5-\frac{2}{w+1}$ 23. 14/3 24. -7/12
25. 96 26. 7

Unit 3 Handout #1: Absolute Value Functions

1. Find all solutions to the following equations. Be sure to check your answers!

a. $|x-2|=5$

b. $2|x|-3=7$

c. $3|x-1|+4=19$

d. $\frac{3}{4}|x-1|+\frac{1}{2}=2$

e. $7-2|x-3|=1$

f. $2|3-2x|+1=11$

g. $6-2|x+1|=8$

h. $\frac{1}{3}|-x|+1=\frac{7}{4}$

2. Solve each inequality below. Remember: solve the related equation first and then "sign test." Graph your solution on a number line AND write it algebraically. Watch your "ors" and "ands"!

a. $|x-3|+2\le 6$

b. $2|x+5|-1>13$

c. $-\frac{1}{3}|x+1|+2 \geq -4$

d. $|x+7|-1 \leq -4$

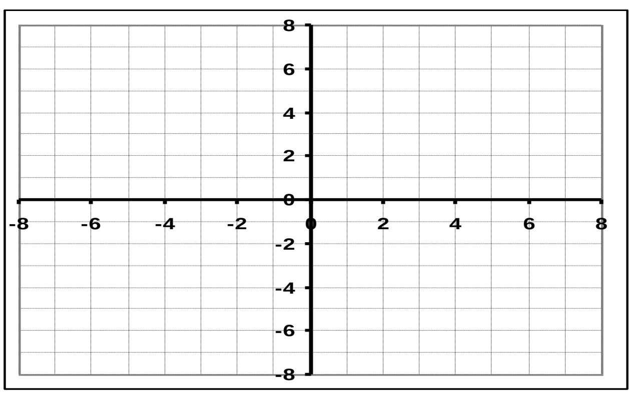
3. Find the zeros (if any) and y-intercept of each function below:

a. $f(x) = -2|x-5|+8$

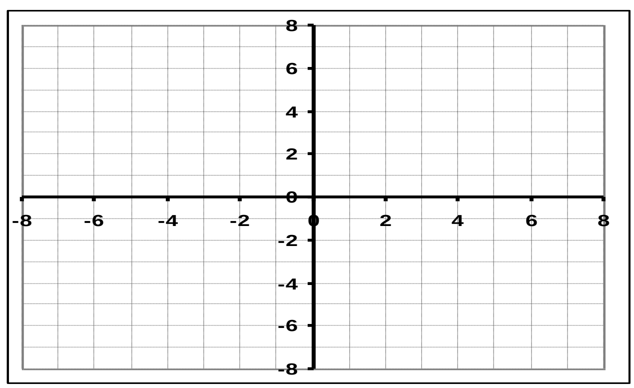
b. $f(x) = \frac{1}{2}|x+4|-3$

4. Sketch the graph of the functions below. You should include the vertex and at least two other points.

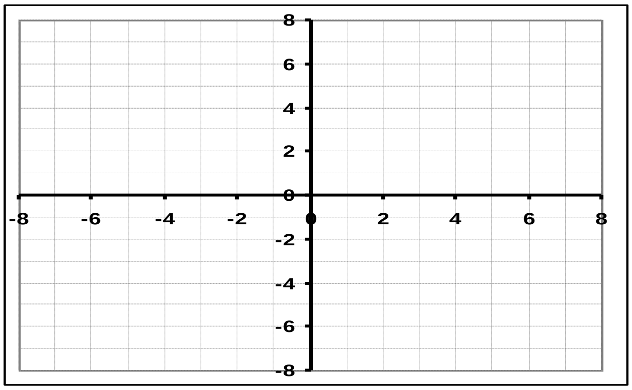
a. $f(x) = |x-1|+2$



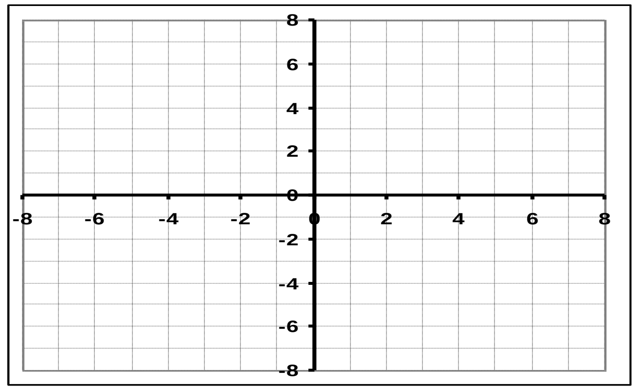
b. $f(x) = -2|x-3|+5$



c. $f(x) = -\frac{1}{2}|x| + 2$



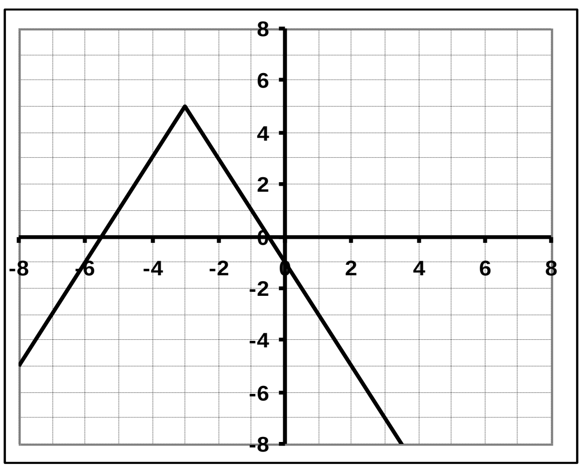
d. $f(x) = \frac{3}{2}|x + 4|$



5. For each function in problem 4 above, **use the graphs** to estimate the following (no algebra yet!):

	Graph *a*	Graph *b*	Graph *c*	Graph *d*
Zeros of $f(x)$				
Solutions to $f(x) = 3$				
Solution to $f(x) \le 3$				

6. What is the equation of the function graphed below?



7. The function $f(x)$ is an absolute value function. Its zeros are -3 and 9 and the solution to $f(x) > 2$ *is* $1 < x < c$.

a. What is the numerical value of *c*?

b. What is the equation of $f(x)$? Hint: graph.

8. A section of railway track is perfectly straight and goes through the Concord Center station. A train starts 30 miles west of Concord and continues through Concord, ending 20 miles east of Concord (at North Station). The train goes a constant speed of 45 miles per hour. Write the function showing its distance from the Concord station as a function of time since its trip began (measured in hours). A sketch may be helpful.

9. Which of the following are true for all values of *a*, *b*, *x*, and *y*?

a. $|2x+6| = 2|x+3|$

b. $|ax+ab| = a|x+b|$

c. $|x+y| = |x|+|y|$

d. $|xy| = |x|\cdot|y|$

Answers

1a. 7, -3 b. 5, -5 c. 6, -4 d. -1, 3 e. 0, 6 f. -1, 4 g. no solutions h. $\pm\frac{9}{4}$

2a. $-1 \le x \le 7$ b. $x < -12 \text{ or } x > 2$ c. $-19 \le x \le 17$ d. never true

3a. zeros are 9, 1 and y-int is (0,-2) b. zeros are -10, 2 and y-int is (0,-1)

4a. vertex is (1,2) & opens up b. vertex is (3,5) & opens down c. vertex is (0,2) & opens down
d. vertex is (-4,0) & opens up

5.

	Graph *a*	Graph *b*	Graph *c*	Graph *d*
Zeros of $f(x)$	none	0.5, 5.5	-4, 4	-4
Solutions to $f(x)=3$	0, 2	2, 4	None	-2, -6
Solution to $f(x) \le 3$	$0 \le x \le 2$	$x \le 2 \text{ or } x \ge 4$	all numbers	$-6 \le x \le -2$

6. $f(x) = -2|x+3|+5$ 7a. 5 b. $f(x) = -0.5\,|\,x-3\,|+3$ 8. $f(t) = 45\,|\,t-\frac{2}{3}\,|$

Unit 3 Handout #2: Absolute Value Graphs

1. For the function $f(x) = -\frac{3}{2}|x+1| + 3$, find the following algebraically:

   a. the coordinates of the vertex

   b. the coordinates of the y-intercept

   c. the zeros

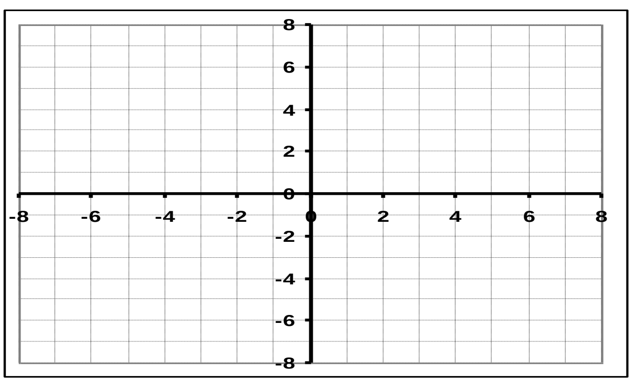
   d. the solution to the inequality $f(x) < 0$

   e. the solution to the inequality $f(x) > -6$

   f. the solution to the inequality $f(x) \leq 4$

2. Sketch the graph of $f(x) = -\frac{3}{2}|x+1| + 3$ below.

   a. State the domain and range of $f(x)$.



   Now **use the graph** to find the following (estimate as closely as possible):

   b. the coordinates of the y-intercept

   c. the zeros

   d. the solution to the inequality $f(x) < 0$

   e. the solution to the inequality $f(x) > -6$

   f. the solution to the inequality $f(x) \leq 4$

   g. Confirm that these are approximately the same as your algebraic solutions in question #1.

3. Sketch a rough graph of the functions below.

a. $h(x) = -2|x-3|$

b. $k(x) = -|x+4|-3$

4. **Algebraically** solve these equations and inequalities based on the function $f(x) = |x-3|-4$.

a. $|x-3|-4=0$

b. $|x-3|-4=-2$

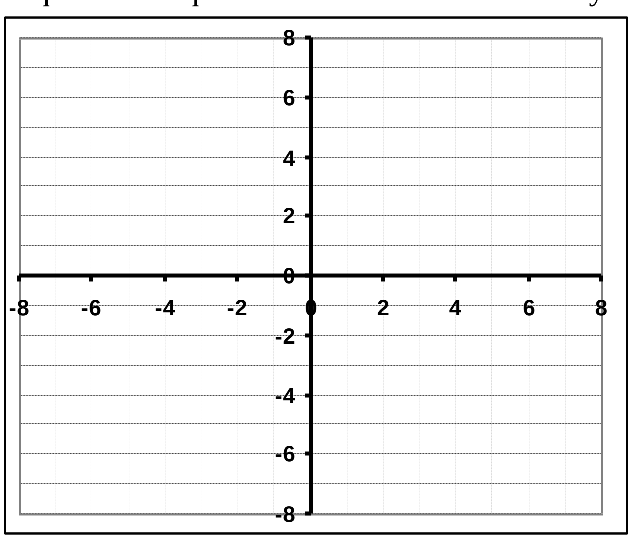
c. $|x-3|-4=-6$

d. $|x-3|-4>1$

e. $|x-3|-4\le -1$

f. $|x-3|-4\le -5$

5. Now graph function $f(x) = |x-3|-4$ and use your graph to solve all of the equations and inequalities in question 4 above. Confirm that your answers are the same (or very similar).



6. Solve the following algebraically. Be sure to check your answers—some may have no solutions or infinite solutions!

a. $3-5|x-1|=2$

b. $-2|x|<4$

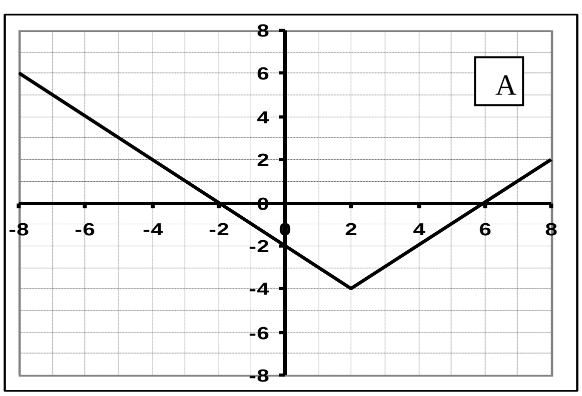
c. $2|2x-5|+3=1$

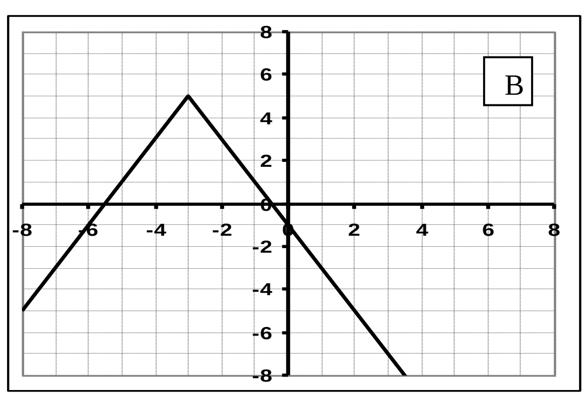
d. $2|x-3|+5<11$

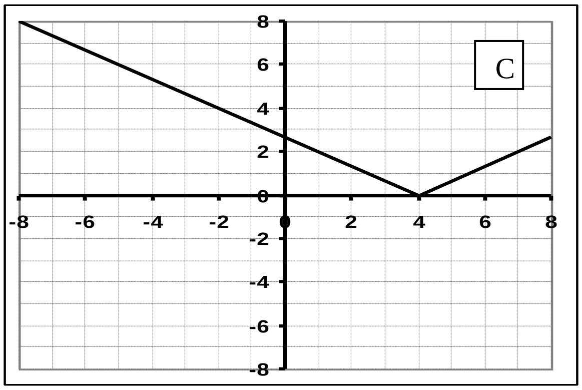
e. $\frac{1}{2}|x+5|+3>5$

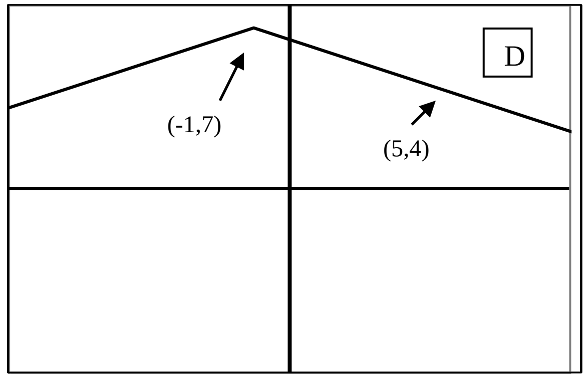
f. $\frac{-1}{2}|2x+4|+3>5$

7. Write the equations of the functions graphed below. Remember, the general form is $f(x)=a|x-h|+k$.









8. An absolute value function's vertex is (4,-2). It also goes through the point (8,6). What is its equation?

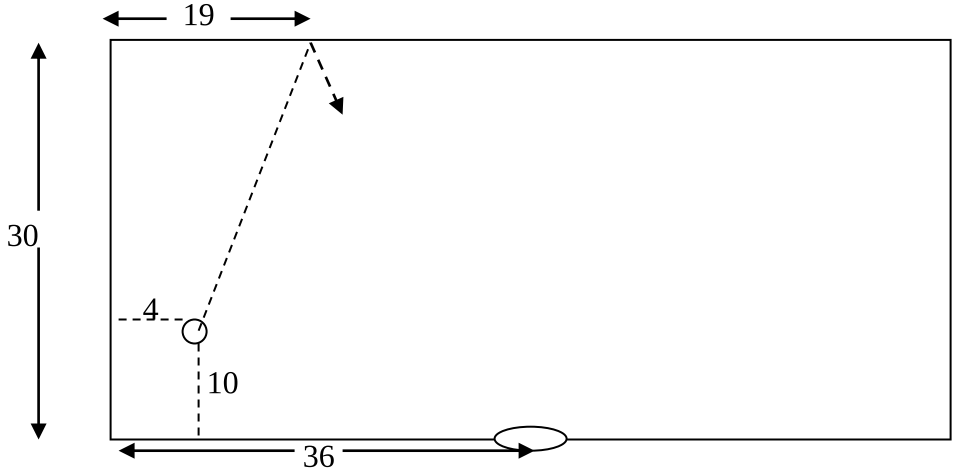
9. Write the equation of an absolute value function whose vertex is (2,-3) and whose *y*-intercept is 2.

10. The mileage (number of miles it travels for each gallon of gas) of a car depends on the speed it is driven. The function $m(r) = 32 - 0.2|r - 60|$ describes that relationship.

a. What is the highest possible mileage and at what speed does that occur?

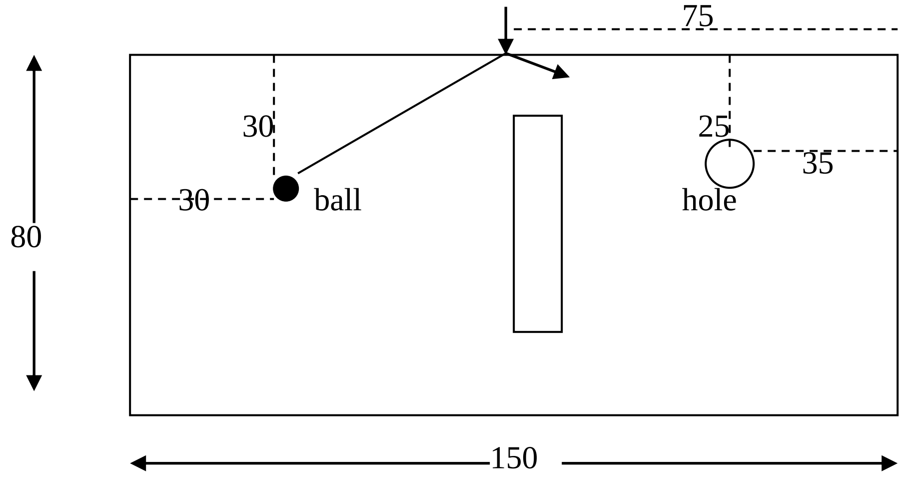
b. For the mileage to be at least 28 miles per gallon, what driving speeds are possible?

11. Pool table. If the center of the ball needs to be within one inch of the center of the pocket, will the shot go in? Write the function of the path of the ball, and solve an equation to see how far from the corner of the table it is when it gets to the bottom of the table. Is it close enough to go in? Make the origin the lower left corner of the table.



12. The solutions to the equation $|x-a|=b$ are x=-2 and x=8, what are *a* and *b*?

13. Ella is playing mini-golf. Here is the layout. The center of the ball needs to be within one inch of the center of the hole for the ball to go in. All distances are in inches.



a. Assume the origin is the lower-left hand corner of the green. Write the coordinates of the ball and the hole.

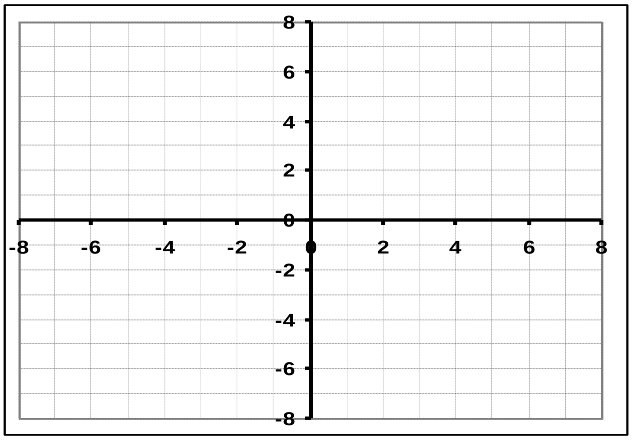
b. Ella decides to bank the shot off the top side of the putting green, 75 inches from each end. Write the coordinates of this point. Then write the equation of the absolute value function that describes the ball's path.

c. Using your equation, how far from the bottom of the green will the ball be when its x-coordinate is the same as the hole's center?

d. Will the ball go in the hole?

e. For the ball to go right in the center of the hole, how far from the left end of the green does the ball need to bounce off the wall?

14. Graph the two functions $f(x)=|x-3|-1$ *and* $g(x)=\frac{1}{2}x-1$ below.



b. Use your graph to estimate the solutions to $|x-3|-1=\frac{1}{2}x-1$.

c. Use your graph to estimate the solutions to $|x-3|-1>\frac{1}{2}x-1$.

d. Solve the equation in part b algebraically. Think of the absolute value function as two lines and find the equation of each line.

15. For different values of *k*, the inequality $-2|x+3|+7\le k$ will have different solutions. Which of the following are possible solutions to the inequality? You do not need to find *k*. Note: more than one may be true, so circle all that apply. For the ones that cannot be true, explain why.

a. $-5\le x\le -1$

b. $x>0$ *or* $x<-6$

c. $x\le -4$ *or* $x\ge -2$

d. all numbers *x*

e. there are no solutions (no *x* makes it true)

f. $x\ge 2$ *or* $x\le -7$

g. $x\ge -3$

16. Find all solutions to the following equation: $\left||x+3|-4\right|=2$. If you can, sketch the graph of $y=\left||x+3|-4\right|$

17. Given $f(x)=2|x-3|+2|x+2|+3|6-x|$. What are the highest and lowest outputs of $f(x)$ for $-3\le x\le 8$

ANSWERS

1a. (-1,3) b. (0,1.5) c. 1, -3 d. x<-3 or x>1 e. -7<x<5 f. all numbers 2. same as #1
3. vertex (3,0) opens down steep b. vertex (-4,-3) opens down
4a. 7, -1 b. 1, 5 c. none d. x>8 or x<-2 e. 0≤x≤6 f. none
5. same answers as #4 (vertex is (3,-4) and opens up)
6a. 4/5, 6/5 b. all numbers c. no solutions d. 0<x<6 e. x<-9 or x>-1 f. no solutions
7a. $f(x)=|x-2|-4$ b. $f(x)=-2|x+3|+5$ c. $f(x)=\frac{2}{3}|x-4|$ d. $f(x)=-0.5|x+1|+7$
8. $f(x)=2|x-4|-2$ 9. $f(x)=2.5|x-2|-3$
10a. 32 miles per gallon when speed is 60 mph b. 40 to 80 mph
11. path of ball is $f(x)=-\frac{4}{3}|x-19|+30$; hits x-axis when x=41.5 so too far to the right 12. a=3 b=5
13a. ball is (30,50) and hole is (115,55) b. vertex is (75,80) so $y=\frac{-2}{3}|x-75|+80$
c. when x=115 y=53.33 d. too low…
e. vertex must be (w,80) and right slope = -1*left slope so 25/(w-115) = -1*30/(w-30) and w=840/11
14b. they appear to meet at (2,0) and (6,2) so solutions (x-values) are x=2 and x=6; when plugged in, they make the equation true c. x<2 or x>6 d. the right branch is y=x-4 and the left branch is y=2-x
15. c and d are the only possible ones 16. -9, -5, -1, 3 17. (-3,41) and (3,19)

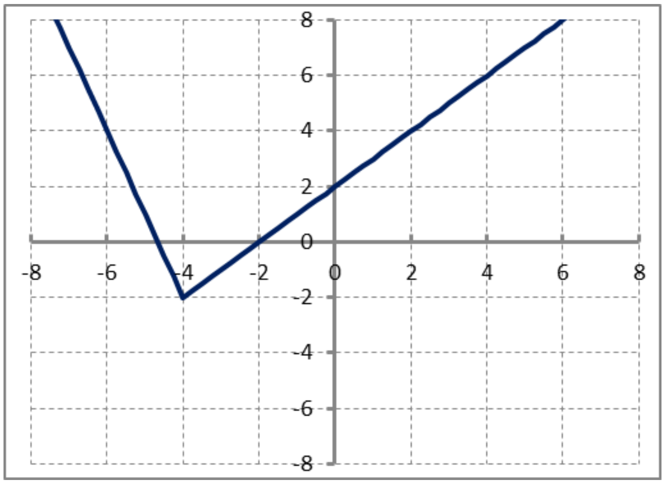
Unit 3 Handout #3: Applications of Absolute Value

1. What are the area and perimeter of the triangular region bounded by the graphs of $y=\frac{1}{2}|x+3|+2 \quad and \quad y=5$?

2. What are the area and perimeter of the region bounded by the graphs of $y=2|x-5|+4$, $y=0$, $x=1$, and $x=7$?

3. The function $f(x)$ is an absolute value function. The solution to $f(x)<5$ is $1<x<7$. The solutions to $f(x)=1$ are x=3 and x=5. What is the equation of $f(x)$?

4. This graph is the sum of a "basic" absolute value function and a linear function, so it looks like $f(x) = a|x+b|+cx+d$. What is its equation? Hint: b should be obvious; then use slopes to find a and c.



5. There are two different functions in the form $f(x) = a|x-h|+k$ that have a range of $y \le 8$ and go through the points (2,-4) and (6,4). What are they?

6. Solve the following equation: $2|x-0.5|+3=|x+6|$. It may be helpful to think of different ranges of x values and how to eliminate the different absolute value expressions in the equation for these x's. I'd look at $x<-6$, $-6<x<0.5$, and $x>0.5$ separately. Think about why! Or perhaps just sketch a graph and see how that helps!

7. Solve the inequality $|x+3|+|x-5|\le 10$.

Answers

1. area is 18 and perimeter is $12+6\sqrt{5}$ 2. area is 44 and perimeter is $26+\sqrt{80}+\sqrt{20}=26+6\sqrt{5}$

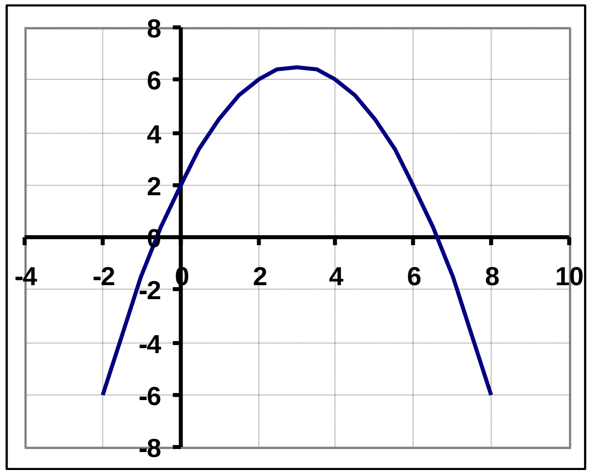
3. $f(x)=2|x-4|-1$ 4. $f(x)=2|x+4|-x-6$ 5. $f(x)=-2\,|\,x-8\,|+8$ and $f(x)=-4\,|\,x-5\,|+8$

6. -2/3 or 4 7. $-4\le x\le 6$

Practice Problems for Units 1-3 Test

Part I: "Plain Vanilla Problems"

1. Use the graph of the function *f(x)* below to answer the following questions:



a. What is *f*(6)?

b. What are the solutions to *f(x)*=6?

c. What are the zeros of *f(x)*?

d. What is *f*(0)?

e. What are the domain and range of *f(x)*?

f. What values of *x* solve the inequality *f(x)*>2.

2. Answer these questions about the function *g(x)*= -1.5*x* + 4
   a. What is/are the zero(s) of *g(x)*?
   b. What is *g*(3)?
   c. Solve: *g(x)*=18.
   d. Solve: *g(x)* < 7.

3. The function *h(x)* shows the total number of baseball cards in *x* packs. What is the difference between *h*(20) and *h*(*x*)=20?

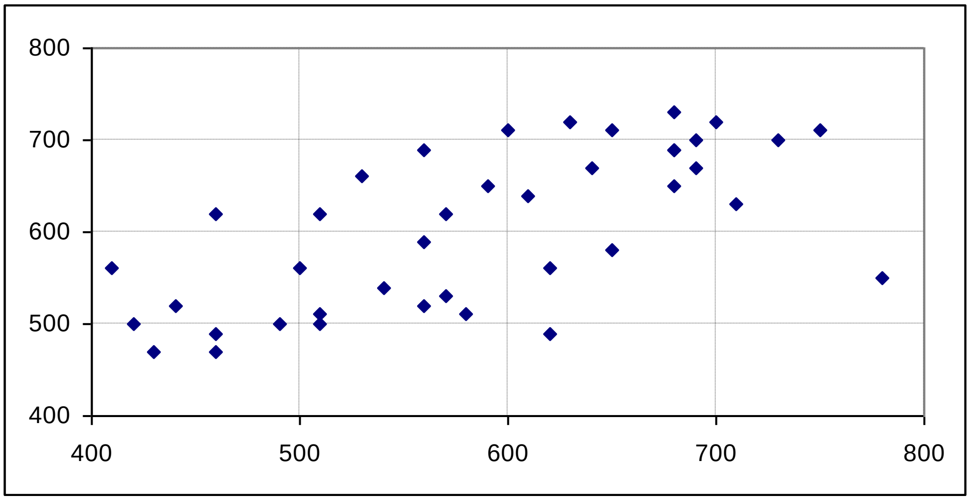
4. After printing 200 pages, a printer has 50 ml of toner left. After printing 350 pages, there are only 30 ml of toner left. In the relation between pages printed (*x*) and toner left (*y*), what is the slope? What is the meaning of the slope in this situation (what are its units)?

5. Write the equation of each linear function described below:
   a. *f*(5)=11 and *f*(-2)=8.
   b. *g(x)* has a zero of 7. Also, *g*(2)=13.
   c. A taxi charges one rate for the first mile and a different rate for each additional mile. If a six-mile ride costs $20 and a nine-mile ride costs $29, then write the function C(*r*) showing the cost for an *r*-mile ride.

6. Liz is running a race. Sixteen minutes after the start of the race she has 4 miles until the finish line. Thirty minutes after the start of the race she has 2 miles until the finish line. Assume a linear relationship.
   a. Write an equation showing the distance Liz has to go *D(t)* as a function of the time since the start of the race (*t*).
   b. What are the units of the slope?
   c. Graph the equation.
   d. Evaluate *D*(7). What meaning does it have in this problem?
   e. What is the zero of *D(t)* and what meaning does it have in this problem?
   f. How long is the race?

7. The graph below shows the math and verbal SAT scores for a group of 40 students at Jefferson High School. Each point represents one student: the *x*-coordinate is the math SAT score and the *y*-coordinate is the verbal SAT score.

a. Is the correlation between the two variables positive, negative, or roughly zero?
b. Sketch the best-fit line.
c. Write an approximate equation for it.
d. Use this equation to estimate the verbal score of a student who gets 620 on the math.



8. Sketch a graph of each of the functions below. Then **use the graphs** to estimate the zeros and y-intercepts of the functions. State the domain and range as well.

a. $f(x) = 2|x-2| - 4$ b. $g(x) = -3|x| + 6$ c. $h(x) = -\frac{2}{3}|x+4| - 1$ d. $f(x) = -\frac{3}{5}x + 4$

9. Solve the following algebraically. For the inequalities, you must show the solution on a number line AND write it symbolically (using < and > signs). Some may have no solution!

$a.\ \frac{1}{2}|x-5| + 1 = 2$

$b.\ 2|x+3| + 3 = 9$

$c.\ 3 - |x-1| = 4$

$d.\ 2|x+2| + 11 = 11$

$e.\ |x-3| < 2$

$f.\ \frac{3}{4}|x+4| + 7 < 4$

$g.\ -2|x| < 6$

$h.\ 2|x-6| - 5 \geq 2$

$i.\ 0.5|2x-1| + 4 > 3$

$j.\ -2|x+1| - 4 \leq -10$

k. $\frac{-2}{3}(x-2) + 4 = \frac{11-4x}{7}$

10. Write the equations of the absolute value functions described.
a. The vertex is (0,6) and it goes through the point (-4,-6).
b. The vertex is (-4,3) and the *y*-intercept is 5.

11. Algebraically find the zeros and y-intercept of each of the following functions (if any).

a. $f(x) = -2x + 7$ b. $g(x) = \frac{2}{3}|x-5| - 2$ c. $h(x) = -2|x+3| - 8$

12. Pears cost 70 cents per pound more than apples do. Four pounds of pears costs $4.20 more than three pounds of apples. How much is one pound of apples?

More challenging Questions:

13 How long green beans stay fresh (after they are picked) is a function of the temperature they are stored at. The best temperature is 40 degrees F; at this temperature they last 18 days. Each 3 degrees away from this temperature (in either direction) reduces their life by one day. Write the function that describes how long they stay fresh as a function of the temperature they are stored at.

14. A triangle has vertices of A (-2,-1), B (1,5), and C (9,0). Is this a right triangle? Try to justify your answer in two (or more different ways).

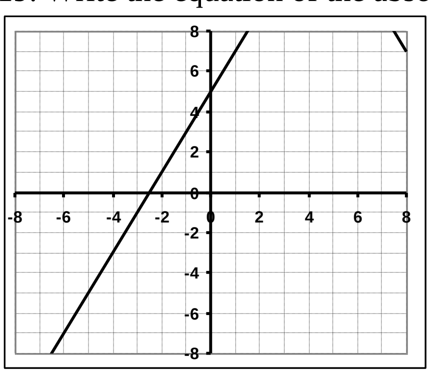
15. The range of an absolute value function is $f(x) \le 6$. The points (0,1) and (2,4) are on the graph. Can you find two possible equations of the function?

16. The region R is bound by the x-axis, the y-axis, the line $x = 6$ and the graph $y = 0.5|x-2|+3$. Find the area and perimeter of R. Note: you don't need the formula for trapezoid, since you can use rectangles and triangles.

17. The line through the points (5,*a*) and (7,2*a*-3) never intersects the line through the points $(2,-4)\ and\ (-5,7)$. What is the value of *a*?

18. Anna and Karina both like to run, but Anna runs at twice the speed of Karina. One day, she gives Karina a 1 mile head start and also waits for Karina to have run for ¼ hour before she starts to run. One hour after Karina starts, Anna is 1.5 miles ahead of her. What is Karina's speed? Hint: use rate* time=distance.

19. Write the equation of the absolute value function graphed below:



20. Draw the graph of any function $f(x)$ where the domain is $x \ge -4$, the range is $-2 \le f(x) \le 6$, the zeros are -1 and 4, and the solution to $f(x) > 5\ is\ 6 < x < 7$.

21. The distance from a point to a line is defined as the distance from the point to the closest point on the line. The shortest segment connecting the point to the line is thus perpendicular to the line (draw it, you'll see!). How far is the point (3,-1) from the line $y = 0.5x + 4$?

22. A rectangle's corners are A (5,3), B(11,3), C(5,13), and D(11,13). The line $y = 0.5x + b$ intersects the left side of the rectangle at point E and the right side at point F. The area of trapezoid ABFE is 20. What is *b*? Hints: draw a diagram! Also, the area of a trapezoid is the average of the two parallel bases times the height.

Answers

1. a. 2 b. x=2, 4 c. about 6.8 and -0.8 d. 2 e. Domain: $-2 \le x \le 8$; range approx $-6 \le y \le 6.5$

1f. $0 < x < 6$ 2. a. x=8/3 b. -0.5 c. -28/3 d. x>-2

3. h(20) is cards in 20 packs; h(x)=20 shows that x packs have 20 cards.

4. slope = -2/15 means every page takes $2/15^{th}$ of a ml of toner

5a. $f(x) = \frac{3}{7}x + \frac{62}{7}$ b. $f(x) = \frac{-13}{5}x + \frac{91}{5}$ c. C(r)=3r+2

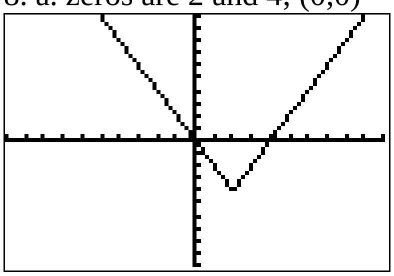
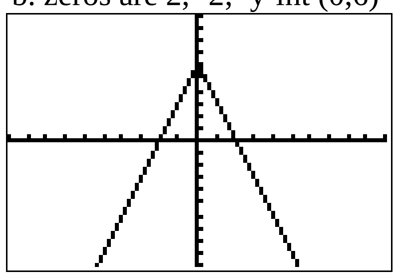
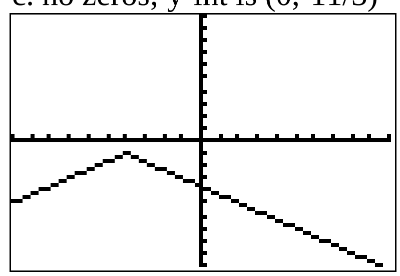
6. a. $D(t) = -\frac{1}{7}t + \frac{44}{7}$ b. miles per minute

d. $D(7)$=5.28 miles; 7 minutes into the race she has this far to go

e. t=44 is zero; after 44 minutes she is finished f. when t=0 D(t)= 44/7 or 6.28 miles to go

7. a. correlation is positive c. one reasonable one is y=0.5x+300 d. if x=620 y=610

8. a. zeros are 2 and 4; (0,0) b. zeros are 2, -2; y-int (0,6) c. no zeros; y-int is (0,-11/3)

d. y-int is (0,4) and zero is about 7 (20/3)

9. a. 3, 7 b. 0, -6 c. no solutions d. -2
   e. 1<x<5 f. no solutions g. all numbers h. $x \le 2.5$ or $x \ge 9.5$
   i. all numbers j. $x \le -4$ or $x \ge 2$ k. 79/2

10. a. $f(x) = -3|x| + 6$ b. $f(x) = \frac{1}{2}|x+4| + 3$

11. a. y-int is (0,7) and zero is x=3.5 b. y-int is (0,4/3) and zeros are 2, 8
   c. y-int is (0,-14) and no zeros (vertex is below x-axis and it opens down)

12: $P = 0.7 + A$ *and* $4P = 3A + 4.2$ so A=$1.40 13. $f(t) = \frac{-1}{3}|t - 40| + 18$

14. No: slopes are 2, 1/11, and -5/8 so none are opposite reciprocals so no right angle
   Also lengths of sides don't work in the Pythagorean Thm, so not a right triangle

15. if the vertex is right of x=4 then a=-3/2 and we get $f(x) = -\frac{3}{2}|x - \frac{10}{3}| + 6$

If the vertex is between the points then a=-7/2 (up and down 7 while across 2) so

$f(x) = -\frac{7}{2}|x - \frac{10}{7}| + 6$

16. perimeter is 4+6+5+ $\sqrt{5} + 2\sqrt{5} = 3\sqrt{5} + 15$ area is 23

17. lines must be parallel so $\frac{a-3}{2} = \frac{11}{-7}$ and a=-1/7

18. $A = 2K$ *and* $0.75A = 1.5 + 1 + K$ so A=2.5 and K=5 mph (step-by-step solution is below)
Let A be Anna's speed and K be Karina's speed.

We know that $A = 2K$.

We also know that the distance Karina runs in 1 hour is 2.5 miles less than the distance Anna runs in 45 minutes (Karina starts a mile ahead and ends 1.5 miles behind). So $0.75A = 2.5 + 1(K)$.

The substituting in A=2K we get $0.75(2K) = 2.5 + 1(K)$ *so* $K = 5$ and A=2.5

19. $f(x) = -2|x - 4.5| + 14$

21. The connecting segment has slope -2 (to make them perpendicular).. the line with slope -2 thru (3,-1) is $y = -2x + 5$. They meet at (2/5, 21/5). So the distance is $\sqrt{\left(\frac{13}{5}\right)^2 + \left(\frac{26}{5}\right)^2} = \frac{13\sqrt{5}}{5}$ (don't stress if you can't yet simplify that radical!)

22. I set w to be the length of segment AE; since the slope of the line is 0.5 and AB is 6, BF must be w+3. The average base is then w+1.5 so (w+1.5)6=20 and w=11/6 so E is (5,11/6) thus the line is $y = 0.5x - \frac{2}{3}$

Unit 4 Handout #1: Solving Equations by Square Roots and Factoring Refresher

These are some questions on topics you should have seen before. They should help refresh your memory, or alert us to any areas that somehow went uncovered.

I. Solving quadratic equations by taking square roots: Solve the following equations by isolating the squared term and then taking square roots. Most of them will have two solutions, although some may have one or none. Like absolute value equations, this is **"Isolate and Undo."**

Example: $2(x-3)^2+5=29$

$$2(x-3)^2=24$$
$$(x-3)^2=12$$
$$x-3=\pm\sqrt{12}=\pm\sqrt{4}\sqrt{3}=\pm2\sqrt{3}$$
$$x=3\pm2\sqrt{3}$$

Problems: solve the following equations, if possible.

1. $3(x-1)^2+5=17$

2. $\frac{-1}{2}(x+7)^2+10=1$

3. $3(x-2)^2+5=2$

4. $\frac{2}{3}(x+4)^2+7=23$

II. Factoring out Monomials: Factor the following expressions. In each one, there is a common monomial that needs to be factored out. (monomial means “one term”). You only need to factor out the monomial, nothing else!

Example: $12x^3-9x=3x(4x^2-3)$

Problems: factor the following:

5. x^2-8x

6. $3x^2+6x$

7. $5x^2+10$

8. $2x^2-6x-2$

9. $5x^3+10x^2$

10. $3x^3-6x^2+30x$

III. Factoring Trinomials into two binomials: Factor the following expressions. Since these are not equations, you should **not** have solutions (such as x=3 or x=-2).

Examples: factor $x^2 - 2x - 15$

$(x+\quad)(x-\quad)$ two numbers whose product is -15 and sum is -2

$(x+3)(x-5)$

factor $2x^2 + 3x - 5$

$(2x \pm \quad)(x \pm \quad)$ numbers must be 5,-1 or -5,1 → use trial and error

$(2x+5)(x-1)$

Factor $x^2 - 16$ this is the "difference of squares" → it has a unique pattern

$(x+4)(x-4)$ the +4 & -4 (from "inner" and "outer") result in no x term

Problems: Factor each of these into two binomials.

11. $x^2 + 5x + 4$

12. $x^2 + 8x + 12$

13. $x^2 - 4x + 3$

14. $x^2 - x - 12$

15. $x^2 - 10x + 16$

16. $x^2 - 36$

17. $x^2 + x - 20$

18. $x^2 - 4x + 4$

19. $x^2 - 3x - 10$

20. $x^2 + 3x - 18$

21. $x^2 - 7x + 12$

22. $x^2 - 11x - 26$

23. $2x^2 + 7x + 3$

24. $2x^2 - x - 3$

IV. Solving quadratic equations by factoring: Solve the following equations by factoring. You want to set one side to zero before factoring.

Example: solve the equation $x^2+4x+2=7$

$x^2+4x-5=0$	set one side to zero
$(x+5)(x-1)=0$	factor other side
$x+5=0 \quad or \quad x-1=0$	set each factor to 0. If two numbers multiply to 0; one must be 0
$x=-5 \; or \; 1$	solve the two linear equations

Problems: solve the following equations.

25. $x^2=9x+10$

26. $x^2+4x=0$

27. $2x^2+12=10x$

28. $x^2=8x$

29. $x^2-6x-16=0$

30. $(x+2)^2=2x+7$

31. $(x+1)(x-4)=14$

32. $2x^2+x=6$

33. $\frac{1}{2}x^2+6x+16=0$ (multiply by 2 first!)

34. $0=5x^2+11x+6$

35. Why can't you "isolate and undo" to solve these equations as you did in part I above?

36. Why is it necessary to set one side to zero before solving an equation by factoring?

37. Solve the inequality by solving the related equation and "sign-testing": $x^2 - x - 6 \geq 0$

V. Factoring Higher-Degree Polynomials

There are two techniques that we can use to factor higher-degree polynomial functions

#1: Monomial factors.

Example: factor $2x^3 - 4x^2 - 16x$

Take the monomial factor $2x$ out first: $2x(x^2 - 2x - 8)$

Then factor the other part, if possible: $2x(x-4)(x+2)$

#2: Quadratic form (aka "hidden quadratic")

Example: factor $x^4 - 3x^2 - 4$

You can write this as $(x^2)^2 - 3(x^2) - 4$

It is now like part III above: something squared minus three somethings minus 4

So it factors into $(x^2+1)(x^2-4)$ and then into $(x^2+1)(x-2)(x+2)$

Some people substitute $u = x^2$ and write it as $u^2 - 3u - 4 = (u-4)(u+1) = (x^2-4)(x^2+1)$

Note: it can also be used for something like $x^6 - 3x^3 - 10 = (x^3-5)(x^3+2)$

But NOT something like $x^4 - 3x - 10$ since the powers need to be 2*n*, *n*, and 0.

Note #2: You can factor $x^5 - 5x^3 + 6x$ by taking an *x* out first and then recognizing quadratic form. It becomes $x(x^2-2)(x^2-3)$

Factor the following fully. Some may have quadratic factors that are themselves not factorable.

38. $-3x^3 + 6x^2 + 9x$

39. $x^4 - 5x^2 + 4$

40. $x^4 - 5x^3 + 4x^2$

41. $x^4 - 2x^2 - 15$

42. $-x^5 + x^3 + 12x$

Solve the following. Remember to treat the inequalities like equations and then "sign test".

43. $x^4 - 11x^2 + 19 = 1$

44. $x^4 - 8x^2 = 2x^3$

45. $x^3 - x > 0$

46. $x^3 - x^2 \geq 0$

ANSWERS

1. 3 or -1	2. $-7 \pm 3\sqrt{2}$	3. no solutions	4. $-4 \pm 2\sqrt{6}$
5. $x(x-8)$	6. $3x(x+2)$	7. $5(x^2+2)$	8. $2(x^2-3x-1)$
9. $5x^2(x+2)$	10. $3x(x^2-2x+10)$	11. $(x+1)(x+4)$	12. $(x+6)(x+2)$
13. $(x-1)(x-3)$	14. $(x-4)(x+3)$	15. $(x-2)(x-8)$	16. $(x+6)(x-6)$
17. $(x+5)(x-4)$	18. $(x-2)(x-2)$	19. $(x+2)(x-5)$	20. $(x-3)(x+6)$
21. $(x-3)(x-4)$	22. $(x+2)(x-13)$	23. $(2x+1)(x+3)$	24. $(2x-3)(x+1)$
24. 10, -1	26. 0, -4	27. 2, 3	28. 0, 8
29. 8, -2	30. 1, -3	31. 6, -3	32. -2, 3/2
33. -8, -4	34. -1, -6/5	37. $x \geq 3$ *or* $x \leq -2$	
38. $-3x(x-3)(x+1)$	39. $(x-1)(x+1)(x-2)(x+2)$	40. $x^2(x-4)(x-1)$	
41. $(x^2-5)(x^2+3)$	42. $-x(x-2)(x+2)(x^2+3)$	43. 3, -3, $\pm\sqrt{2}$	
44. 0, 4, -2	45. x>1 or -1<x<0	46. x=0 or x>1	

Unit 4 Handout #2: Graphing Parabolas

1. Which of the following functions are more likely to be linear, and which quadratic. Briefly explain your reasoning.

a. The population of a city as a function of time.

b. The height of a ball thrown as a function of time since it was thrown.

c. The frequency of motor vehicle accidents by drivers of different ages (from 16 to 90)

d. The amount of tomatoes a plant produces as a function of the rainfall.

e. The value of a Chevy Corvette as a function of its age.

f. The times at a 5k race for women as a function of their age (from 10 to 80)

g. The path of a volleyball from when it is served (underhand?) until it lands.

For each quadratic function below, find the <u>equation</u> of the axis of symmetry, the coordinates of the vertex, the y-intercept, the number of x-intercepts, and specify whether it opens up or down:

	Axis of Symm	**Vertex**	**Y-Int**	**Opens**	**# of x-ints**
2. $f(x) = 2x^2 + 8x - 5$					
3. $f(x) = -x^2 + 5x - 3$					
4. $f(x) = \frac{1}{2}(x-3)^2 + 4$					
5. $f(x) = -2(x+4)^2 + 2$					

6. $f(x) = (x+1)^2 - 6$					
7. $f(x) = -2x^2 + 7$					
8. $f(x) = (x-1)(x-5)$					
9. $f(x) = 2(x+1)(x-7)$					
10. $f(x) = -2x(x+6)$					
11. $f(x) = -\frac{1}{2}(x-3)(x+2)$					

In 12-15, rewrite each of these quadratic functions in standard form ($f(x) = ax^2 + bx + c$).

12. $f(x) = -(x-4)^2 + 11$

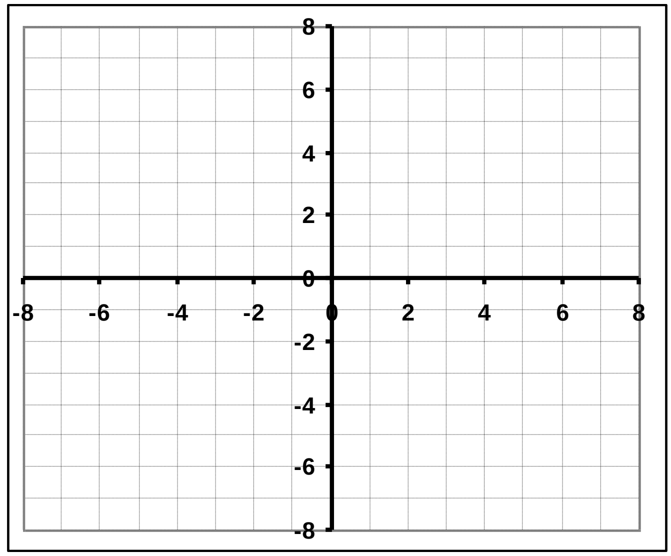
13. $f(x) = \frac{1}{2}(x+2)^2 - 3$

14. $f(x) = 3(x+4)(x+3)$

15. $f(x) = -\frac{1}{2}(x-1)(x+4)$

16. Answer the following questions about $f(x) = x^2 - 4x - 2$.

a. Sketch a graph below of the function.



b. Use the graph to get approximate solutions to the following equations and inequalities:

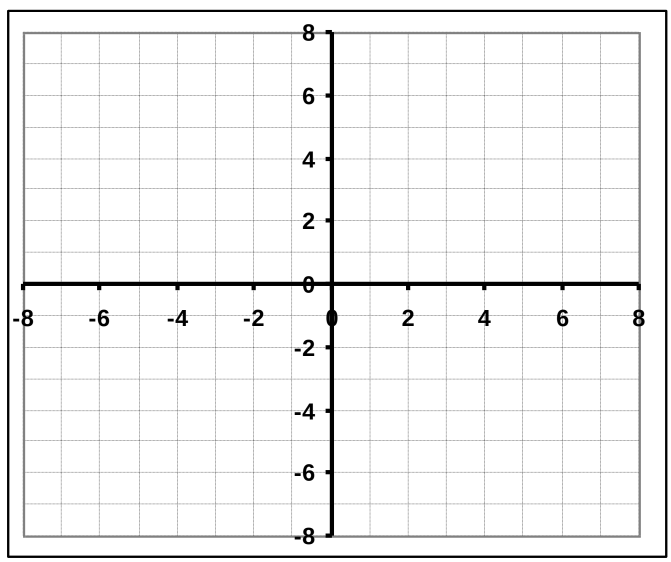
i. $x^2 - 4x - 2 = 2$

ii. $x^2 - 4x - 2 = -1$

iii. $x^2 - 4x - 2 < 3$

iv. $x^2 - 4x - 2 < -7$

17. Graph the function $f(x) = -\frac{1}{2}(x+2)^2 + 7$. Label the coordinates of **five** points. Then **<u>use your graph</u>** to estimate solutions to the following equations and inequalities.



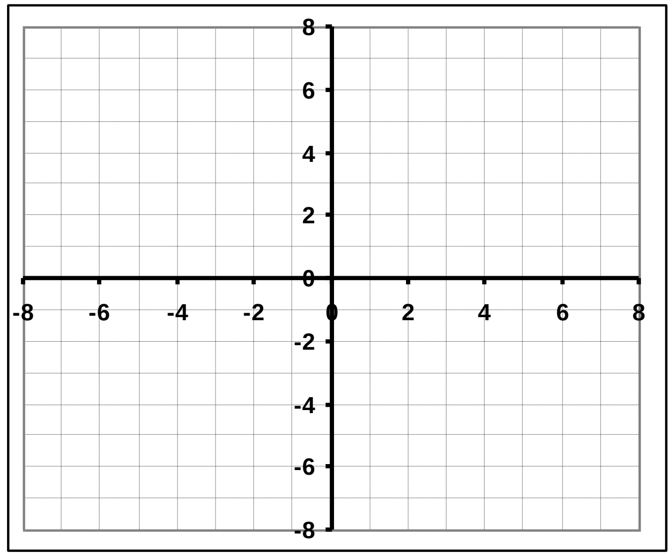
a. What are the zeros of $f(x)$?

b. $-\frac{1}{2}(x+2)^2 + 7 = 3$

c. $-\frac{1}{2}(x+2)^2 + 7 > 0$

d. $-\frac{1}{2}(x+2)^2 + 7 < -1$

18. Graph the function $f(x) = -2(x-1)(x+3)$. Label the coordinates of **five** points. Then use your graph to estimate solutions to the following equations and inequalities.



a. $-2(x-1)(x+3) = 4$

b. $-2(x-1)(x+3) < -2$

c. $-2(x-1)(x+3) > 6$

19. Answer the following questions.

a. A projectile is launched. Its height (in meters) as a function of time (in seconds) is given by the equation $h(t) = -5t^2 + 20t + 15$. What was its maximum height and when did it attain it?

b. The function $p(x) = -2x^2 + 12x + 20$ describes the profits a company earns if it charges x dollars for its product. What are its highest possible profits, and what price should it charge to earn them?

20. Solve the following by factoring. Remember, you must set one side to zero before you factor!

a. $(x+2)^2 = 2x+7$

b. $(x+1)(x-4) = 14$

c. $\frac{1}{2}x^2+6x+16=0$

d. $8+3x=5x^2+14x+14$

21. On the axes below, sketch the graphs of $f(x)=-\frac{1}{2}x+5$ and $g(x)=(x-3)^2-5$. Use the graphs to estimate solutions to the following:

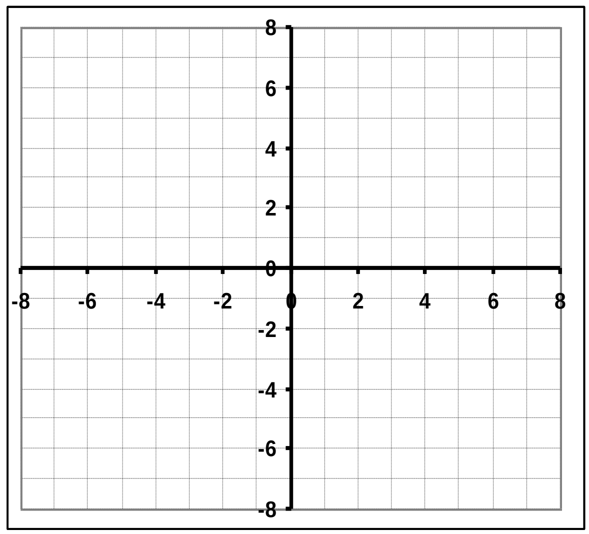
a. $(x-3)^2-5=2$

b. the zeros of $g(x)$

c. $(x-3)^2-5<0$

d. $(x-3)^2-5=-\frac{1}{2}x+5$

e. $(x-3)^2-5<-\frac{1}{2}x+5$



22. Write the equation of each parabola described below. Think about what form is best to use.

a. Vertex is (2,-3) and goes through (0,5)

b. Vertex is (-2,5) and goes through (-3,2)

c. Zeros are 4 and 2 and goes through (1,6)

d. Zeros are -1 and 5 and y-intercept is -2.

e. $f(x) = ax^2 + bx + c$ where $f(0) = 5,\ f(1) = 8,\ and\ f(2) = 19$. Hint: write 3 equations and solve.

f. The solution to $f(x) \geq 3$ is $1 \leq x \leq 7$ and the range of $f(x)$ is $f(x) \leq 16$.

g. The y-intercept of $f(x)$ is 12 and for $g(x) = x + 4,$ the solution to $f(x) \leq g(x)$ is $2 \leq x \leq 6$.

ANSWERS
#1: all parabolas except a
#2-11: axis of symmetry is vertical line (x=) through the vertex.
2. vertex (-2, -13) y-int (0,-5) 3. vertex (2.5, 3.25) y-int (0,-3) 4. vertex (3,4) y-int (0, 8.5)
5. vertex (-4,2) y-int (0,-30) 6. vertex (-1,-6) y-int (0, -5) 7. vertex (0,7) y-int (0,7)
8. vertex (3, -4) y-int (0,5) 9. vertex (3, -32) y-int (0,-14)
10. vertex (-3, 18) y-int (0,0) 11. vertex (0.5, 3.125) y-int (0, 3)
12. $-x^2 + 8x - 5$ 13. $0.5x^2 + 2x - 1$ 14. $3x^2 + 21x + 36$ 15. $-0.5x^2 - 1.5x + 2$
16a. x is about -0.8 or 4.8 b. about -0.2, 4.2 c. -1<x<5 d. no solutions
17a about 1.8 and -5.8 b. about 0.8 and -4.8 c. about -5.8<x<1.8 d. x>2 or x<-6
18. vertex is (-1,8) a. about 0.5, -2.5 b. about x>1.5 or x<-3.5 c. -2<x<0
19a. t=2 and h(t)=35 meters b. x=3 and profits are $38
20a. 1, -3 b. 6, -3 c. -8, -4 d. -1, -6/5
21a. about 0.3 and 5.7 b. about 0.8 and 5.2 c. approximately 0.8<x<5.2
d. x is about -0.5 or 5.5 e. parabola below line so -0.5<x<5.5
22a. $f(x) = 2(x-2)^2 - 3$ b. $f(x) = -3(x+2)^2 + 5$ c. $f(x) = 2(x-4)(x-2)$
d. $f(x) = (2/5)(x+1)(x-5)$ e. $f(x) = 4x^2 - x + 5$
f. $f(x) = -\dfrac{13}{9}(x-4)^2 + 16$ g. $f(x) = \dfrac{2}{3}x^2 - \dfrac{13}{3}x + 12$

Unit 4 Handout #3: Simplifying Radicals and Applied Quadratics Problems

1. Why is the axis of symmetry $x = \dfrac{-b}{2a}$? (part I)

   a. Given the quadratic function $f(x) = ax^2 + bx + c$, find the *y*-intercept.

   b. Now find the other *x*-value whose output is the same as the *y*-intercept.

   c. Given symmetry, the vertex must lie on which vertical line?

2. Simplify the following radicals and expressions involving radicals: (section 4.5 in the book)

a. $\sqrt{50}$ b. $\dfrac{\sqrt{54}}{6}$ c. $\dfrac{\sqrt{48}}{2}$ d. $\sqrt{\dfrac{16}{25}}$ e. $\dfrac{-3+\sqrt{18}}{3}$

3. Find the vertex of the following parabolas. Then find the zeros and y-intercept algebraically:

a. $f(x) = -3x^2 - 12x + 36$ b. $f(x) = 2(x+3)^2 - 8$

4. Solve the following systems by factoring. They can be solved by substituting for *y*. Most will have two solutions because both equations are not linear. (Graphically they correspond to the intersections of a parabola and a line).

a. $\begin{array}{l} y = x^2 + 2x + 3 \\ y = x + 9 \end{array}$

b. $\begin{array}{l} y = (x+3)(x+1) \\ y = \dfrac{1}{2}x + 3 \end{array}$

5. Some students throw a snowball upwards from the roof of the building; it lands on the ground not far from the building. Its height (in meters) t seconds after it is thrown is given by the equation $h(t) = -5t^2 + 10t + 15$. Solve the following algebraically, but graph this in your calculator to check yourself. Be careful to set an appropriate viewing window. (hit the window button and try different numbers for x-min, x-max, y-min, and y-max).

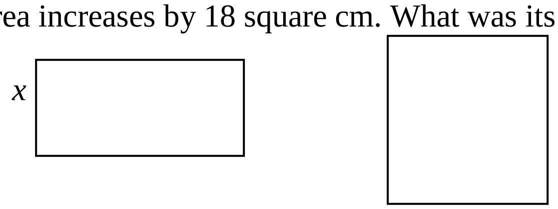
a. What was its height after 2 seconds?

b. How high was it thrown from?

c. When did it reach its maximum height and what was its maximum height?

d. When did it land?

6. A rectangle's length is 5 cm more than its width. If you double its width and reduce its length by 4 then its area increases by 18 square cm. What was its original width?



7. Solve the following inequalities. When solving the related equations, think about whether it is more efficient to solve with square roots or by factoring.

a. $x^2 > 25$

c. $x^2 - 11x > x$

b. $x^2 - 6x \le 7$

d. $-2(x+1)^2 + 10 < 8$

8. A quadratic function $f(x)$ has a vertex of (2,-4) and goes through the point (4,4).

a. Sketch a rough graph.

b. Write its equation.

c. Find its zeros algebraically (think about the best technique).

d. For what x values is $f(x) = 12$?

e. For what x values is $f(x) < 14$?

9. The zeros of a quadratic function $f(x)$ are -3 and 5. It also goes through the point (4,7).

a. Sketch a rough graph.

b. Write its equation.

c. Find its vertex.

d. Find the solutions to the equation $f(x) = 12$.

10. The function $f(x)$ is a quadratic function of the form $f(x) = ax^2 + bx + c$. You are told that $f(0) = 5, \quad f(1) = 4$, *and* $f(-2) = 19$.

a. Given the first point, you should know what the constant c is. Find it.

b. Given this *c*, plug in each of the other two points to get 2 equations involving *a* and *b*.

c. Solve those two equations, giving you the function $f(x)$.

11. Given that $f(x) = -5x^2 + bx + c$ and $f(1) = 10$ *and* $f(2) = 5$, find $f(x)$.

12. A ball is dropped off a tall building. Its height (in feet) t seconds after it is dropped is given by the equation $h(t) = 832 - 16t^2$. Answer the following algebraically. Use to calculator to graph the function and make sure your answers are reasonable.

a. How tall was the building?

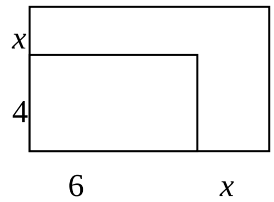
b. How high was the ball 5 seconds after being dropped?

c. When was the ball's height equal to 576 feet?

d. When did the ball land? (decimal answer)

e. When was the ball's height at least 688 feet?

13. A rectangle is 6 by 4 (length by width). Its length and width are each increased by *x* units and its area is doubled. What is *x*?



14. The path of an Olympic long-jumper can be given by the equation $y = -0.02x^2 + 0.50x$ where *y* is her height and *x* is her horizontal distance from the start of her jump (both are in feet).

a. How far did she jump?

b. What was the maximum height she reached, and how far from the start was she?

c. Someone mistakenly left a box 2½ feet high 18 feet past where her jump began. Does she clear it?

15. A bridge over a road is shaped like a parabola (opening down). It is 20 feet high at its highest and the roadway is 100 feet wide. Can a rectangular object 40 feet wide and 16.5 feet high (a wide trailer, for example) fit through? Explain.

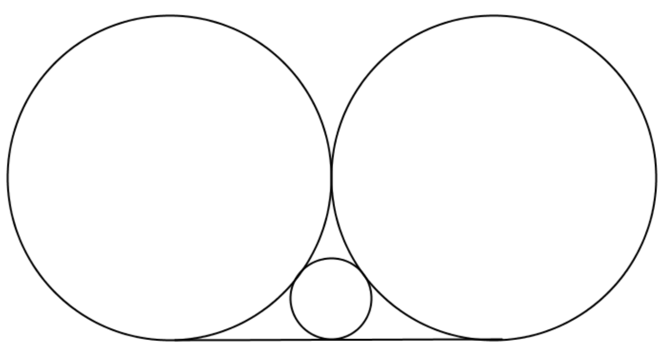
16. Anna and Lena were in a rowboat one day and Lena saw a water lily. She told Anna that they could use it to determine the depth of the water because it was rooted to the lake's bottom. When the water lily was pulled straight up it was 10 inches above the water's surface. When they rowed 5 feet away, the water lily was exactly at the water's surface. How deep was the lake?

17. One way to solve the quadratic equation $(x-4)^2+5(x-4)+6=0$ is to multiply each term out, combine like terms, and then try to factor. But you could also solve it by recognizing that it is quadratic in (x-4), factor it to solve for (x-4) and then solve for x. Do it. Then use a similar technique to solve these equations.

a. $(x-2)^2-3(x-2)-10=0$ b. $(x+1)^2-4(x+1)-12=0$ c. $2(x^2-1)^2-3(x^2-1)+1=0$

18. Given the points A (0,4), B (10,0) and C(3,w). What are the four values of w that make ABC a right triangle? The right angle can be at any of the vertices.

19. In the diagram below, two circles are tangent to each other and tangent to a line segment of length one unit. A small circle is tangent to both larger circles and the line segment. What is the radius of the small circle? (from jamestanton.com)



20. In his book *Mathletics*, Wayne Wilson says the winning percentage (w) of a baseball team can be approximated by the formula $w \approx \frac{(RS)^2}{(RS)^2 + (RA)^2}$, where RS is the total runs scored for the team over the season, and RA is the total runs scored by their opponents in their head-to-head games. For a team to attain a winning percentage of 60% (0.6), they must score what percent more runs than their opponents? (hint: pick a number for RA)

ANSWERS

2a. $5\sqrt{2}$ b. $\sqrt{6}/2$ c. $2\sqrt{3}$ d. 4/5 e. $-1+\sqrt{2}$

3a. vertex is (-2,48); zeros are -6 and 2; y-int is (0,36)

b. vertex is (-3,-8); zeros are -1 and -5; y-int is (0,10)

4a. $x^2 + x - 6 = 0$ so (-3,6) and (2,11) b. $(x+3)(x+1) = \frac{1}{2}x + 3$ so (0,3)and (-3.5, 1.25)

5a. $h(2) = 15$ meters b. $h(0) = 15$ c. vertex is $(1,20)$ so 20 meters after 1 second

d. $h(t) = 0 \quad when\ t = 3\ or\ -1$ (-1 makes no sense; before launch, so t=3 seconds)

6. $x(x+5) + 18 = 2x(x+1)$ so x=6 (x=-3 makes no sense because can't have negative dimensions).

7a. x<-5 or x>5 b. -1≤x≤7 c. x<0 or x>12 d. x<-2 or x>0

8b. $f(x) = 2(x-2)^2 - 4$ c. $2 \pm \sqrt{2}$ d. $2 \pm 2\sqrt{2}$ e. -1<x<5

9b. $f(x) = -(x+3)(x-5)$ c. (1, 16) d. 3, -1

10a. c=5 b. $a + b + 5 = 4 \quad and \quad 4a - 2b + 5 = 17$ c.a=2 and b=-3 so $f(x) = 2x^2 - 3x + 5$

11. $f(x) = -5x^2 + 10x + 5$

12a. 832 feet b. 432 feet c. after 4 seconds d. after 7.21 seconds e. 0≤ t ≤3 seconds

13. $(x+4)(x+6) = 48\ so\, x = 2$

14a. y=0 when x=0 (start) or x=25 so 25 feet b. vertex after 12.5 feet horiz her height was 3.125 feet high c. after 18 feet horiz, her height was 2.52 feet, so she barely cleared it

15. If the origin is the road midway beneath the bridge, the bridge's equation is $y = -\frac{x^2}{125} + 20$ and the height 20 feet from the center is 16.8 feet, so it can make it through.

16. 175 inches 17a. 0, 7 b. 5, -3 c. $\pm\sqrt{2}, \pm\sqrt{1.5}$ 18. 11.5, -17.5, -3, 7 19. 1/8 20. about 22.5%

Unit 4 Handout #4: Completing the Square, Complex Numbers, & Quadratic Formula

1. Solve the following by completing the square. Simplify all radicals fully.

a. $x^2 + 2x = 17$

b. $x^2 - 8x + 1 = 9$

c. $x^2 - 5x + 3 = x - 1$

d. $x^2 + 10x + 3 = 8$

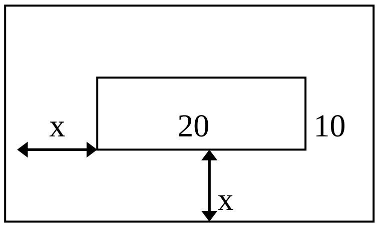
e. $x^2 + x = 5$

f. $2x^2 + 12x = 4x + 2$

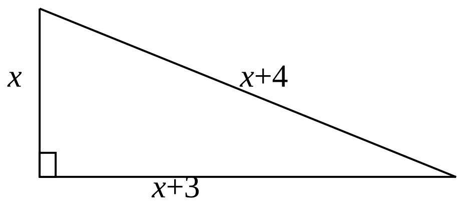
g. $-5x^2 + 30x + 10 = 40$

h. $-5t^2 + 20t + 5 = 10$

2. Amy has a painting: its dimensions are 20 inches wide by 10 inches high. She puts a border of uniform width around it. It turns out that the area of the picture is just one third of the total area of the picture and frame. What is the area of the total picture and frame? Then find the width of the border. Hint: write area of large rectangle in terms of *x*.



3. Find all possible values of *x* in the diagram below.



4. The height of a cannonball (in meters) *t* seconds after launch is given by $h(t) = -5t^2 + 40t + 45$.

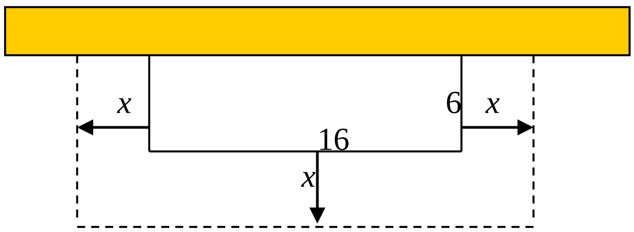
a. What is $h(3)$ and what does it mean?

b. What is its maximum height and when does it reach it?

c. How long after launch does it land?

d. When is the cannon ball's height at least 65 meters? For how many seconds does this occur? (Convert answer to decimal)

5. A rectangular garden is 16 meters by 6 meters, with one of the longer sides against a stone wall. Josie wants to double the area of the garden by increasing each of the three sides (not next to the wall) the same amount. How much must she increase them?



6. Rewrite the following in vertex form by completing the square:

a. $f(x) = x^2 + 6x + 2$ b. $f(x) = -x^2 + 3x + 11$ c. $f(x) = 2x^2 + 12x + 3$

7. Why $x = -\dfrac{b}{2a}$ (part 2)? Think of writing $f(x) = ax^2 + bx + c$ in vertex form (but you don't actually need to do the whole thing!). What is the value of h in the equation $f(x) = a(x-h)^2 + k$?

8. Simplify the following radicals:

a. $\sqrt{-16}$ b. $\sqrt{-24}$ c. $\sqrt{-4} + \sqrt{-9}$

d. $\dfrac{\sqrt{-32}}{2}$ e. $\dfrac{\sqrt{-27}}{6}$ f. $\dfrac{4 \pm \sqrt{-4}}{2}$ g. $\dfrac{6 \pm \sqrt{-18}}{3}$

9. For each part below, write in the form $a+bi$.

$a.$ $(3+4i)-(2-6i)$

$e.$ $(2-i)(3+2i)$

$b.$ $4(2+5i)$

$f.$ $(2+2i)(4-i)$

$c.$ $(3-2i)+2(5+i)$

$g.$ $(3+2i)^2$

$d.$ $(1+6i)-3(-2-4i)$

$h.$ $(5+2i)(5-2i)$

10. Write in the form $a+bi$ by multiplying numerator and denominator by the denominator's conjugate.

a. $\dfrac{2}{2+3i}$

b. $\dfrac{2-i}{2+i}$

c. $\dfrac{-4+i}{3+4i}$

11. Find all solutions, real or complex, by taking square roots or using completing the square.

a. $(x-3)^2+5=-11$

b. $\dfrac{-1}{3}(x+4)^2+5=13$

c. $x^2+6x+2=5$

d. $x^2-4x+12=3$

12. Solve with the quadratic formula. You should simplify all answers; this means reduce fractions and simplify radicals, when possible. Remember, it is often easiest to remove fractions, divide out common terms, and have a positive coefficient on the x^2 term before using the quadratic formula.

a. $x^2 + 2x - 7 = 0$

b. $x^2 - 3x = 2x + 5$

c. $2x^2 - 3x - 11 = 0$

d. $(x+2)^2 = 5x + 8$

e. $10x^2 + 20x - 60 = 0$

f. $\frac{x^2}{3} - 2x = \frac{x}{2} + 1$

g. $-x^2 - 4x + 9 = x^2 + 1$

h. $-2x^2 + 10x = 2$

i. $-5x^2 + x + 1 = 0$

j. $\frac{1}{2}x^2 - 2x = 1$

13. Why $x = -\frac{b}{2a}$ (part 3)? Use the quadratic formula to find the zeros of the function $f(x) = ax^2 + bx + c$. Now find their average. What relevance does this have in finding the equation of the axis of symmetry?

14. A fire hose shoots water in a parabolic arc. Let *x* be the horizontal distance from the hose's nozzle and *y* be the corresponding height of the stream of water (both measured in feet). The equation describing this function is $y = -0.016x^2 + 0.5x + 4.5.$ (Calculator encouraged—answer as decimals!)

a. What is the significance of the 4.5?

b. What is the water's greatest height?

c. How far does the water reach (horizontally) from the end of the hose?

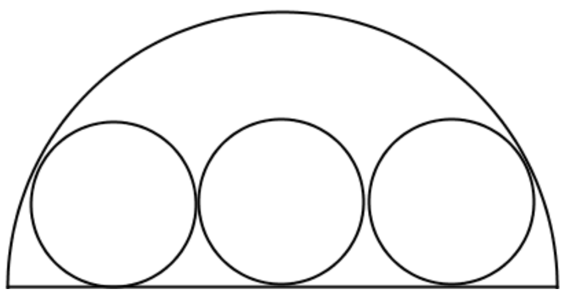
d. Will the stream go over a 6-foot high fence that is located 28 feet from the nozzle? Explain.

15. Use completing the square to find the solutions to the following equations:

a. $x^2 + vx = w$

b. $x^2 - x\sqrt{24} + 2 = 0$

16. Three congruent circles are placed inside a semi-circle with radius 2. They are all tangent to the diameter of the semi-circle, tangent to the adjacent circles, and the end ones are tangent to the semi-circle. What is the radius of these smaller circles?



17. For what values of *x* is the expression below equal to 1?

$$\left(x^2 + 11x + 29\right)^{x^2 - 2x - 8}$$

18. Three forms of the parabola are $y = a(x-h)^2 + k$, $y = a(x-p)(x-q)$, *and* $y = ax^2 + bx + c$.

a. Express c in terms of a, p, and q. (ie, for any given parabola written in all three forms, how does c relate to a, p, and q?)

b. Express p and q in terms of a, b, and c.

c. Express the sum $p+q$ in terms of a and b.

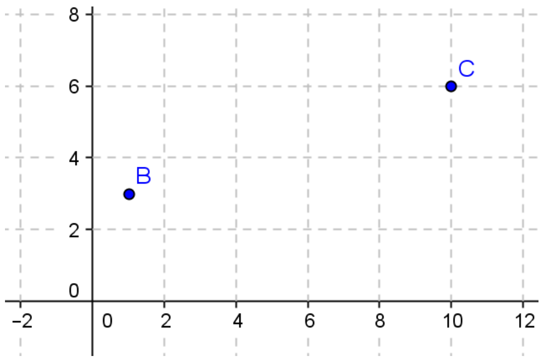
d. Express h in terms of p and q.

e. Express h in terms of a and b.

f. Express pq in terms of a and c.

g. Express k in terms of a, b, and c.

19. Triangle ABC is a right triangle with hypotenuse BC. Point B is at (1,3) and point C is at (10,6). Is it possible for point A to be on the *x*-axis? If so, where? If not, why not?



ANSWERS

1a. $-1\pm 3\sqrt{2}$ b. $4\pm 2\sqrt{6}$ c. $3\pm\sqrt{5}$ d. $-5\pm\sqrt{30}$ e. $-\frac{1}{2}\pm\frac{\sqrt{21}}{2}$ f. $-2\pm\sqrt{5}$ g. $3\pm\sqrt{3}$ h. $2\pm\sqrt{3}$

2. 600 square inches: $(20+2x)(10+2x)=600$ *so* $x^2+15x-100=0$ *and* $x=5$ $(-20\,can't\,be)$

3. $x^2+(x+3)^2=(x+4)^2$ *so* $x^2-2x=7$ *and* $(x-1)^2=8$ so $x=1+2\sqrt{2}$

4a. $h(3)=120$; 3 secs after launch the ball’s hgt is 120 m; b. vertex: 4 secs after launch hgt is 125 m

c. $-5t^2+40t+45=0$ *or* $t^2-8t-9=0$; factor to get 9 seconds (-1 is not in the reasonable domain)

d. $-5t^2+40t+45\ge 65$ *so can be* $t^2-8t=-4$ *complete square to get* $t=4\pm 2\sqrt{3}$ so

$0.54\le t\le 7.46$ or 6.92 seconds 5. $(2x+16)(x+6)=192$ *so* $x=-7+\sqrt{97}$ *or* $2.85m$

6a. $f(x)=(x+3)^2-7$ b. $f(x)=-(x-1.5)^2+13.25$ c. $f(x)=2(x+3)^2-15$

8a. $4i$ b. $2i\sqrt{6}$ c. $5i$ d. $2i\sqrt{2}$ e. $\frac{i\sqrt{3}}{2}$ f. $2\pm i$ g. $2\pm i\sqrt{2}$

9 *a*. $1+10i$ *b*. $8+20i$ *c*. 13 *d*. $7+18i$ *e*. $8+i$ *f*. $10+6i$ *g*. $5+12i$ *h*. 29

10a. $\frac{4}{13}-\frac{6}{13}i$ b. $\frac{3}{5}-\frac{4}{5}i$ c. $-\frac{8}{25}+\frac{19}{25}i$ 11a. $3\pm 4i$ b. $-4\pm 2i\sqrt{6}$ c. $-3\pm 2\sqrt{3}$ d. $2\pm i\sqrt{5}$

12a. $1\pm 2\sqrt{2}$ b. $\frac{5\pm 3\sqrt{5}}{2}$ c. $\frac{3\pm\sqrt{97}}{4}$ d. $\frac{1\pm\sqrt{17}}{2}$ e. $-1\pm\sqrt{7}$ f. $\frac{15\pm\sqrt{273}}{4}$

g. $-1\pm\sqrt{5}$ h. $\frac{5\pm\sqrt{21}}{2}$ i. $\frac{-1\pm\sqrt{21}}{-10}$ *or* $\frac{1\pm\sqrt{21}}{10}$ j. $2\pm\sqrt{6}$

13. The axis of symmetry is midway between and 2 x’s with the same y

14a. nozzle is 4.5 feet above ground level b. y=8.40625 when x=15.625 c. y=0 when x=38.546 feet

d. f(28)=5.956 so not quite 15a. $x=\frac{-v}{2}\pm\sqrt{w+\frac{v^2}{4}}$ b. $\sqrt{6}\pm 2$ 16. $\frac{-1+\sqrt{5}}{2}$

17. ans -4, -7, -2, 4, and -6 – wow.. lots of them! 19. at x=4 or 7..

Unit 4 Handout #5: Practice and Gravity Problems

1. Find all real and imaginary solutions to each equation below. Be sure to simplify your answers. Think carefully about what might be the best way to solve each. My suggestions:

#1. square roots if possible (only one *x* in the equation)

#2. try to factor

#3. quadratic formula or complete-the-square (only try complete-the-square if "b" term is even).

a. $x^2 + 5 = -31$

b. $x^2 + x = -3$

c. $2(x+4)^2 + 2 = -78$

d. $(x+1)^2 = 3x - 6$

e. $x^2 + 4x + 11 = 5$

f. $x^2 - 3x + 1 = 2x + 25$

g. $2x^2 + 6x + 3 = -x^2 - 5x + 3$

h. $-2(x+1)^2 + 9 = 51$

2. The height above the water level of a diver (in meters) t seconds after she jumps is given by the equation $h(t) = -5t^2 + 5t + 3$. How long after she jumps does she hit the water (decimal answer)?

Physics lesson #1: When an object is dropped (not thrown upwards or downwards, but simply dropped) we call this "free fall". The equation relating height (in meters) to time since drop is approximately $h(t) = -5t^2 + h_0$, where h_0 is the initial height.

3. An object is dropped from a tower 250 meters high.

   a. How high is it after 4 seconds?

   b. When does it land?

   c. How long does it take to get half-way down? In other words, when was its height 125 meters?

4. An object is dropped from a tower of unknown height (call this h_0). It lands after 7 seconds.

   a. Using $h(t) = -5t^2 + h_0$, find h_0.

   b. Instead, if it landed 8 seconds after it was dropped, then how high was the tower?

   c. Instead, if 5 seconds after it dropped it was 25 meters high, then how high was it dropped from?

Physics lesson #2: When we are relating the height of an object (in meters) to the time since it was launched/dropped, the function is always a quadratic function. The coefficient of the t^2 term will always be -5. (It is actually -4.9, but in the interest of simplicity, we will round it). This is due to the acceleration caused by the earth's gravity. On other planets the coefficient will be different. But for us, we will use an equation that looks like $h(t) = -5t^2 + bt + c$. Here, b&c depend on the initial height and whether the object was dropped or launched.

5. A ball's initial height is 10 meters. After 3 seconds the ball is 25 meters high.

a. Write the equation for the ball's height as a function of time. [$h(t) = -5t^2 + bt + c$]

b. When does the ball land?

c. What was the ball's maximum height, and when was it attained?

6. A different ball's initial height was 20 meters. It landed after 5 seconds.

a. Write the equation for the ball's height as a function of time. [$h(t) = -5t^2 + bt + c$]

b. What was the highest the ball got and when did it get there?

7. A ball was launched from an unknown height. After 1 second its height was 18 meters and after 3 seconds its height was 8 meters. Given the two points, use the equation $h(t) = -5t^2 + bt + c$ to write two equations involving b & c. Then solve that system to find the ball's height equation.

8. A ball was launched from unknown height and reached its maximum height of 200 meters 3 seconds after launch. Write its height equation and determine when it lands. Think about the best form to use!

Answers

1a. $\pm 6i$ (sq roots) b. $\frac{-1 \pm i\sqrt{11}}{2}$ (quad form.) c. $-4 \pm 2i\sqrt{10}$ (sq roots) d. $\frac{1 \pm 3i\sqrt{3}}{2}$ (quad form.)
e. $-2 \pm i\sqrt{2}$ (compl sq) f. 8, -3 (factor) g. 0, -11/3 (factor) h. $-1 \pm i\sqrt{21}$ (sq roots)

2. 1.422 seconds 3a. 170 meters b. after $\sqrt{50} = 5\sqrt{2}$ seconds (about 7.07 seconds) c. 5 seconds
4a. 245 meters b. 320 meters c. 150 meters
5a. $h(t) = -5t^2 + 20t + 10$ b. $h(t) = 0$ *when* $t = 2 + \sqrt{6}$ or 4.45 seconds c. 30m after 2 seconds
6a. $h(t) = -5t^2 + 21t + 20$ b. after 2.1 seconds it got 42.05 meters high
7. $b + c = 23$ *and* $3b + c = 53$ so $h(t) = -5t^2 + 15t + 8$

8. $h(t) = -5t^2 + 30t + 155$ or $h(t) = -5(t-3)^2 + 200$. It lands after $3 + 2\sqrt{10}$ or 9.32 seconds.

Unit 4 Handout #6: More Challenging Quadratics Problems

1. Depending on the value *k* takes, the equation $x^2 - 6x + 7 = k$ may have two real solutions, one real solution, or two imaginary/complex solutions. There is only one value of *k* for which the equation will have one real solution. What is that value of *k*?

2. Depending on the value *k* takes, the parabola $y = x^2 + 3x + 1$ intersects the line $y = -x + k$ zero times, one time, or two times. For what one value of *k* do the graphs intersect just one time?

3. We know that $i = \sqrt{-1}$. But what is the square root of *i*? It is a complex number and can be written in the form $a + bi$. Here's a cool way to find it. Assume some complex number (a+b*i*) is the square root of *i*. That means that $(a+bi)^2 = i = 0 + 1i$. Now expand the left side, set the real parts equal and the imaginary parts equal, and you should have the two square roots of *i*! Check your answer by squaring them and see what you get.

4. Use a similar technique as in problem #3 above to find the two square roots of $-3+4i$

5. Solve for *x* in terms of *p*: $x^2 - 2px - 8p^2 = 0$

6. Can a rectangle have a perimeter of 100 feet and an area of 100 square feet? Explain.

7. Can a rectangle have a perimeter of 100 feet and an area of 700 square feet? Explain.

8. Solve the following quadratic equation by completing the square: $x^2 - 6x + 1 = 10$.

9. Given that $f(x)$ is a quadratic function and the solutions to $f(x) = 0$ are $2 \pm \sqrt{5}$, give a possible function for $f(x)$. Hint: think about doing question #8 in reverse!

10. Is there more than one possible function $f(x)$ in question #9 above that has the given zeros? How many?

11. The solutions to a quadratic equation are x=0.75 and x=-0.5. Write any possible such equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.

12. If $x = 2$ is a solution to $kx^2 - 7x + 2 = 0$ then what is k and what is the other solution?

13. Find the equation of the quadratic function that goes through the points (2,10), (-5, -32), and (0,8).

14. A tunnel is shaped like a parabola that opens down. It is 20 feet wide and has a maximum height of 15 feet. There is a one-lane road in it, as well as a sidewalk.

a. If the bottom left point of the tunnel is at the origin, then what is the equation of the tunnel?

b. The pedestrian walkway needs to be able to accommodate someone 6 feet tall. How far from the left edge of the tunnel can it be?

c. Can a truck that is 10 feet wide and 12 feet tall (across its entire width) fit through the tunnel? Assume it drives in the center of the tunnel.

d. What is the maximum width that a truck 10 feet high can be and still fit through the tunnel?

15. You have 60 feet of string and decide to make a rectangle out of it, using all 60 feet as the perimeter of the rectangle.

a. If one side of the rectangle is 5 feet, then what are the rectangle's dimensions? What is its area?

b. If one side of the rectangle is x feet, then what are the rectangle's dimensions (in terms of x)? What is its area (in terms of x)?

c. Use your answer to part *b* above to determine what x should be to make the area of the rectangle as large as possible. What is the largest possible area?

d. Would you be able to make the area larger if you made a circle instead of a rectangle? If so, by how much?

16. A farmer has a long fence along one side of his property. He wants to build a rectangular enclosure for horses using this fence as one side. He has 100 meters of fencing for the other three sides.

a. Let x be the width of the enclosure. What is the length in terms of x? (Assume the existing fence is one of the lengths). What is the area in terms of x?

b. What dimensions maximize the area of the enclosure and what is the maximum area possible?

17. The revenues of a company are defined as the sales price times the number of items sold. Sally Ann's bake shop sells chocolate chip cookies. If they sell the cookies for 50 cents each, then they sell 600 cookies per week. For every penny they increase the price, they sell 5 fewer cookies per week. For example, at 51 cents, they sell 595 cookies per week and their revenues are $303.45.

a. Complete the following table:

Price	**Sales**	**Revenues per week**
52 cents		
60 cents		
100 cents		

b. Define *x* as the number of cents over 50 cents they charge. Now add a row to the bottom of the table where the price is 50+*x* cents. Fill in the sales and weekly revenues in terms of *x*.

c. At what price or prices will their revenues be zero? What are price and sales at these points?

d. What are the highest possible revenues they can get from the chocolate-chip cookies and what price do they need to charge to get it?

18. The solutions to the equation $x^2 - 6x + 4 = x - 10$ are $\frac{7 \pm i\sqrt{7}}{2}$. What does that tell us about the graphs of $f(x) = x^2 - 6x + 4$ *and* $g(x) = x - 10$? Graph on your calculator if you like.

19. Graph the line $2y+5x=40$ in the first quadrant only. We are going to try to fit rectangles under this line, so that two sides of the rectangle will lie on the coordinate axes and one corner of the rectangle will lie on the graph of this line.

a. If the base of the rectangle is 2 units wide (so one side is on the line x=2) then what is the rectangle's area?

b. If the base of the rectangle is 7, what is its area?

c. Let the base of the rectangle be w units wide—write the area of the rectangle in terms of w. What is the maximum possible area? What x value maximizes the area?

20. The functions $f(x)$ *and* $g(x)$ are both quadratic functions. The zeros of $f(x)$ are 1 and -3 and its y-intercept is -6. The vertex of $g(x)$ is (5,28) and it also goes through the point (2,19). Write the equations of both functions and then algebraically determine exactly where their graphs intersect.

21. What is the area of the largest possible rectangle that will fit in the first quadrant under the graph of $y = 8 - 3x$? Two sides of the rectangle will lie on the coordinate axes and one corner of the rectangle will lie on the graph of this line

22. Point A's coordinates are (6,0). Point B is somewhere on the line $y = 0.5x + 3$.

a. Let the *x*-coordinate of point B be *w*. What is the distance from A to B in terms of *w*?

b. What value of *w* minimizes the distance from A to B? Hint: since distance is always positive, you can minimize the distance by minimizing the distance squared. In other words, the *x*-value corresponding to the minimum value of $\sqrt{f(x)}$ is the same as the *x*-value corresponding to the minimum value of $f(x)$.

c. Show that the distance from A to the line is minimized when $\overline{AB}$ is perpendicular to the line $y = 0.5x + 3$.

23. Is there any point on the line $y = 2x - 2$ that is exactly 3 units away from the point (-1,3)? If so, what is/are its *x*-coordinates?

24. A quadratic function whose zeros are $1 \pm \sqrt{3}$ is $f(x) = x^2 - 2x - 2$ (there are others, such as $f(x) = 5x^2 - 10x - 10$, but the given one is the ***monic*** quadratic—meaning the coefficient of the quadratic term is 1). What is the monic quadratic function whose zeros are $\sqrt{3} \pm 1$?

25. Consider the function $f(x) = -x^2 + 6x$. If P and Q are the points of intersection of the graph of *f* with the *x*-axis and R is a point on the portion of the graph above the *x*-axis, what is the maximum area of triangle PQR?

26. Sketch the graph of $y = -0.5|x-4|+7$. A rectangle is tucked under the graph in the first quadrant. The base of the rectangle is on the x-axis and the upper corners are on the graph of the absolute value function.

a. If one corner is (2,6), what is the area of the rectangle?

b. If the area of the rectangle is 14, then what are the coordinates of its lower left-hand corner?

27. Sometimes teachers are willing to weight the final exam differently for different students in the semester grade depending on whether it helps the student. For example, a teacher may decide the final exam counts as either 20% of the course grade or 30%, letting it be different for different students based on which gives each the highest course grade.

But a mean teacher once said that the final exam would count ***more*** the ***worse*** it was. The weight of the final exam in the course grade was $w(x) = 100 - x$, where x was the final exam grade. So a person getting 98 on the final had the final count as 2% of the course grade (with the other 98% being the test/quiz/homework average). But a person getting a 60 on the final would have it count as 40% of the course grade. So mean! Going into the final, Sally has an average of 84.

a. If Sally gets a 90 on the final exam, then what is her course grade?

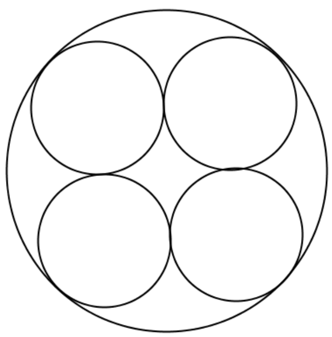
b. If Sally gets a 98 on the final exam, then what is her course grade?

c. What grade on the final exam would give Sally the best overall grade in the course?

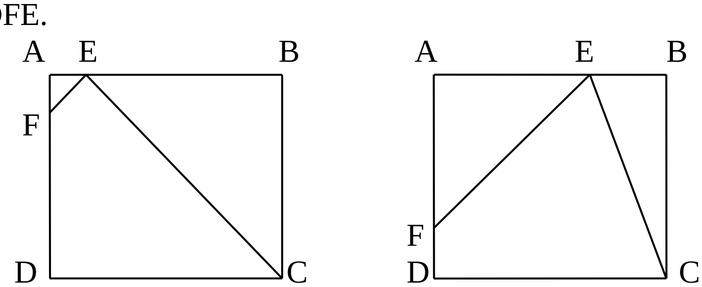
d. If Andy's grade, going into the final exam, is a. What final exam grade gives him the highest course grade?

28. The lower two vertices of a square lie on the x-axis and the upper two vertices of the square lie on the parabola $y = 15 - x^2$. What is the area of the square?

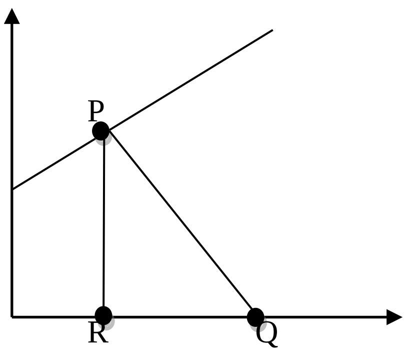
29. In the diagram below, a circle with diameter 6 has four congruent circles inside it, each tangent to each other and to the larger circle. What is the radius of the smaller circles?



30. ABCD is a square with side length one. Points E and F are on sides AB and AD such that AE=AF. What is maximum possible area of CDFE? The diagrams below show a few possible quadrilaterals CDFE.



31. Point P is on the line $y = x + 5$. Point Q's coordinates are fixed at (7,0). Point R is on the *x*-axis directly below point P. As P slides between (0,5) and (7,12), what is the maximum possible area of triangle PRQ?



Answers

1. $(x-3)^2 = 0$ has one solution so *k*=-2 2. *k*=-3 3. $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, \ -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

4. $1+2i \ and \ -1-2i$ 5. 4*p*, -2*p*

6. $xy = 100 \ and \ 2x + 2y = 100 \ \ yields \ y^2 - 50y + 100 = 0 \ so \ y = 25 \pm \sqrt{525}$; yes!

7. $xy = 700 \ and \ 2x + 2y = 100 \ \ yields \ y^2 - 50y + 700 = 0$ and this has no real solutions

8. $3 \pm 3\sqrt{2}$ 9. $(x-2)^2 = 5 \ so \ x^2 - 4x - 1 = 0 \ so \ f(x) = x^2 - 4x - 1$

10. an infinite number, because it can be $f(x) = a(x^2 - 4x - 1)$ -- and there are an infinite number of parabolas that have any two zeros—they can open up or down and be any steepness

11. $(2x+1)(4x-3) = 0 \ so \ 8x^2 - 2x - 3 = 0$ 12. k=3 so x=1/3 also

13. c=8 so $4a + 2b + 8 = 10$ and $25a - 5b + 8 = -32$ solving the system yields $y = -x^2 + 3x + 8$

14a. $f(x) = -0.15x(x-20)$ *or* $f(x) = -0.15(x-10)^2 + 15$ b. $-0.15(x-10)^2 + 15 = 6 \ so \ 10 - 2\sqrt{15}$
 c. no; truck needs to go thru (5,12) and (15,12) and plugging 5 in for x gives height of 11.25 feet
 d. tunnel is 10 feet high when $-0.15(x-10)^2 + 15 = 10 \ so \ 10 \pm 10/\sqrt{3}$ so can be $20/\sqrt{3}$ feet wide

15a. 5 by 25; area is 125 b. x by (30-x); area is $30x - x^2$ c. vertex of parabola is highest area, so x=15 by 15 and area is 225 sq ft d. circle is better: with circumference of 60 radius is 30/pi and area is about 286.5 16a. 100-2x ; $100x - 2x^2$ b. x=25 so 25 by 50 and area is 1250 sq meters

17a. 1st row is 590 cookies and revenue is \$306.8
 2nd row is 550 cookies for revenue of \$330; last row is 350 cookies for revenue of \$350
 b. 50+x cents means 600-5x cookies and revenue is $(50 + x)(600 - 5x)$
 c. zeros are x= -50 (giving cookies away) and x=120 (charging \$1.70 and selling none)
 d. vertex is midway between zeros so x=35 and they charge 85 cents, sell 425 for revenue of \$361.25

18. Since the only solutions have imaginary components, the graphs never intersect.

19a. 2 by 15 so 30 b. 7 by 2.5 so 17.5 c. $Area = w(20 - 2.5w)$ maximum when x=4 and area=40

20. $f(x) = 2(x+3)(x-1) \ and \ g(x) = -(x-5)^2 + 28$; they meet at (-1,-8) and (3,24).

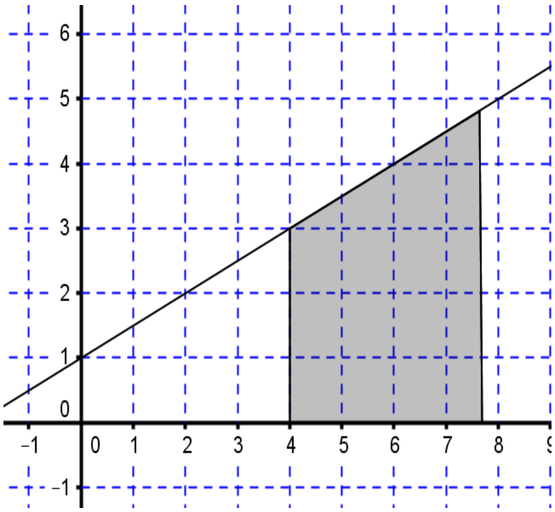
21. area is 16/3 22a. $\sqrt{(w-6)^2 + (0.5w+3)^2}$ b. 18/5 c. slope from A to (18/5, 24/5) is -2 so perp to line 23. no: the solution to the quadratic 5x^2-18x+17 has imag parts

24. $f(x) = x^2 - 2\sqrt{3} \cdot x + 2$ 25. 27 26a. 24 b. $-3 + \sqrt{35}$ 27a. 84.6 b. 84.28 c. 92 d. 50+0.5a

28. 36 29. $3\sqrt{2} - 3$ 30. 5/8 31. Area=18 when x=1

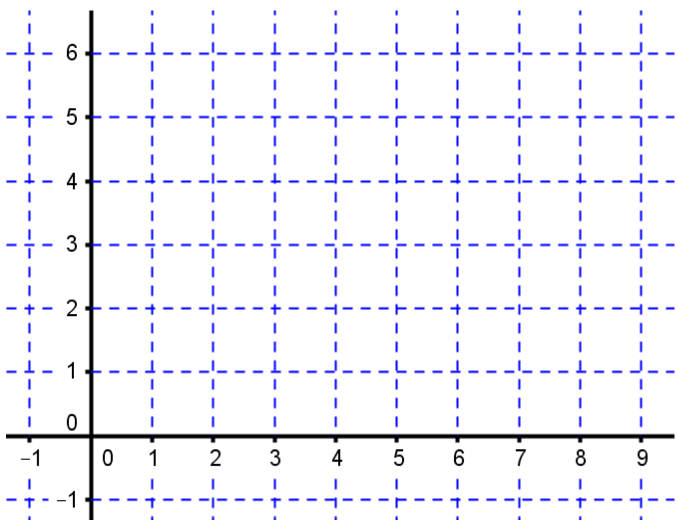
Accelerated Algebra 2: Area Questions in Quadratics

1. The area of the trapezoid is 14. Its left edge is the line x=4. Its right edge is the line x=a. What is the value of a?



2. Sketch a graph of $f(x) = 0.5|x-2|+2$.

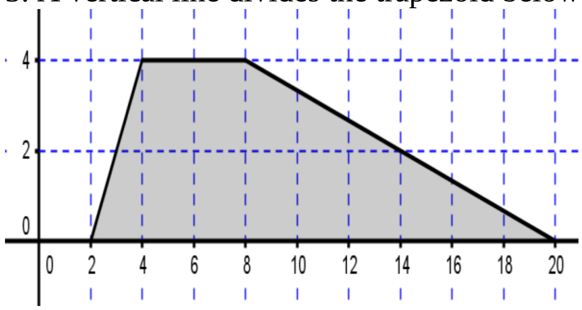
a. Find the area of the region bound by the graphs of $f(x)$, the x-axis, the y-axis, and the line $x = 8$.



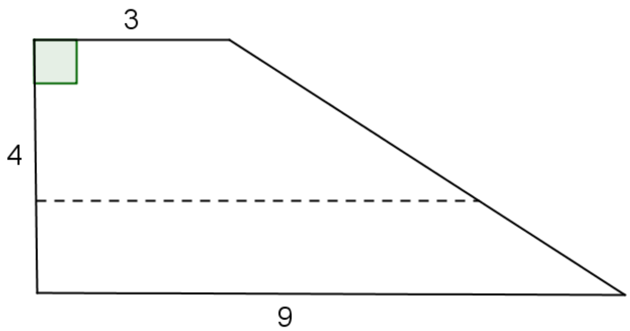
b. A vertical line divides this region into two regions of equal area. Find its equation.

c. A line through the origin divides this region into two regions of equal area. Find its slope.

3. A vertical line divides the trapezoid below into two regions of equal area. Find its equation.



4. The trapezoid below has bases of 3 and 9 and a height of 4. The dashed line below is parallel to the bases and divides the trapezoid into two regions of equal area. How long is it?

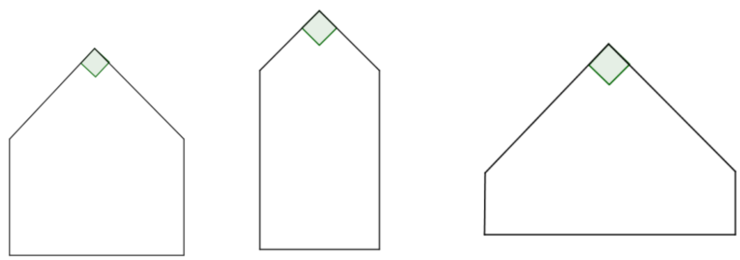


5. The shapes below all contain isosceles right triangles where the hypotenuse is one side of a rectangle. Imagine a shape like this has a perimeter of 100 (the sum of the lengths of the five sides):

a. If the legs of the triangle are 10, then find the dimensions of the shape and its area.

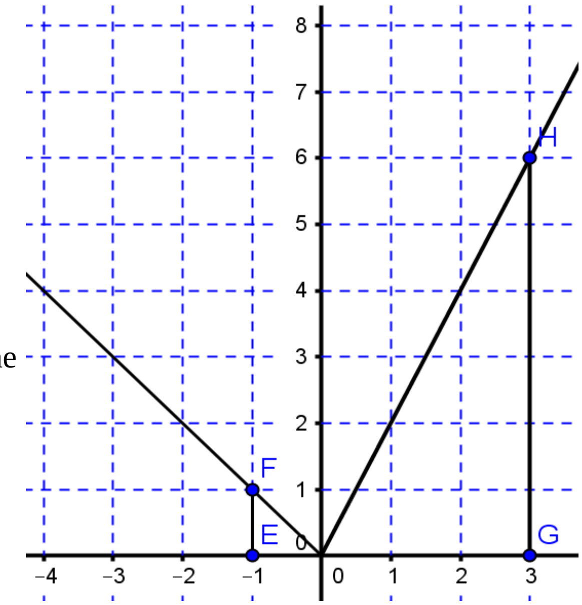
b. If the legs of the triangle are x, the find its dimensions and area.

c. Out of all such shapes with a perimeter of 100, what is the largest area possible?



6. In the diagram below, two line segments meet at the origin. Points E and G are on the *x*-axis at -1 and 3 and points F and H are always directly above E and G.

a. What is the sum of the areas of the triangles whose vertices are E, F, and the origin and G, H, and the origin?

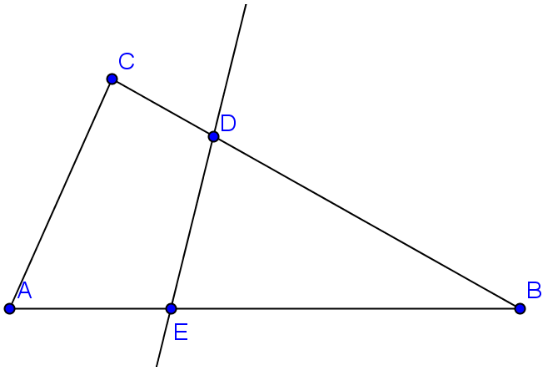


b. Now place E and G in different places on the *x*-axis such that the length of segment EG is still 4 units but the combined area of the triangles is 11 square units. What is the *x*-coordinate of E?

c. Now place E and G in different places on the *x*-axis such that the length of segment EG is still 4 units. Your goal is to minimize the combined areas of the triangles. What is the *x*-coordinate of E? Note: F and H are always directly above E and G.

d. In part *c*, find the *y*-coordinates of F and H when the combined area is minimized. Is it a coincidence that they are the same? Can you explain geometrically why this must be the case?

7. In the diagram below, AC has slope 3, BC has slope -½, and DE has slope 4. AB is 14 units long and the altitude from C to side AB is 6 units long. If DE cuts ΔABC into two pieces of equal area, then find the length of AE.



Answers

1. $-2+2\sqrt{23}$ 2a. 26 b. $-2+4\sqrt{3}$ c. 13/32 3. $x=20-2\sqrt{33}$ 4. $3\sqrt{5}$

5a. $-50+400\sqrt{2}$ b. $x^2\left(-0.5-\sqrt{2}\right)+x\left(50\sqrt{2}\right)$ c. about 653 when $x=\dfrac{50\sqrt{2}}{1+2\sqrt{2}}$

6a. 9.5 b. $\dfrac{-8+\sqrt{34}}{3}$ c. -8/3 d. magic! We'll talk about it in class...

7. $14-1.5\sqrt{42}$ Hint: you may want to define the altitude from D of the smaller angle as *k* and write the area of triangle BED in terms of *k*.

Unit 4: Fundamentals Review Problems

1. For each of these parabolas, find the vertex, x-intercepts (if any), and y-intercept.

a. $f(x) = -2x^2 - 4x + 4$ b. $g(x) = -(x+3)(x-5)$ c. $h(x) = -\frac{1}{2}(x+2)^2 + 8$

2. Sketch a graph of the parabolas above. Label at least three points on each one.

3. Factor the following expressions completely:

$a.\ x^2 - x - 42$

$b.\ 16x^2 - 9$

$c.\ 6t^2 - 36t$

$d.\ 2x^2 - x - 6$

$e.\ -3x^3 + 12x$

$f.\ -x^2 - 3x - 2$

$g.\ -2x^3 + 6x^2 + 20x$

$h.\ 3x^2 - 2x - 5$

4. Find the value of c that makes the trinomial a perfect square; write the trinomial as a perfect square: (ie, completing the square)

$a.\ x^2 + 4x + c$

$b.\ x^2 - \frac{2}{7}x + c$

$c.\ x^2 + 3x + c$

$d.\ x^2 - 6x + c$

5. Write the equation of each parabola described.
a. Vertex is (5,-2) and goes through (3,4)
b. The x-intercepts are 4 and –6 and goes through (0,5)
c. The y-intercept is 5 and the highest point is (-2,8).
d. The only zero is -4 and the y-intercept is 8.

6. Write the following expressions in $a + bi$ form:

a. $(25 + 15i) - 2(3 - 2i)$

b. $(2 - 9i)(5 - 4i)$

c. $2(3 + 2i) + 4(-2 - i)$

d. $(4 - 3i)^2$

e. $\frac{1+i}{2-4i}$

7. Simplify the following expressions completely:

a. $\sqrt{75}$ b. $\frac{3 \pm \sqrt{27}}{3}$ c. $\sqrt{-40}$ d. $\frac{3\sqrt{-8}}{2}$ e. $\frac{-2 \pm \sqrt{24}}{2}$

8. Solve by completing the square:

a. $x^2 - 6x + 1 = 14$ b. $x^2 + 3x = x - 13$

9. Write in standard form:

a. $f(x) = -\frac{1}{2}(x+1)^2 - 3$ b. $f(x) = -2(x-3)(x+1)$

10. Solve the following equations in the most efficient way. Think about it before you jump in and solve it! If only one x-term then "isolate & undo." If no constant term, factor!

$a.\ x^2 + 7x = 30$

$b.\ x^2 - 90 = 0$

$c.\ x^2 - 12x = -28$

$d.\ -x^2 + 7x - 19 = 0$

$e.\ 2x^2 + 28x = -48$

$f.\ 4x^2 = -29x$

$g.\ 2x^2 + 5 = -11$

$h.\ \frac{1}{2}x^2 + \frac{3}{4}x = -4$

$i.\ 3x^2 + 6x = -3$

$j.\ 7x^2 + 28 = 3x^2 - 12$

$k.\ \frac{1}{4}(x-8)^2 + 2 = 9$

$l.\ 2x^2 + 8x = 24$

$m.\ (x-3)(x+4) = 3(x+2)$

11. Solve the following inequalities:

$a.\ x^2 - 3x - 10 > 0$

$b.\ 2x^2 - 7x + 3 \le 0$

$c.\ 2x^2 + 5 > 0$

$d.\ 2(x-1)^2 - 5 < 13$

12. Answer the questions about the function $f(x) = x^2 - 6x - 8$:

a. What are its zeros?

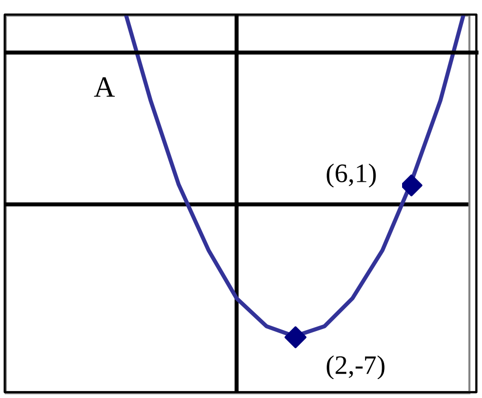
b. Where does it intersect the line y=-8?

13. Cindy’s rectangular room is 6 feet longer than it is wide. If its area is 216 square feet then what are its dimensions?

14. Use the diagram to answer the questions. The vertex of the parabola is (2,-7) and the line is y=8.

a. What is the equation of the parabola?

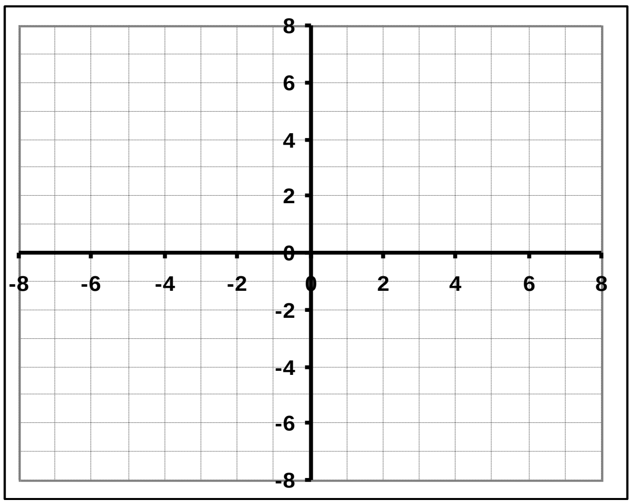
b. What are the exact coordinates of point A in the diagram?



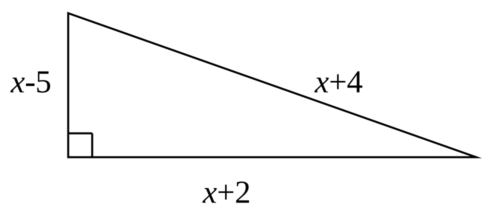
15. A function is defined as $f(x) = 2(x-3)^2 - 12$
   a. What is its vertex?
   b. What are its zeros?
   c. What is its y-intercept?
   d. Solve the equation $f(x) = -14$. What does this tell us about the graph of $f(x)$?

16. Sketch graphs of the two functions below and then algebraically find the exact **coordinates** of all points of intersection.

$y = x^2 - 4x - 8$ $\quad\quad$ $y = x - 2$



17. Solve for *x*:



18. A square gets modified. The length is doubled and the width is reduced by 5. The resulting rectangle's area is 16 smaller than the original square. What was the length of the square's side?

19. A ball is thrown upwards from an initial height of 8 meters. It lands 4 seconds later.
   a. What is the equation for the ball's height as a function of time? [$h(t) = -5t^2 + bt + c$ you need to find b and c from the information given.]
   b. What was the highest the ball got, and when did it get there?
   c. When was the ball exactly 15 meters high?
   d. For how many seconds was the ball's height more than 15 meters? [hint: sketch a rough graph]

20. A football kicker kicks the ball from the origin or a graph. The path of the ball is given by the equation $y = -\frac{1}{90}x^2 + \frac{4}{3}x$, where x and y are both in feet.

a. What is the maximum height of the ball?

b. How far away from where it is kicked does it land?

c. The goal post is located 105 feet from where the ball is kicked. The cross-bar is ten feet high. Does the ball clear the cross-bar?

21. A penny is dropped off a tower 600 feet high. Its height (in feet) t seconds after it is dropped is given by the equation $h(t) = -16t^2 + 600$.

a. When does it land?

b. How high is it 2 seconds after it is dropped?

c. When is it exactly 150 feet high?

d. When is it at least 344 feet high?

22. A picture is 3 inches longer than it is wide. When a 2 inch border is put all around it, the total area (picture and frame) is equal to 70 square inches. What are the dimensions of the picture?

23. Keyes North Atlantic is a company that repairs furnaces and air-conditioners. If the temperature is very high or very low it makes a lot of money. If the temperature is moderate it loses money, as its employees get paid despite having very little work to do. The company's profits (in hundreds of dollars per day) as a function of the temperature (Celsius) are given by the equation $P(c) = \frac{1}{2}(c-10)(c-30)$.

a. Sketch a graph of this function, labeling the vertex and zeros.

b. What are coordinates of the vertex, and what meaning do they have in the problem?

c. What are the profits if it is just freezing out?

d. At what temperatures are profits $62.5 hundred dollars per day?

e. Use symmetry to determine if the profits are higher if the temperature is 6 degrees or 32 degrees.

24. The profits of a farmer (in thousands of dollars) depend on the rainfall over the growing season (in inches). The function p(x) is a quadratic function that describes this relationship. It turns out that the highest profits the farmer may attain are $32 thousand dollars: this happens when there are 10 inches of rain. Also, if there are 3 inches of rain the profits are $7.5 thousand dollars.

a. Based on the fact that $p(3) = 7.5$ and the maximum possible profits of 32 are attained when x=10, write the equation of the function p(x). [Hint: use **vertex** form]

b. Sketch a graph of this. Label the vertex and at least 2 other points.

c. What are the zeros of p(x) and what is the meaning of these points?

d. What are the coordinates of the y-intercept? What is the meaning of this point?

e. If the farmer's profits were 14 thousand dollars then how much did it rain?

f. If the farmer's profits were 5 thousand dollars then how much did it rain?

25. A landlord has 60 apartments to rent. If she charges $1000 per month rent, she can rent them all. For each $100 increase in monthly rent, three additional apartments go unrented. The money she takes in is the product of the monthly rent and the number of apartments rented. What is the most she can take in, and what rent should she charge? Hint: define *x* as the number of $100's above $1000 the rent is.

Answers

1. a. vertex is (-1,6); y-intercept is 4 and x-intercepts are $-1\pm\sqrt{3}$
   b. vertex is (1,16); y-intercept is 15 and x-intercepts are 5 and -3
   c. vertex is (-2,8); y-intercept is 6 and x-intercepts are 2 and –6

3. a. (x-7)(x+6) b. (4x+3)(4x-3) c. 6t(t-6) d. (2x+3)(x-2)
   e. (-3x)(x+2)(x-2) f. –(x+2)(x+1) g. -2x(x-5)(x+2) h. (3x-5)(x+1)

4. a. 4; $(x+2)^2$ b. 1/49; $(x-1/7)^2$ c. 2.25; $(x+1.5)^2$ d. 9; $(x-3)^2$

5. a. $y=1.5(x-5)^2-2$ b. y = (-5/24)(x-4)(x+6) c. $y=(-3/4)(x+2)^2+8$ d. $f(x)=0.5(x+4)^2$

6. a. $19+19i$ b. $-26-53i$ c. -2 d. $7-24i$ e. $-0.1+0.3i$

7a. $5\sqrt{3}$ b. $1\pm\sqrt{3}$ c. $2i\sqrt{10}$ d. $3i\sqrt{2}$ e. $-1\pm\sqrt{6}$

8. a. $3\pm\sqrt{22}$ b. $-1\pm 2i\sqrt{3}$ 9a. $f(x)=-0.5x^2-x-3.5$ b. $f(x)=-2x^2+4x+6$

10. a. –10, 3 (factor) b. $\pm 3\sqrt{10}$ (square roots) c. $6\pm 2\sqrt{2}$ (complete sq)
    d. $\left(7\pm 3i\sqrt{3}\right)/2$ (QF) e. -2, -12 (factor or complete sq) f. 0, -29/4 (factor)
    g. $\pm 2i\sqrt{2}$ (sq root) h. $\left(-3\pm i\sqrt{119}\right)/4$ (QF) i. –1 (factor)
    j. $\pm i\sqrt{10}$ (sq root) k. $8\pm 2\sqrt{7}$ (sq root) l. 2, -6 (factor)
    m. $1\pm\sqrt{19}$ (QF or complete sq)

11. a. x<-2 or x>5 b. $0.5\le x\le 3$ c. all x (no boundary points) d. -2<x<4

12. a. $3\pm\sqrt{17}$ b. x=0, 6

13. (x)(x+6)=216; by completing the square x=12 so 12x18.

14a. $f(x)=0.5(x-2)^2-7$ b. $0.5(x-2)^2-7=8$ *so* $x=2\pm\sqrt{30}$

15a. (3,-12) b. $3\pm\sqrt{6}$ c. (0,6) d. $3\pm i$; it never hits the line y=-14 (or no point has a y of -14)

15. $x-2=x^2-4x-8$ *so* $x^2-5x-6=0$ $x=6$ *or* -1 *so* $(6,4)$ *and* $(-1,-3)$

17. $(x-5)^2+(x+2)^2=(x+4)^2$ *so* $x^2-14x+13=0$ and x=13 (but not 1)

18. $x^2-16=2x(x-5)$ so x=8 or 2; 2 makes no sense so x=8.

19a. $h(t)=-5t^2+18t+8$ b. after 1.8 seconds it was 24.2 meters c. $t=\dfrac{18\pm\sqrt{184}}{10}=3.16, 0.44$
  d. between 0.44 and 3.16 seconds, so for about 2.72 seconds

20. a. y-value of vertex: x=60 and y=40; so 40 feet b. y=0 so x=120
    c. when x=105 y=17.5, so yes it does (since 17.5>10)

21. a. 6.12 seconds b. h(2)=536 feet c. 5.30 seconds d. t≤4 seconds

22. (x+7)(x+4)=70 so $x^2+11x-42=0$ so $(x+14)(x-3)=0$ *and* $x=3$; 3 by 6

23b. vertex: x=-b/(2a) = 20 degrees: P(20)=-50 or $5000 loss.
c. 0 degrees: P(0)=150 or $15,000 profit
d. 62.5 = - ½ (c-10)(c-30) so c=5 or 35
e. vertex is at 20 degrees: 6 is 14 units away and 32 is 12 units away; so profits higher at 6

24. a. vertex=(10,32) and goes through point (3,7.5) so $p(x)=-\frac{1}{2}(x-10)^2+32$
c. p(x)=0 so $-\frac{1}{2}(x-10)^2+32=0$ and x=2 or 18
d. y-int is where x=0 so no rain means $18,000 loss
e. $14=-\frac{1}{2}(x-10)^2+32$ so 4 or 16 inches f. $5=-\frac{1}{2}(x-10)^2+32$ so $x=10\pm 3\sqrt{6}$ inches

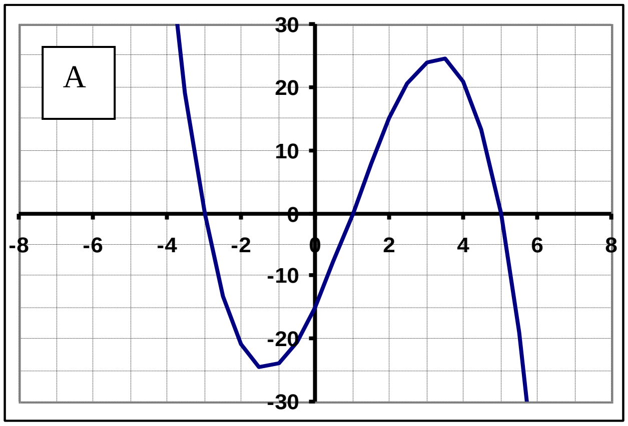
25. rent is 1000+100x and apartments rented is 60-5x. Maximize (60-3x)(1000+100x) and vertex is where x=5 and rent is $1500; she takes in (1500)(45)=$67500 per month

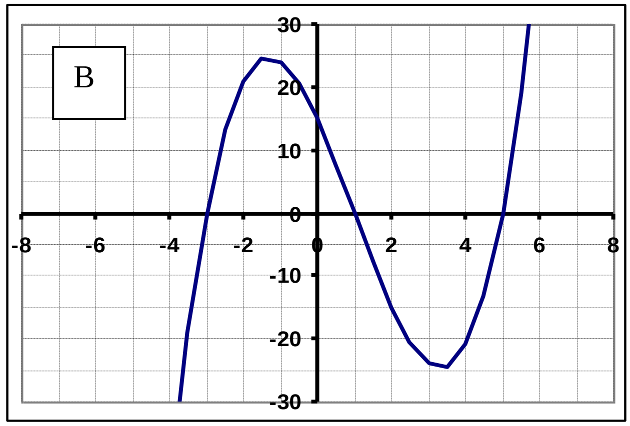
Unit 5 Handout #1: Graphs of Polynomial Functions

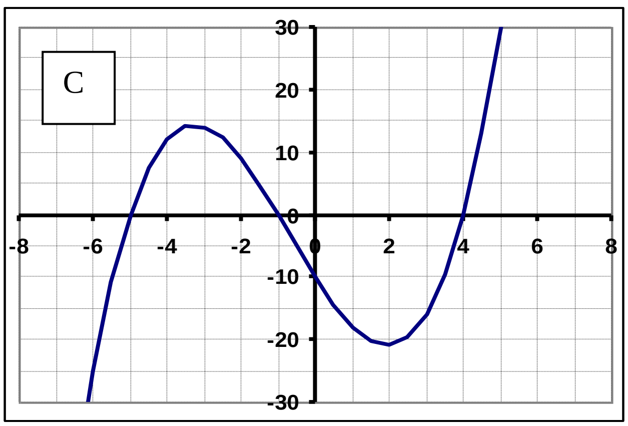
1. The four functions below have each been graphed. Match the graphs with the functions without using your calculator. **Use the zeros and end behavior as a guide!**

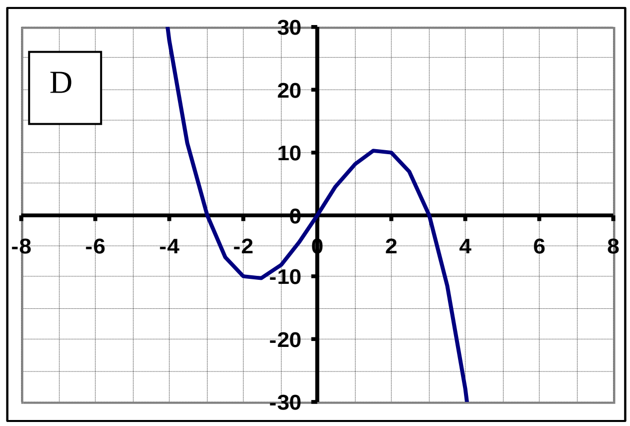
$f(x) = (x+3)(x-1)(x-5)$ $\qquad$ $h(x) = -x(x+3)(x-3)$

$g(x) = 0.5(x+5)(x+1)(x-4)$ $\qquad$ $k(x) = -1(x+3)(x-1)(x-5)$





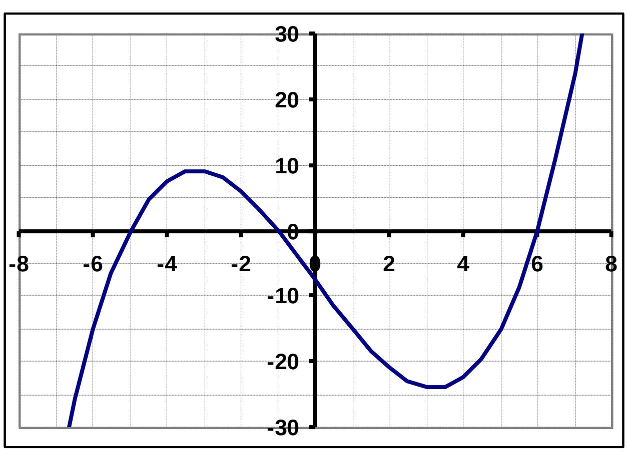




2. The graph below could be the graph of which of the following functions? **Hint: x-intercepts.** Circle all that apply. Assume a is a constant, and it may have a positive or a negative value.

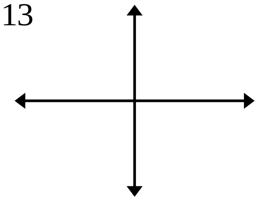
$f(x) = a(x+1)(x+6)(x-5)$ $\qquad$ $h(x) = ax(x+1)(x+5)(x-6)$

$g(x) = a(x+1)(x+5)(x-6)$ $\qquad$ $k(x) = a(x+1)(x+5)(x-6)(x+5)$

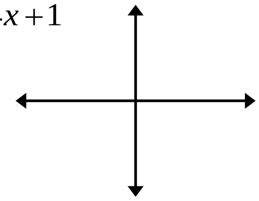


3. Graph the **<u>end behavior</u>** of the following functions.

a. $f(x) = 3x^4 - 7x^3 - 13$



b. $f(x) = -2x^3 + 3x^2 + 4x + 1$

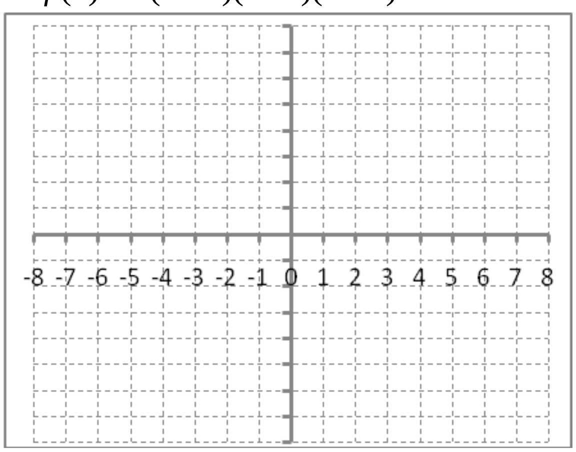


4. Describe the end behavior of the graphs of the following functions. You do NOT need to FOIL out the ones that are factored—all that matters is the term with the highest power of *x*.

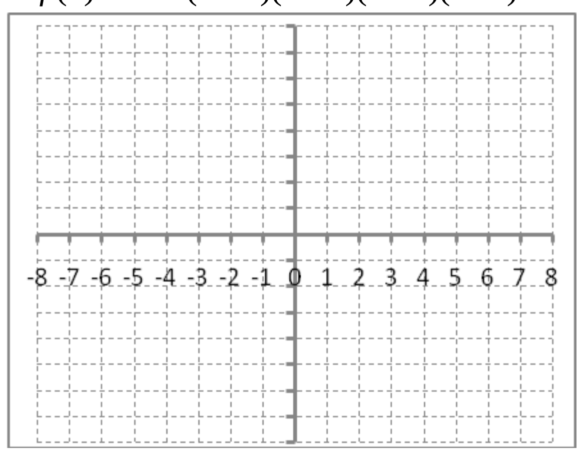
	as $x \to \infty,\ f(x) \to$	as $x \to -\infty,\ f(x) \to$
a. $f(x) = -2x^3 + 6x - 11$		
b. $f(x) = x^4 - 5x^3 - x^2 + 2x + 1$		
c. $f(x) = 2(x-1)(x+3)(x-5)$		
d. $f(x) = -x^2(x+7)(x-3)$		
e. $f(x) = 3x^2(x-1)^3(x+2)$		

5. Sketch a graph of each of the functions below. Plot the *x*-intercepts and make sure that your end behavior is appropriate. You do not need to plot any additional points. Don't worry about the *y*-scale.

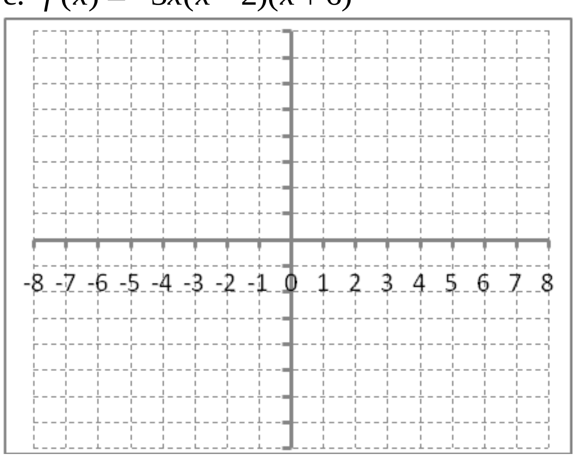
a. $f(x) = 2(x-5)(x+1)(x+4)$



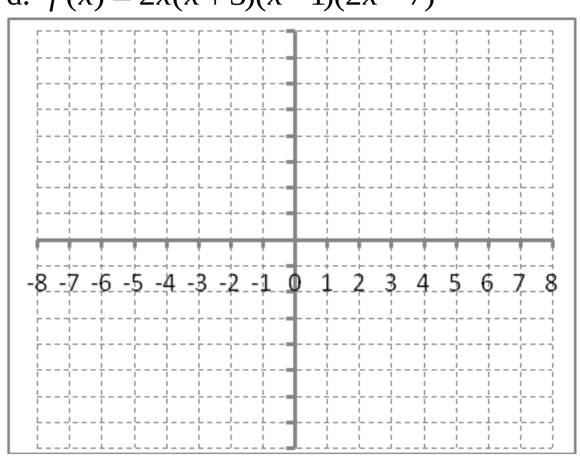
b. $f(x) = -0.5(x+6)(x+2)(x-5)(x-1)$



c. $f(x) = -3x(x-2)(x+6)$

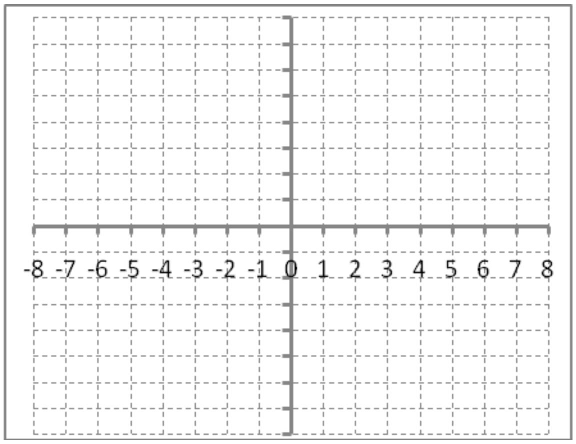


d. $f(x) = 2x(x+3)(x-1)(2x-7)$

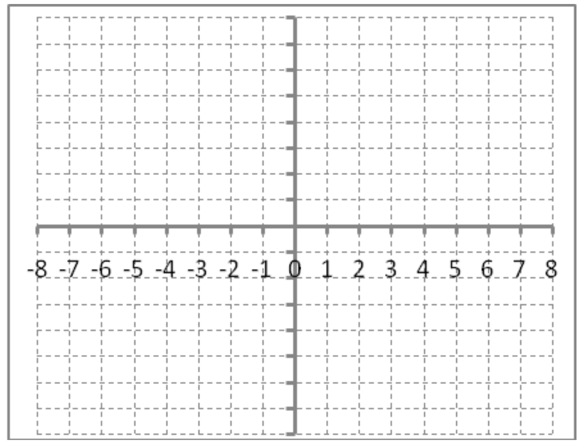


On the next two, you will need to factor the quadratic terms to find all the zeros.

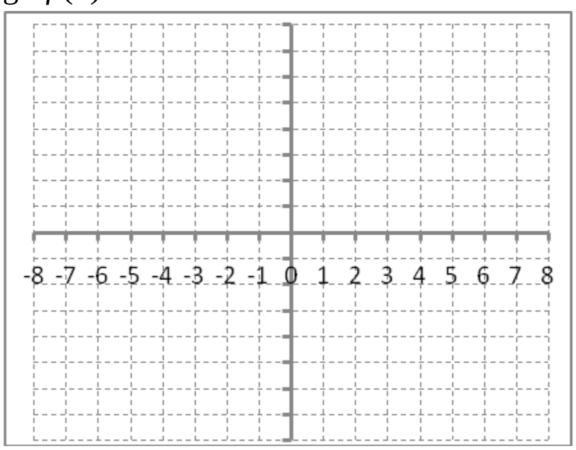
e. $f(x) = (x-5)(x+1)(x^2 - 3x + 2)$



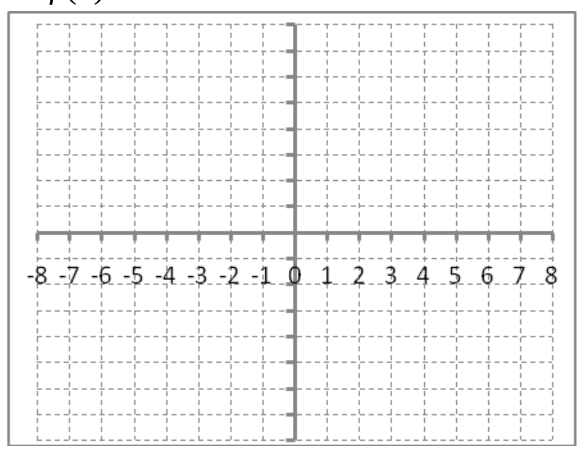
f. $f(x) = -3(x^2 - 2x - 8)(2x^2 + 9x - 5)$



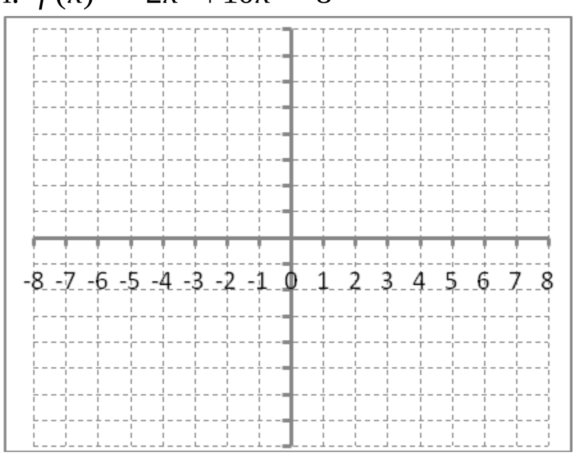
g. $f(x) = x^3 - 2x^2 - 15x$



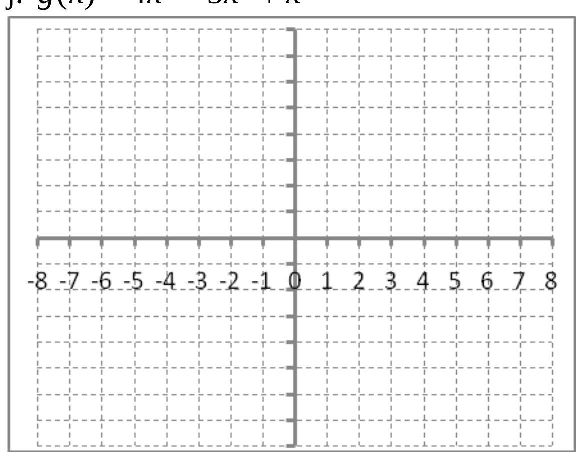
h. $f(x) = -3x^3 + 27x$



i. $f(x) = -2x^4 + 10x^2 - 8$



j. $g(x) = 4x^5 - 5x^3 + x$



6. Find all real and imaginary/complex zeros of the following functions algebraically. You do NOT need to sketch their graphs. **Then determine how times each graph hits the *x*-axis**.

a. $f(x) = -2x^3 + 4x^2 + 48x$

b. $f(x) = (2x-1)(x^2 + 2x - 35)$

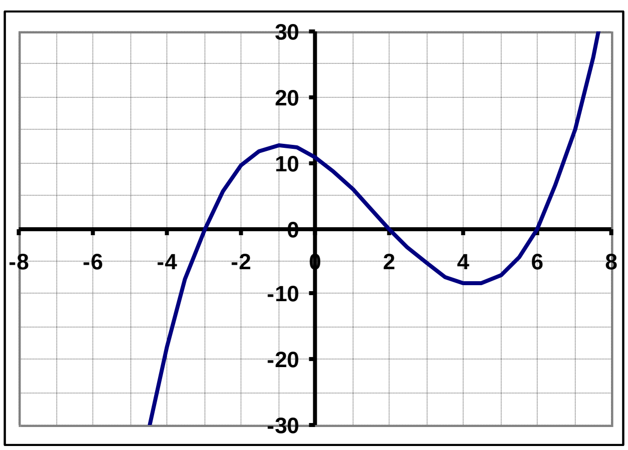
c. $f(x) = -2x(x+3)(x^2 + 4x - 6)$

d. $f(x) = (2x-1)(x^2 + x + 3)$

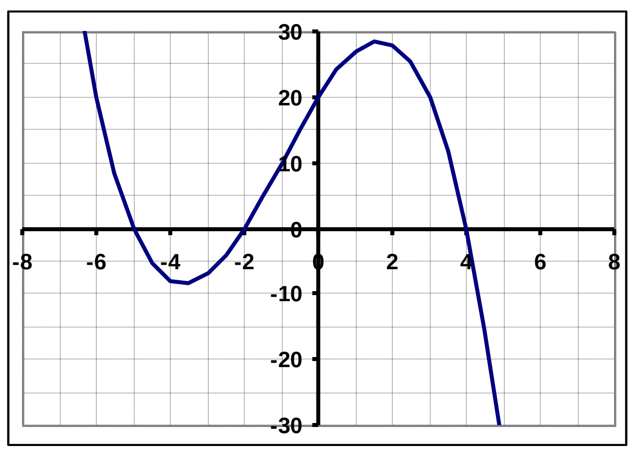
e. $f(x) = 2x^4 - 6x^2 - 20$

7. Write a possible equation for each graph below. Use the form $f(x) = a(x - \_\_)(x - \_\_)$..... You do not need to find the value of *a*, determine whether it is positive or negative.

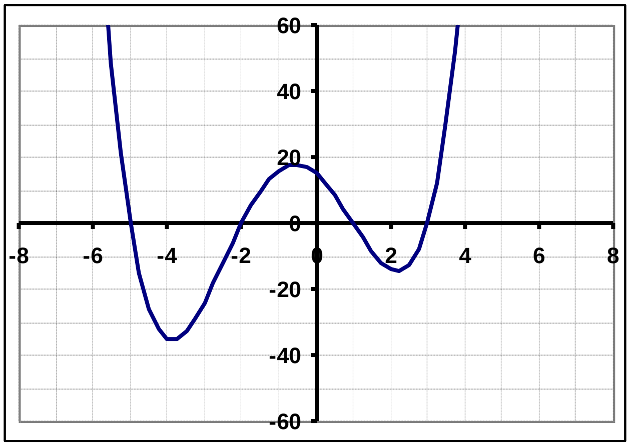
a.



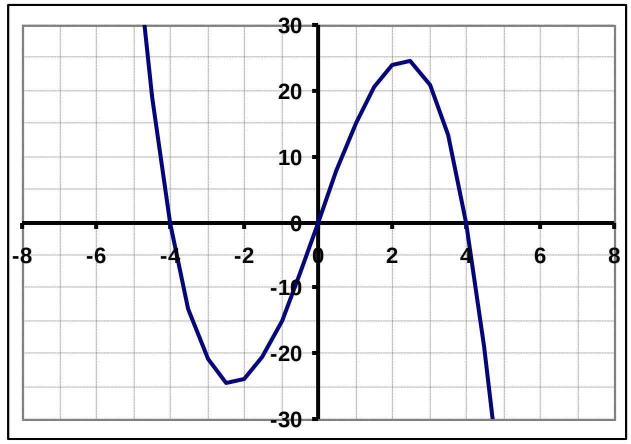
b.



c.



d.



8. A polynomial function has zeros of *x*=2, -2, and 3. Its y-intercept is 24. What is its equation?

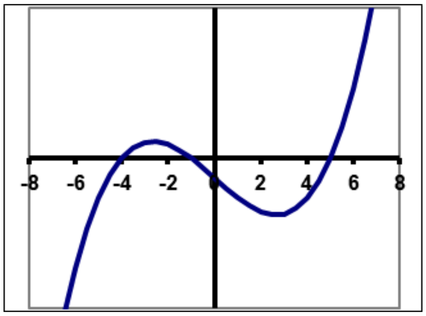
9. A 4th-degree polynomial function has zeros of $x = 2 \pm \sqrt{3}$, 0, *and* -1. It goes through the point (1,-6). What is its equation? Note: you may leave it partially factored—the first two zeros I gave you yield a quadratic factor; you do not need to FOIL this with the other factors.

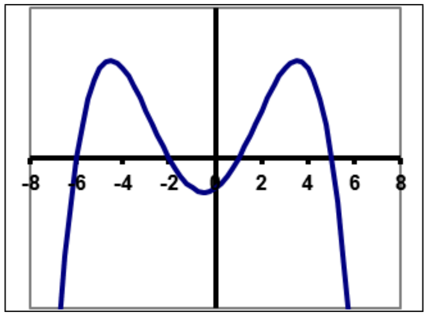
Answers

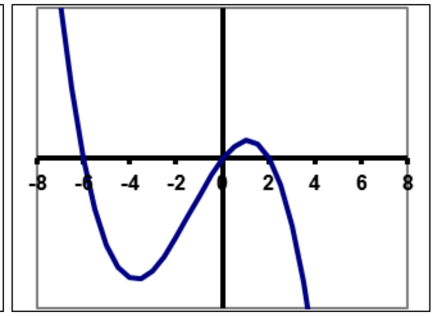
1. f(x) is B g(x) is C h(x) is D k(x) is A   2. Must be g(x)

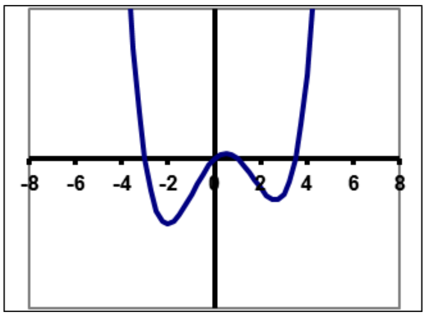
3a. up left and up right b. down right and up left 4a. -∞, ∞ b. ∞, ∞ c. ∞, -∞ d. -∞, -∞ e. ∞, ∞

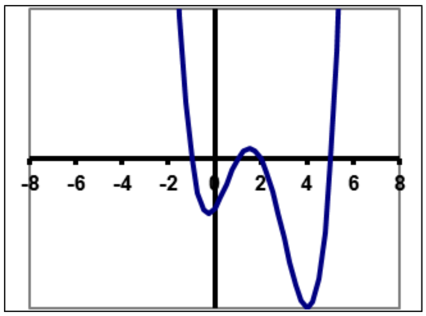
5. these are the general shape and there is no scale on the y-axis…

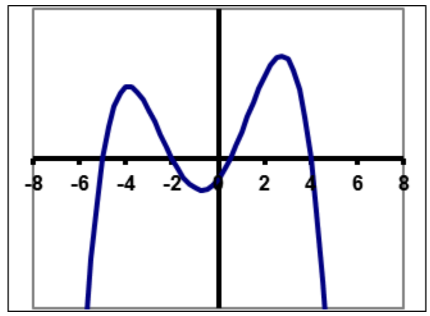
a.


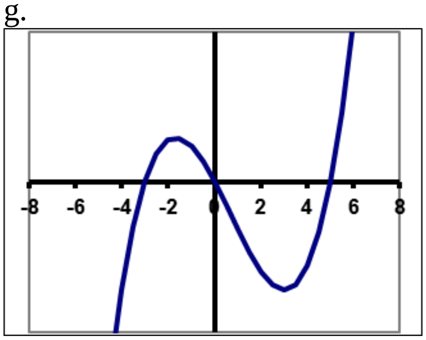
b.


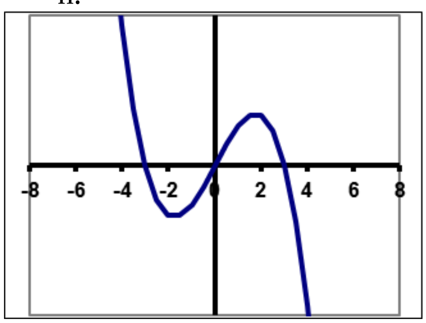
c.


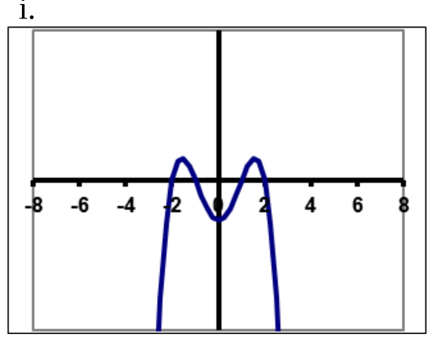
d.


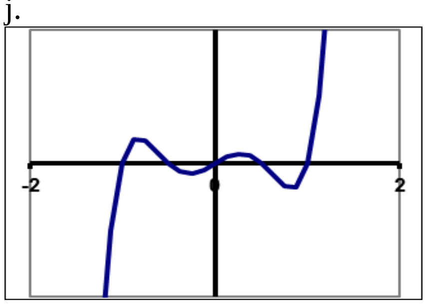
e.


f.


g.


h.


i.


j.


6a. 0, 6, -4 so 3 times b. ½, 5, -7 so 3 times c. 0, -3, $-2\pm\sqrt{10}$ so 4 times

d. ½ , $\dfrac{-1\pm i\sqrt{11}}{2}$ so only once e. $\pm\sqrt{5}, \pm i\sqrt{2}$ so twice

7a. $f(x)=a(x+3)(x-2)(x-6)\quad a>0$ b. $f(x)=a(x+5)(x+2)(x-4)\quad a<0$

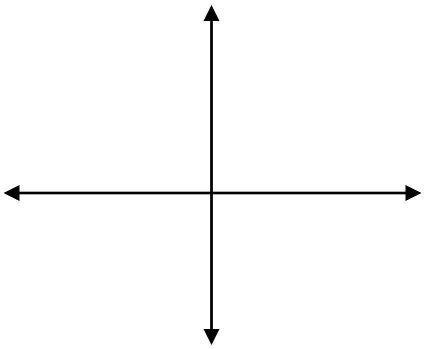
c. $f(x)=a(x+5)(x+2)(x-1)(x-3)\quad a>0$ d. $f(x)=ax(x+4)(x-4)\quad a<0$

8. $f(x)=2(x-2)(x+2)(x-3)$ 9. $f(x)=1.5x(x+1)(x^2-4x+1)$

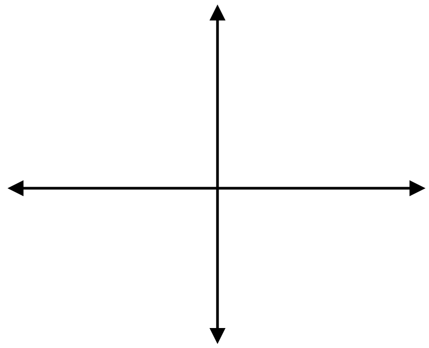
Unit 5 Handout #2: Graphs of Polynomial Functions

1. Sketch graphs of the following. You **DO NOT** need to find the y-intercepts. Make sure your end behavior is appropriate, and watch the multiple (double and triple) roots.

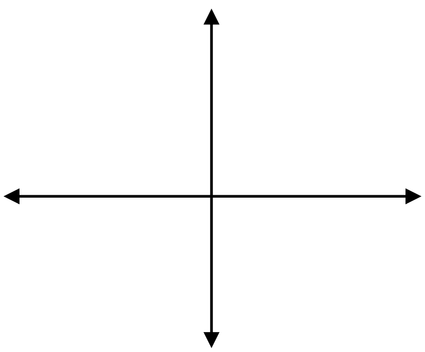
a. $f(x) = -2x(x-1)(x+3)^2$



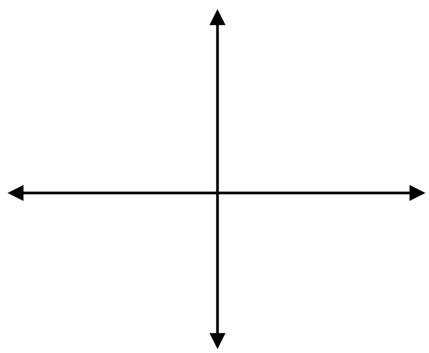
b. $f(x) = 2(x+2)^2(x-3)^2$



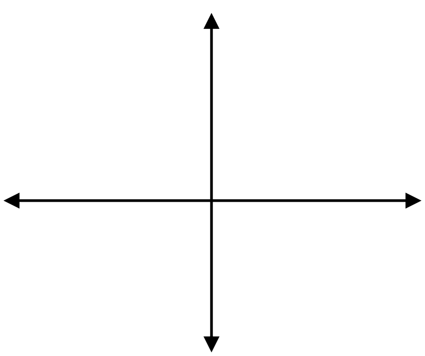
c. $f(x) = -3(x-5)^2(x+3)$



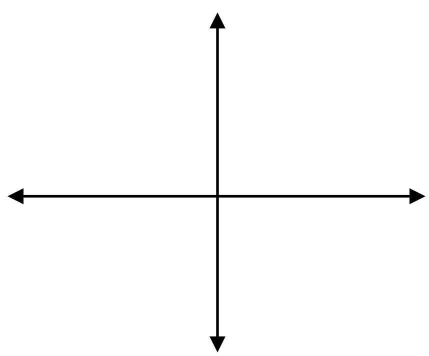
d. $f(x) = -2(x-3)^3(x+2)^2$



e. $f(x) = x^3$



f. $f(x) = 2(x-3)(x+1)(x-5)^2$



2. Sketch graphs of the following. You **DO NOT** need to find the y-intercepts. Make sure your end behavior is appropriate, and watch the multiple roots. All terms should be factorable.

a. $f(x) = x^4 - 2x^3 - 8x^2$

b. $f(x) = 2(x^2 - 9)(x^2 - 2x - 3)$

c. $f(x) = -x^5 + 4x^3$

d. $f(x) = (x^3 - x)(x^2 - x - 12)$

e. $f(x) = -2x(x^2 - 5x + 6)(2x^2 - x - 15)$

f. $f(x) = (x^2 - 4)^2$

g. $f(x) = -x^4 + 4x^2 - 4$

3. Some (but not all) cubics can be written in the form $f(x)=a(x-h)^3+k.$ Starting with the graph of $f(x)=x^3$ (from question 1*e* above) and remembering the impact of a, h, and k on absolute value functions and quadratics in vertex form, graph each of these. Also find their *x*-intercepts (don't worry about their imaginary/complex zeros).

a. $f(x)=(x-3)^3+8$

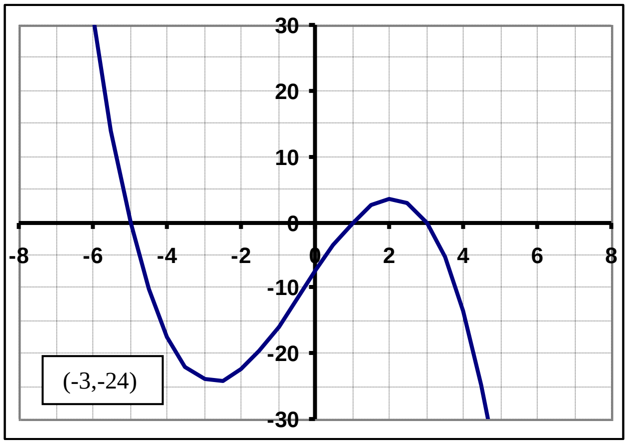
b. $f(x)=-2x^3-2$

c. $f(x)=0.5(x+2)^3-4$

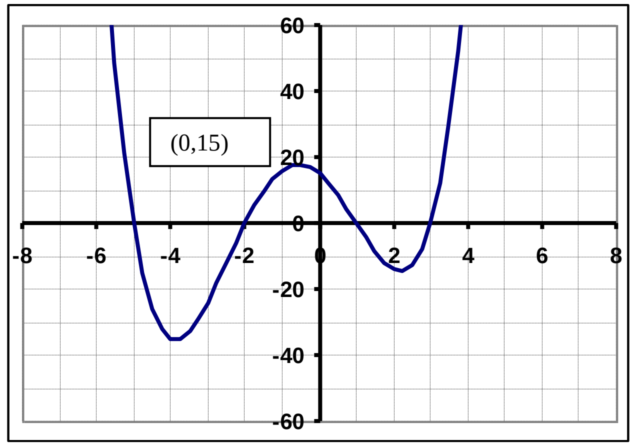
d. $f(x)=-\frac{1}{3}(x+1)^3-9$

4. Write the equation of each graph below. Use the form $f(x) = a(x - \_\_)(x + \_\_)...$ Use the point that is labeled to find the exact value of a. Assume the lowest possible degree (all bounces are doubles…)

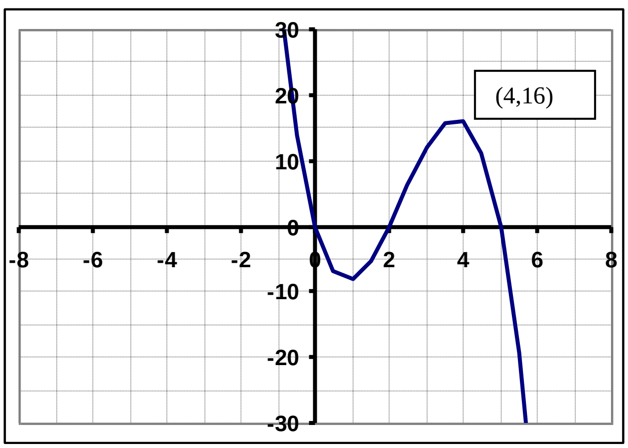
a.



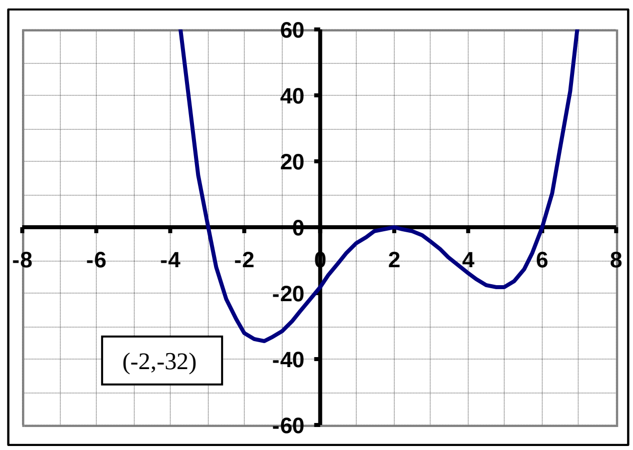
b.



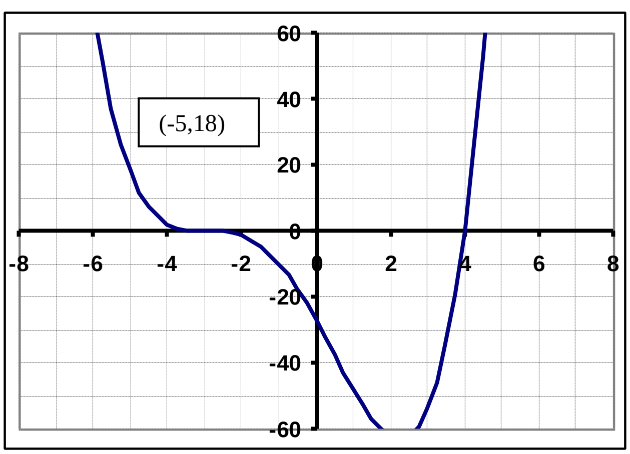
c.



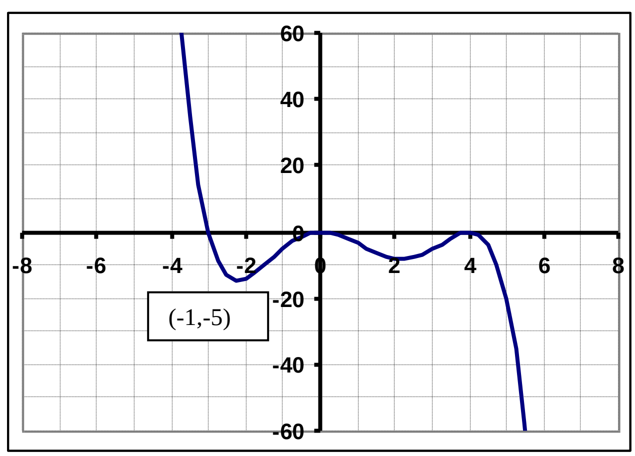
d.



e.



f.



5. Write the equations of the polynomial functions described:

a. A cubic function has zeros of 1, -2, and 3. Its y-intercept is -24.

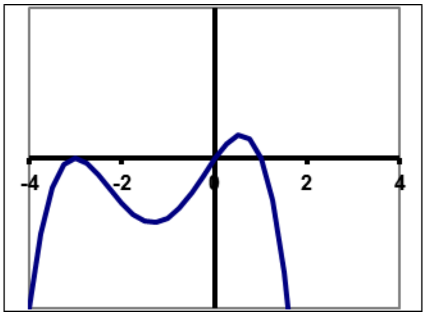
b. A 4th degree polynomial functions has zeros at $\pm 1 \textit{ and } \pm 2$. It goes through the point (0,8).

c. A 3rd degree polynomial function has zeros of $\pm 2i \textit{ and } 3$. It goes through the point (-1,40).

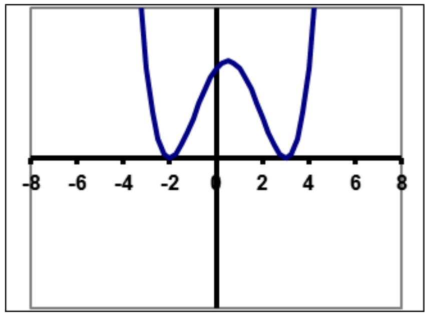
d. The zeros of a quartic are $2 \pm \sqrt{5}$ and $\pm i\sqrt{2}$. It goes through the point (0,10).

Answers

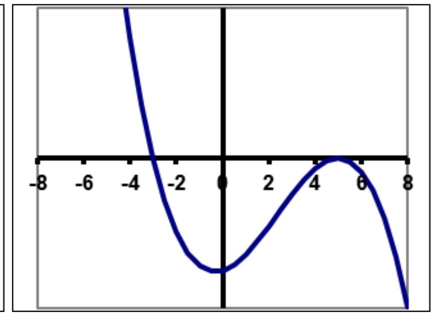
1a.



b.



c.



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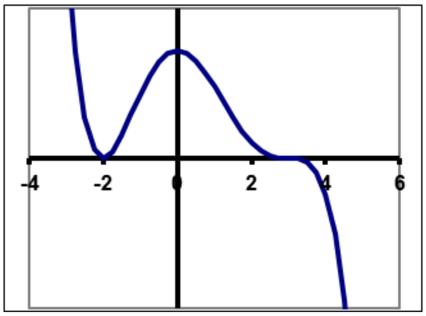

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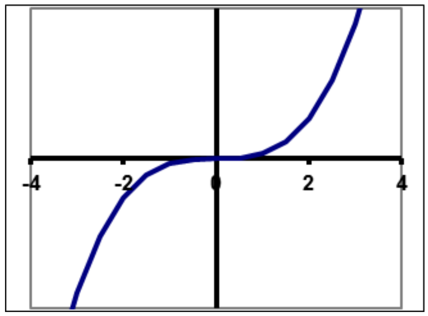

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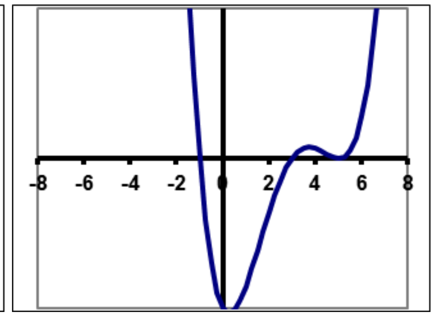
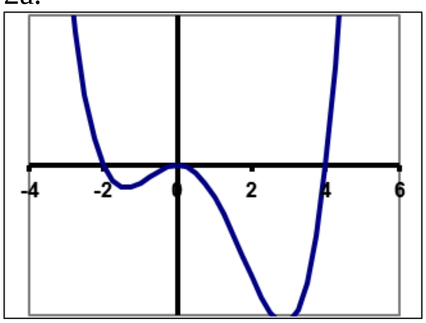


2a.



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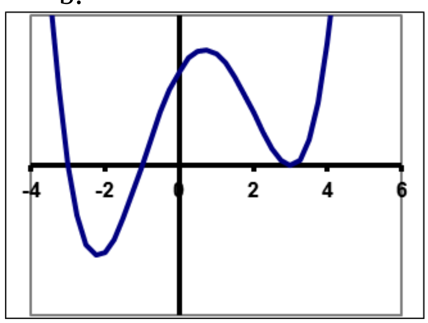

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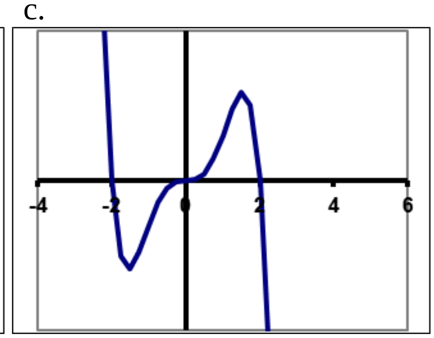

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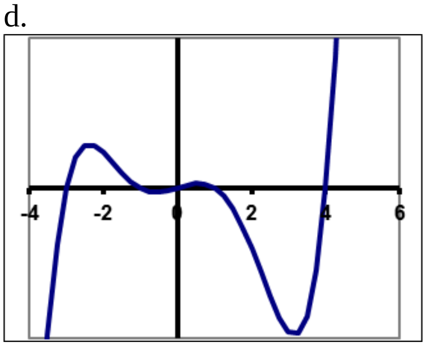

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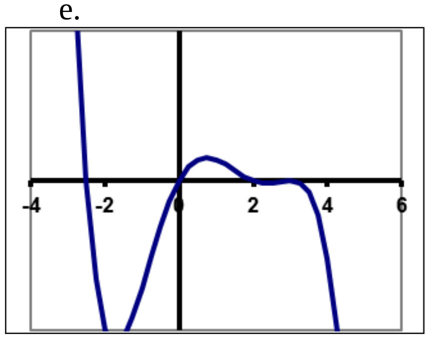

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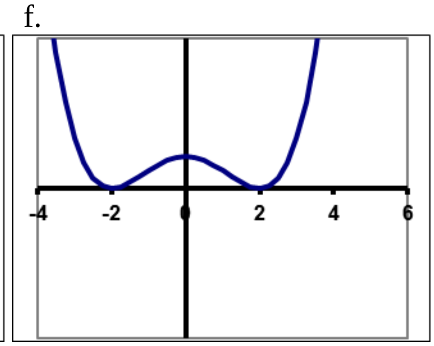


2g. zeros are $\sqrt{2}, \sqrt{2}, -\sqrt{2}, -\sqrt{2}$

3a. 3 right and 8 up; x-intercept is 1 b. flip and stretch vertically, down 2 x-int is -1
c. compress vert, 2 left and 4 down x-int is 0 d. flip and compress vert, left 1, down 9 x-int is -4

4a. $f(x) = -0.5(x+5)(x-1)(x-3)$ b. $f(x) = 0.5(x+5)(x+2)(x-1)(x-3)$

c. $f(x) = -2x(x-2)(x-5)$ d. $f(x) = 0.25(x+3)(x-2)^2(x-6)$

e. $f(x) = 0.25(x+3)^3(x-4)$ f. $f(x) = -0.1x^2(x+3)(x-4)^2$

5a. $f(x) = -4(x-1)(x+2)(x-3)$ b. $f(x) = 2(x^2-1)(x^2-4)$ c. $f(x) = -2(x^2+4)(x-3)$

d. $f(x) = -5(x^2+2)(x^2-4x-1)$

Unit 5 Handout #3: Polynomial Equations and Inequalities and Graphs

1. Solve the following equations by factoring. Find all **real and imaginary solutions**. Remember, the total number of solutions should be the degree (but this counts double-roots as two, triple-roots as three, etc.). Also, whenever factoring, make sure one side is zero!

a. $x^4 - 5x^2 + 4 = 0$

b. $\dfrac{x^4 - 11x^2}{2} + 14 = 2$

c. $\dfrac{x^4 - 3x^2}{2} = 5$

d. $x^4 = 9$

e. $\dfrac{-11}{17}(x^4 - 1)(x^2 + 2x + 1) = 0$

f. $(2x - 1)(x^2 - 9)(x + 2) < 0$

g. $x^5 + 4x \le 5x^3$ h. $-3x^2(x^2 - 4x - 5) > 0$ i. $x^4 \ge 5x^2$

2. Find all points of intersection between the following curves algebraically. Remember, since you are finding intersection points, you need to show both the x and the y-coordinates. Also remember that only real solutions are necessary, since imaginary numbers do not show up on the coordinate plane.

a. $\begin{array}{l} y = x^5 - 5x^3 + 4x + 5 \\ y = 5 \end{array}$ b. $\begin{array}{l} y = x^4 + 2x^3 - 4x^2 + 5x \\ y = 6x^3 + x^2 + 5x \end{array}$

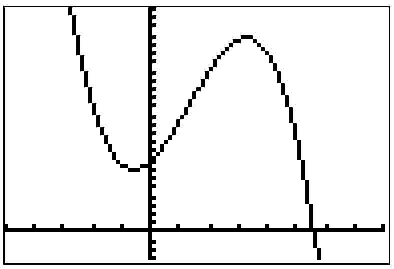
3. Let $f(x)$ be a polynomial function. The solution to $f(x) \le 0$ is $x \le -3$ *or* $x = 2$. We also know that $f(0) = 12$.

a. Could $f(x)$ be 3rd degree? If so, what is $f(x)$? b. Could $f(x)$ be 4th degree? If so, what is $f(x)$?

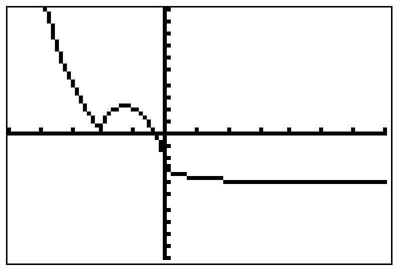
c. Could $f(x)$ be 5th degree? If so, what are **two** possible functions $f(x)$?

4. Which of the following graphs might be those of polynomial functions? For any that cannot be, briefly describe why not.

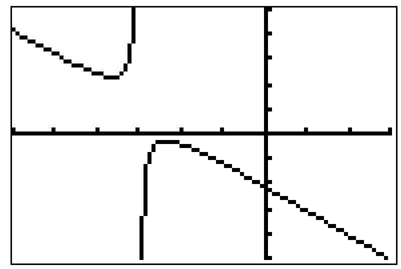
a.



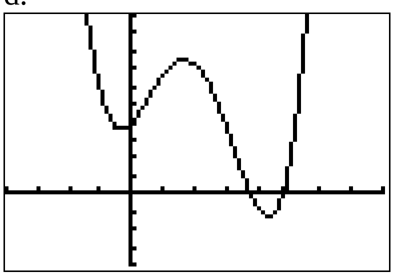
b.



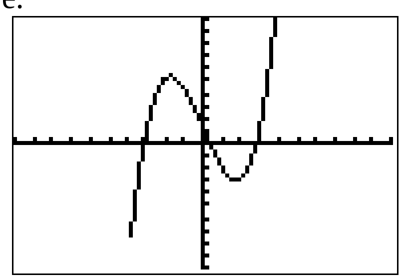
c.



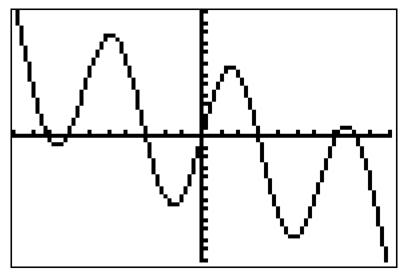
d.



e.



f.



5. For any in the prior question that are polynomial functions, what do you know about the degree and the leading coefficient?

6. A cubic polynomial function with real coefficients may have …. (more than one may be correct)
a. Three real zeros.
b. Two real zeros and one zero with an imaginary component.
c. One real zero and two zeros with imaginary components.
d. Three zeros with imaginary components.
e. More than three zeros.

7. A polynomial function has one real zero (a single root) and 4 zeros with imaginary components. What degree is it, and how many times does it intersect the x-axis?

8. The solutions to the equation $x^3 - x - 14 = x^2 + x - 2$ are $x = 3$ and $x = -1 \pm i\sqrt{3}$. What does this tell us about the graphs of the functions $f(x) = x^3 - x - 14$ *and* $g(x) = x^2 + x - 2$? (Choose one)

a.) These two graphs never intersect.
b.) These two graphs intersect at exactly one point.
c.) These two graphs intersect at exactly two points.
d.) These two graphs intersect at exactly three points.

9a. The domain of any polynomial function is __________.
b. If a polynomial function has an odd degree, then its range **must** be _________.
c. If a polynomial function has an even degree, then its range **cannot** be _________.

10. A polynomial function has 4 turns and *as* $x \to \infty$ $f(x) \to -\infty$. It touches the *x*-axis once (at a single root). We know that its degree is ____ and at least _________, its leading coefficient is ______, it has _____ real zeros, and at least _____ zeros with imaginary components.

11. Write the equation of each polynomial function described: (hint: it may help to graph). Assume the lowest degree possible.
a. $f(x) > 0$ *when* $x < -2$ *or* $1 < x < 3$ and y-intercept is -8

b. $f(x) \le 0$ *when* $-2 \le x \le 0$ *or* $x \ge 5$ and goes through (-1,-10)

c. $f(x) > 0$ *when* $x > 3$ *or* $x < -4$ *or* $-1 < x < 3$ and y-int is 24

d. $f(x) \geq 0$ *when* $x = -2$ *or* $1 \leq x \leq 3$ and y-intercept is -6.

e. The zeros are $2 \pm i\sqrt{3}$ *and* ± 2 and it goes through the point (1,7)

f. The zeros are $\pm\sqrt{5}$ *and* $\pm i\sqrt{2}$ and the y-intercept is -40.

g. The zeros are $3 \pm i$ *and* $-2 \pm \sqrt{7}$ and the y-intercept is 100.

12. A third degree polynomial function's graph must intersect any given line. True or false? Explain.

13. A third degree polynomial function's graph must intersect any given parabola. True or false? Explain.

14. A fourth degree polynomial function's graph must intersect any given parabola. True or false? Explain.

ANSWERS

1a. 2, -2, 1, -1 b. $\pm\sqrt{3},\pm 2\sqrt{2}$ c. $\pm\sqrt{5},\ \pm i\sqrt{2}$ d. $\pm i\sqrt{3},\pm\sqrt{3}$
e. $1,-1,i,-i,-1,-1$ f. -3<x<-2 or 0.5<x<3 g. x≤-2 or -1≤x≤0 or 1≤x≤2
h. -1<x<0 or 0<x<5 (can also say -1<x<5 but not x=0) i. $x\le -\sqrt{5}\ or\ x=0\ or\ x\ge\sqrt{5}$

2a. $x^5-5x^3+4x+5=5\ so\ x^5-5x^3+4x=0$ points are (0,5), (1,5), (2,5), (-1,5), and (-2,5)
b. (0,0), (-1,-10), and (5,800)

3a. yes; $f(x)=(x+3)(x-2)^2$ b. no c. $f(x)=0.25(x+3)(x-2)^4\ or\ f(x)=(1/9)(x+3)^3(x-2)^2$

4a. yes b. no: has a horiz asymptote c. no; asymptotes d. yes e. no; domain is not all reals f. yes

5a. odd degree 3 or higher; negative leading coeff d. even degree at least 4; positive leading coeff
f. odd degree 7 or higher; negative leading coeff

6. a and c are OK; zeros with imaginary parts must come in pairs

7. Fifth degree and intersects the x-axis at its only real zero (once)

8. b -- intersections are real solutions to the system of equations

9a. all reals b. all reals c. all reals

10. We know that its degree is _odd_ and at least _5_, its leading coefficient is _negative_, it has _one_ real zeros, and at least _four_ zeros with imaginary components.

11a. $f(x)=-(4/3)(x+2)(x-1)(x-3)$ b. $f(x)=(-5/3)x(x+2)(x-5)$
c. $f(x)=(2/3)(x+4)(x+1)(x-3)^2$ d. $f(x)=-0.5(x+2)^2(x-1)(x-3)$
e. $f(x)=(-7/12)(x^2-4)(x^2-4x+7)$ f. $f(x)=4(x^2-5)(x^2+2)$
g. $f(x)=(-10/3)(x^2-6x+10)(x^2+4x-3)$

Unit 5 Handout #4: Polynomial Division and the Factor Theorem

1. Do the following long division:

a. $\dfrac{x^3-3x^2+4x+8}{x+1}$

b. $\dfrac{5x^3-8x+5}{x-1}$

c. $\dfrac{x^4+3x^3-5x^2+2x+11}{x^2-3x-1}$

d. $\dfrac{x^3+8}{x+2}$

2. In each of these, one zero or factor is given. Do long division to find another factor. Then either continue factoring or use the quadratic formula or completing the square to find all zeros of the function. They may be real or have imaginary components.

a. $f(x)=x^3-x^2-10x-8$ $\quad f(-2)=0$

b. $f(x)=x^3+x^2-14x+6$ $\quad f(3)=0$

c. $f(x) = 2x^3 - 11x^2 + 17x - 6$ $(2x-1)$ is a factor

d. $x^3 + x = -10$ $x = -2$ is one solution

e. $f(x) = x^4 - x^3 - 19x^2 + 14x + 40$ $(x^2 - 3x - 5)$ is a factor

f. $x^4 - 4x^3 - x^2 - 16x - 20 = 0$ $x = 2i$ is one solution.

g. One cube root of 64 is 4. The other two are complex numbers. Find them.

3. For each part below, graph the function on your calculator and find one zero that is an integer. Then use long division to help you factor it and find the other zeros algebraically.

a. $f(x) = x^3 + 7x^2 + 7x - 6$

b. $f(x) = 2x^3 + 7x^2 + 7x + 2$

4. One zero of the function $f(x) = x^4 - 2x^3 - 9x^2 + 8x + 20$ is $x = 1 + \sqrt{6}$.

a. Given that all of the coefficients of $f(x)$ are rational numbers (no square roots), we know the conjugate must be a zero as well. What is the conjugate of $1 + \sqrt{6}$?

b. The two zeros you have identified can give you one quadratic factor of $f(x)$. What is it?

c. Use long division to find the other quadratic factor and then other two zeros.

5. Given that two zeros of $f(x) = 3x^4 - x^3 - 21x^2 - 11x + 6$ are 1/3 and -2, find the other two zeros.

6. Solve the following inequalities algebraically. For the inequalities, work just in the real number system. For the equations, find all real and complex solutions.

a. $2x^2(x+1)(x-3)^3 \geq 0$

b. $x^5 - 2x^3 = 15x$

7. The solution to $f(x) > 0$ is $-2 < x < 0$ *or* $1 < x < 3$.

a. What is one possible $f(x)$?

b. Could $f(x)$ be a 5^{th} degree polynomial function? If so, give one possible $f(x)$.

c. Could $f(x)$ be a 6^{th} degree polynomial function? If so, give one possible $f(x)$.

SOME WORD PROBLEMS – like box of water with cubes in it….

ANSWERS

1a. $x^2 - 4x + 8$ b. $5x^2 + 5x - 3 + \dfrac{2}{x-1}$ c. $x^2 - 6x + 14 + \dfrac{50x+25}{x^2 - 3x - 1}$ d. $x^2 - 2x + 4$

2a. -2, -1, 4 b. $3, -2 \pm \sqrt{6}$ c. ½,2,3 d. -2, $1 \pm 2i$ e. 2,-4, $\dfrac{3 \pm \sqrt{29}}{2}$ f. $\pm 2i, -1, 5$ g. $-2 \pm 2i\sqrt{3}$

3a. -2, $\dfrac{-5 \pm \sqrt{37}}{2}$ b. -2, -1, -1/2

4a. $1 - \sqrt{6}$ b. $(x-1)^2 - 6$ *or* $x^2 - 2x - 5$ c. $x^2 - 4$ *so other zeros are* ± 2

5. one quadratic factor must be $(3x-1)(x+2)$ *or* $3x^2 + 5x - 2$; $f(x) = (3x^2 + 5x - 2)(x^2 - 2x - 3)$ so other zeros are 3 and -1 6a. $x \le -1$ *or* $x = 0$ *or* $x \ge 3$ b. $0, \pm\sqrt{5}, \pm i\sqrt{3}$

7a. $f(x) = ax(x+2)(x-1)(x-3)$ $a < 0$ b. no; end behavior

c. like (a) but any one is a triple root; so $f(x) = ax^3(x+2)(x-1)(x-3)$ $a < 0$, for example

Review Problems for Unit 5

1. Sketch a graph of each:

a. $f(x) = -2(x-2)^2(x+5)$

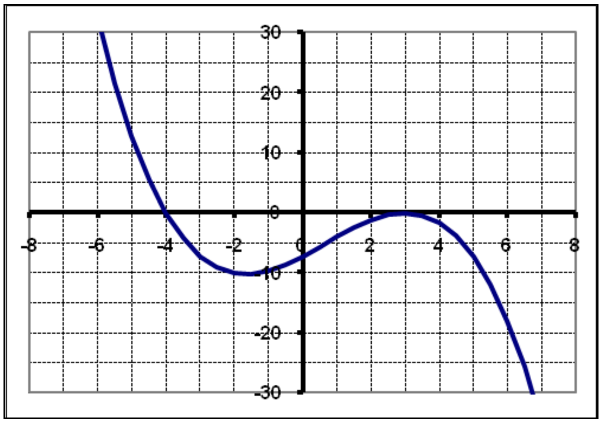
b. $f(x) = 2(x+3)^3(x-2)^2$

c. $f(x) = -2x^4 - 6x^3 - 4x^2$

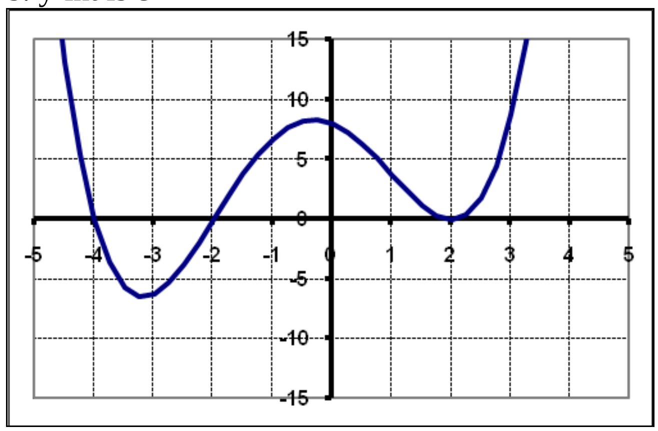
d. $f(x) = x^5 - 5x^3 + 4x$

2. Write the equation of each function:

a. Goes through (1,-4)



b. y-int is 8



3. Write the equation of a 4th degree polynomial function with zeros of $\pm\sqrt{3}$ and $2 \pm i$ and a y-intercept of 12.

4. Do the following division.

a. $\dfrac{x^3 - 3x^2 + 4x + 8}{x+1}$

b. $\dfrac{x^4 - 3x^2 - 10x - 24}{x-3}$

5a. The domain of any polynomial function is __________.
b. If a polynomial function has an odd degree, then its range must be _________.
c. If a polynomial function has an even degree, then its range cannot be _________.

6. The zeros of a fourth-degree polynomial function are $\pm 2i$ *and* $\pm\sqrt{5}$. The y-intercept is 10. What is the function?

7. Given that a zero of the function $f(x) = x^3 + 7x^2 + 14x + 6$ is -3, what are the other two?

8. Solve the following inequalities and graph the solutions on a number line:

a. $-2x^2(x+5)(x-2)^3 < 0$

b. $x^6 - 2x^4 + 1x^2 > 0$

9. $f(x)$ is a polynomial function and the solution to $f(x) \le 0$ *is* $x \le -3$ *or* $-1 \le x \le 2$ *or* $x \ge 3$. The y-intercept of $f(x)$ is -24.
a. What do we know about the degree of $f(x)$ and its leading coefficient?
b. Write two possible equations for $f(x)$; each should have a different degree.

10. Find all solutions, real and with imaginary components, of the following equations:
a. $x^3 - 2x = 10x^2$ b. $x^3 - 8 = 0$ c. $2x^3 - 5x^2 + 9x = 6$ (given x=1 is one solution)

11. Given one zero of each function below, find the others.
a. $f(x) = x^4 - 6x^3 + 5x^2 + 22x - 40$; zero is $x = 2 + i$
b. $h(x) = x^4 - 2x^3 + 38x^2 - 2x + 37$; zero is $1 - 6i$

12. A polynomial function has 3 turns and as $x \to \infty$ $f(x) \to -\infty$. It never crosses the x-axis, but it bounces off the x-axis once. We know that its degree is ____ and at least ________, its leading coefficient is ______, it has _____ unique real zero(s), and at least _____ zero(s) with imaginary components.

ANSWERS
1. you can check the graphs on your calculator. Here are the factored forms of the last two:
c. $f(x) = -2x^2(x+2)(x+1)$ d. $f(x) = x(x+2)(x-2)(x+1)(x-1)$
2a. $f(x) = -0.2(x+4)(x-3)^2$ b. $f(x) = 0.25(x+4)(x+2)(x-2)^2$
3. $f(x) = \frac{-4}{5}(x^2 - 3)(x^2 - 4x + 5)$
4a. $x^2 - 4x + 8$ b. $x^3 + 3x^2 + 6x + 8$
5a. all reals b. all reals c. all reals
6a. $f(x) = -0.5(x^2 + 4)(x^2 - 5)$
7. $-2 \pm \sqrt{2}$ *because* $f(x) = (x^2 + 4x + 2)(x + 3)$
8a. $x < -5$ *or* $x > 2$
b. $x < -1$ *or* $-1 < x < 0$ *or* $0 < x < 1$ *or* $x > 1$ (in other words, everything but -1,0,1)
9a. even degree and negative leading coefficient
b. $f(x) = -\frac{4}{3}(x+3)(x+1)(x-2)(x-3)$ or $f(x) = -\frac{4}{3}(x+3)(x+1)^3(x-2)(x-3)$ or others..
10a. 0, $5 \pm 3\sqrt{3}$ b. 2, $-1 \pm i\sqrt{3}$ c. 1, $\frac{3 \pm i\sqrt{39}}{4}$ since it factors to $(x-1)(2x^2 - 3x + 6)$
11a. $2 - i$, -2, 4 b. $1 + 6i$, $\pm i$
12. We know that its degree is _even_ and at least _4_, its leading coefficient is _negative_, it has _one_ unique real zeros (a double or quadruple root..), and at least _two__ zeros with imaginary components.

Unit 6 Handout #1: Properties of Exponents

PART I
Simplify all expressions. You combine terms with the same base when possible. You may leave your answers as something like 5^7 instead of multiplying it out if you like. **Some may not be simplifiable**.

1. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
2. $x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$
3. $2^3 \cdot 2^4$
4. $4^0 \cdot 4^3$
5. $4 \cdot 4^7$
6. $x^2 \cdot x^5$
7. $x^2 + x^5$
8. $x \cdot x^2 \cdot x^4$
9. $2x^3 \cdot 4x^3$
10. $2x^3 + 4x^3$
11. $2xy \cdot 3x^2 y$
12. $2xy \cdot 2x + 5x^2 y$
13. $2xy \cdot 2y + 5x^2 y$
14. $2^3 + 2^2$
15. $(x^2) \cdot (x^2) \cdot (x^2)$
16. $(x^2)^3$
17. $(2^2)^3$
18. $(3^2)^4$
19. $(4^6)^0$
20. $2x^3 \cdot (x^3)^4$
21. $3(3x)$
22. $(3x)(3x)$
23. $(3x)^3$
24. $(2 \cdot 3)^2$
25. $(-1 \cdot 2)^3$
26. $(-1 + 2)^3$
27. $(2x)^3 - 2x^3$
28. $(-2x)^2$
29. $(-2 + x)^2$
30. $(-2x)^3$

31. $(x^2y^3)^2$

32. $(-x^2y^3)^2$

33. $(x^2-y^3)^2$

34. $(2+3)^2-(2^2+3^2)$

35. $(x+y)^2-(x^2+y^2)$

36. $\dfrac{5\cdot5\cdot5}{5\cdot5}$

37. $\dfrac{5^3}{5^2}$

38. $\dfrac{2^4}{2}$

39. $\dfrac{x^6}{x^2}$

40. $\dfrac{6x^3}{2x}$

41. $\dfrac{x^2y}{xy}$

42. $\dfrac{-2x^3}{2x}$

43. $\dfrac{(-2x)^3}{2x}$

44. $\left(\dfrac{1}{2}\right)\cdot\left(\dfrac{1}{2}\right)\cdot\left(\dfrac{1}{2}\right)$

45. $\left(\dfrac{1}{3}\right)^2$

46. $\left(\dfrac{-2}{3}\right)^3$

47. $\left(\dfrac{2x^3}{3}\right)^2$

48. $\dfrac{8x^3}{2x}\cdot\dfrac{3x}{4}$

49. $\left(\dfrac{-2x}{y}\right)^2\cdot 2y^3$

50. $\left(\dfrac{2xy}{3}\right)^2\cdot\left(\dfrac{3}{x^2y}\right)^3$

51. $\left[\left(\dfrac{2x}{3}\right)^2-\dfrac{2x^2}{9}\right]^2$

52. $\dfrac{(-2q)^3}{(4q^2)^2}$

53. $\left(\dfrac{x^2y}{3x}\right)^2\cdot(3xy)^2$

54. $\left(\dfrac{-2x^2y^3}{3}\right)^2\cdot\left(\dfrac{3x}{y^2}\right)^2$

55. $\left[(3x)^2-3x^2\right]^2$

ANSWERS

1. 2^5 *or* 32
2. x^7
3. 2^7 *or* 128
4. 4^3 *or* 64
5. 4^8
6. x^7
7. *can't simplify*
8. x^7
9. $8x^6$
10. $6x^3$
11. $6x^3y^2$
12. $9x^2y$
13. $4xy^2+5x^2y$
14. $8+4$ *or* 12
15. x^6
16. x^6
17. 2^6 *or* 64
18. 3^8
19. 1
20. $2x^{15}$
21. $9x$
22. $9x^2$
23. $27x^3$
24. 6^2 *or* 36
25. -8
26. 1
27. $6x^3$
28. $4x^2$
29. $4-4x+x^2$
30. $-8x^3$
31. x^4y^6
32. x^4y^6
33. $x^4-2x^2y^3+y^6$
34. $25-13=12$
35. $2xy$
36. 5
37. 5
38. 2^3 *or* 8
39. x^4
40. $3x^2$
41. x
42. $-x^2$
43. $-4x^2$
44. $\frac{1}{8}$
45. $\frac{1}{9}$
46. $\frac{-8}{27}$
47. $\frac{4x^6}{9}$
48. $3x^3$
49. $8x^2y$
50. $\frac{12}{x^4y}$
51. $\frac{4x^4}{81}$
52. $\frac{-1}{2q}$
53. x^4y^4
54. $4x^6y^2$
55. $36x^4$

Part II: Negative Exponents
Simplify when possible. Write without negative exponents. Be careful of addition and subtraction!

1. $\frac{2^3}{2^7}$

2. 3^{-2}

3. $\frac{1}{4^{-2}}$

4. $\frac{2}{x^{-2}}$

5. $\frac{2}{3x^{-2}}$

6. $\left(2^{-3}\right)^2$

7. $\left(xy^2\right)^{-3}$

8. $\left(2x^2\right)^{-1}$

9. $3x^{-2}$

10. $2(5x)^{-1}$

11 $\frac{4^{-3}}{4^{-5}}$

12. $\frac{4^{-3}}{4^5}$

13 $\left(3^{-2}\right)^{-2}$

14. $\left(\frac{1}{2}\right)^{-1}$

15. $\left(\frac{1}{2}\right)^{-2}$

16. $\left(\frac{2}{x}\right)^{-3}$

17. $\left(\frac{4}{5}\right)^{-2}$

18. $\frac{2x^{-2}}{3x}$

19. $(x+y)^{-1}$

20. $(-2x)^{-2}\cdot(4x^3)$

21. $\left(q^{-3}\right)^2\cdot q^0$

22. $3x^3y\cdot 2x^{-2}y^2$

23. $(x^{-2})^3\cdot x^5$

24. $(3xy)^{-1}$

25. $(3x+y)^{-1}$

26. $\frac{2x^{-2}y^3}{x^2y^{-1}}$

27. $\frac{\left(2x^2y\right)^{-2}}{4xy^{-1}}$

28. $(2+3)^{-1}$

29. $\left(2^{-1}+3^{-1}\right)^{-1}$

30. $(x+y)^{-2}$

31. $\left[(2x)^2 - 2x^2\right]^{-1}$

32. $2x^{-2} - x^{-2}$

33. $2x^{-2} \cdot \left(-x^{-2}\right)$

34. $2x^{-2} \cdot x^{-2}$

35. $(2x)^{-2} \cdot x^{-2}$

36. $(-2x)^{-2} \cdot x^{-2}$

37. $3^{-2} + (-2)^3$

38. $\left(\dfrac{2}{3}\right)^{-2} \cdot \left(\dfrac{-3}{4}\right)^{-1}$

39a. Is the expression $\dfrac{x}{x^2}$ the same as $\dfrac{1}{x}$?

b. Is the expression $\dfrac{x}{x^2+1}$ the same as $\dfrac{1}{x+1}$? Try a few number to see and explain why or why not.

40. Is the expression $x^{-2}+3$ equal to $\dfrac{1}{x^2}+3$ *or* $\dfrac{3}{x^2}$? Try a few numbers to see.

41a. Is the expression $\dfrac{x^{-2}}{x}$ the same as $\dfrac{1}{x^3}$?

b. Is the expression $\dfrac{x^{-2}+1}{x}$ the same as $\dfrac{1}{x^3}$? $\dfrac{2}{x^3}$? Neither? Try a few numbers to see.

PART II ANSWERS

1. $2^{-4} = \dfrac{1}{2^4} = \dfrac{1}{16}$
2. $\dfrac{1}{9}$
3. 16
4. $2x^2$
5. $\dfrac{2x^2}{3}$ $(not\ 6x^2)$
6. $\dfrac{1}{2^6} = \dfrac{1}{64}$
7. $\dfrac{1}{x^3y^6}$
8. $\dfrac{1}{2x^2}$
9. $\dfrac{3}{x^2}$
10. $\dfrac{2}{5x}$
11. $4^2 = 16$
12. $\dfrac{1}{4^8}$
13. $3^4 = 81$
14. 2
15. 4
16. $\dfrac{x^3}{8}$
17. $\dfrac{25}{16}$
18. $\dfrac{2}{3x^3}$
19. $\dfrac{1}{x+y}$ *not* $\dfrac{1}{x}+\dfrac{1}{y}$
20. x
21. $\dfrac{1}{q^6}$
22. $6xy^3$
23. $\dfrac{1}{x}$
24. $\dfrac{1}{3xy}$
25. $\dfrac{1}{3x+y}$
26. $\dfrac{2y^4}{x^4}$
27. $\dfrac{1}{16x^5y}$
28. $\dfrac{1}{5}$
29. $\dfrac{1}{\frac{1}{2}+\frac{1}{3}} = \dfrac{1}{\frac{5}{6}} = \dfrac{6}{5}$
30. $\dfrac{1}{(x+y)^2}$ *or* $\dfrac{1}{x^2+2xy+y^2}$
31. $\dfrac{1}{2x^2}$
32. x^{-2} *or* $\dfrac{1}{x^2}$
33. $-2x^{-4} = \dfrac{-2}{x^4}$
34. $\dfrac{2}{x^4}$
35. $\dfrac{1}{4x^4}$
36. $\dfrac{1}{4x^4}$
37. $-7\dfrac{8}{9}$ *or* $\dfrac{-71}{9}$
38. -3

Unit 6 Handout #2: Rational (Fractional) Exponents

Part I:

1. Rewrite in radical notation:

$a.\ 3^{1/4}$

$b.\ 4^{5/3}$

$c.\ x^{1/3}$

$d.\ 2x^{3/4}$

$e.\ (2x)^{4/3}$

2. Rewrite in exponential notation

$a.\ \sqrt{14}$

$b.\ \sqrt[3]{5}$

$c.\ \sqrt[4]{7^3}$

$d.\ \left(\sqrt[4]{7}\right)^3$

$e.\ \sqrt[4]{2x^3}$

3. Evaluate the following **without** your calculator. Some may be impossible in the real number system:

a. $4^{1/2}$ b. $9^{3/2}$ c. $8^{2/3}$ d. $\sqrt[3]{-8}$

e. $-8^{2/3}$ f. $(-8)^{2/3}$ g. $(-4)^{3/2}$

4. Rewrite without negative exponents and without fractional exponents (use radicals instead)

a. $x^{-1/2}$ b. $(x+2)^{1/3}$ c. $3x^{-1/3}$ d. $(3x)^{-1/3}$ e. $x^{-3/2}$ f. $(x^2+9)^{-1/2}$

5. Evaluate the following **without** your calculator. Some may be impossible in the real number system:

a. $9^{-1/2}$ b. $(-9)^{1/2}$ c. $8^{-2/3}$ d. $(-27)^{-2/3}$

e. $\left(\frac{1}{4}\right)^{1/2}$ f. $\left(\frac{1}{4}\right)^{-1/2}$ g. $\left(-\frac{1}{4}\right)^{1/2}$

6. Evaluate the following without your calculator. Some may be impossible in the real number system:

a. $\left(\frac{1}{25}\right)^{-1/2}$ b. $\frac{1}{4^{-3/2}}$ c. $\left(\frac{3}{5}\right)^{-2}$ d. $\left(\frac{9}{4}\right)^{-1/2}$

e. $\left(\frac{8}{27}\right)^{-2/3}$ f. $(3+13)^{-1/2}$ g. $(2^{1/2})^4$ h. $\left(2^4\right)^{1/2}$

i. $64^{1/3}$ j. $64^{-1/2}$ k. $\sqrt[5]{0}+\sqrt[4]{1}$

7. Combine the fractional exponents when the bases are the same, where possible.

a. $3^{1/3}\cdot 3^{1/2}$ b. $3^{1/3}+3^{1/2}$ c. $3^{3/4}\cdot\sqrt[4]{3}$ d. $\frac{13^{3/4}}{13^{1/2}}$

e. $\frac{7^{1/2}}{7^{1/3}}$ f. $\frac{8}{\sqrt[5]{8^2}}$ g. $\frac{2x^{2/3}}{6x^{1/3}}$

Part II

1. Simplify the following. Use the rules of exponents (they apply to fractional exponents too). Variables or numbers with the same base should have exponents combined, when possible.

a. $\frac{x^8}{x^3}$ b. $\frac{x^1}{x^{1/3}}$ c. $\sqrt{x}\cdot x^{1/3}$ d. $\left(x^{1/3}\right)^6$ e. $\left(\sqrt{3x}\right)^4$

2. True/false. Which of the following are true? Test your answer by choosing a value for x and seeing if the statement holds for that value. Hint: exactly 4 of them are true!

a. $(x+2)^2 = x^2 + 4$

b. $(x-3)^{-1} = \frac{1}{x} - \frac{1}{3}$

c. $2^x \cdot 2^y = 2^{xy}$

d. $\left(\sqrt{x}\right)^4 = x^2$ (for x>0)

e. $\frac{3^{2x}}{3^x} = 3^x$

f. $(3 \cdot 2^x)^2 = 9 \cdot 2^{2x}$

g. $\sqrt{x^{36}} = x^6$

h. $\frac{x^2}{x^{-1}+3} = \frac{x^3}{3}$

i. $(x+5)^{-1/2} = \frac{1}{\sqrt{x+5}}$

3. Evaluate without a calculator:

a. 1^{-3}

b. $(-1)^{-3}$

c. $(-1)^{-1/3}$

d. $(-8)^{2/3}$

e. $\left(\frac{2}{5}\right)^{-2}$

f. $(3^2 + 4^2)^{-1/2}$

g. $\left(\frac{1}{9}\right)^{-3/2}$

h. 6^{-2}

i. $\left(\sqrt{7}\right)^{-2}$

4. Simplify. Combine like terms when possible and write without negative exponents.

a. $\frac{5x^{-2}}{25x^{-1}}$

b. $\frac{1}{2xy^{-2}} \cdot (-x^2 y)^2$

c. $\left(\frac{-2x^2}{3y}\right)^{-3}$

d. $\frac{2x^{-3}}{(-2x)^{-2}}$

e. $(x+1)^{-2}$

f. $\left[(3x)^{-1} + 3x^{-1}\right]^{-2}$

g. $(-2x)^3 - 2x^3$

h. $\frac{6x^2 y}{xy^{-1}} \cdot (3x)^{-2}$

i. $(-3x)^3$

j. $\left[(2x-y)^7 + 19\right]^0$

5. Solve the following equations. It is usually easiest to isolate the term with the *x* and take the opposite (reciprocal) power. Find all **real** solutions only. Think about when there may be more than one solution (or no solutions).

a. $x^2 = 6$

b. $(x-1)^{2/3} = 4$

c. $(x+1)^{1/3} = -2$

d. $\dfrac{(x+1)^3}{9} = -3$

e. $(x-3)^{1/2} = 4$

f. $(x-3)^2 = -4$

g. $3(x-1)^{3/2} + 1 = 25$

h. $x^{-2} = 16$

i. $(2x-1)^{-1/2} + 1 = 4$

j. $(x-3)^{-1/2} = \dfrac{1}{5}$

k. $3(x-2)^2 + 5 = 29$

l. $|x| = 5$

m. $2|x-3| - 1 = 9$

n. $\dfrac{1}{2}(x-4)^{-3/2} = 4$

6. Solve by taking roots with your calculator. Watch out for where there may be two answers (or no answers). Also be careful with parentheses. Your answers should be rounded to two decimal places.

a. $x^7 = 18$ b. $x^4 = 18$ c. $2x^5 - 6 = 196$ d. $(x-2)^8 = 2198$

7. Solve by factoring out a monomial.

a. $3\sqrt{x} - 5x = 0$ b. $4x^{2/3} - 9x^{4/3} = 0$ c. $2(x-1) - 3\sqrt{x-1} = 0$

d. $\sqrt{x} - \dfrac{3}{\sqrt{x}} = 0$ e. $x\sqrt{3x} - 2\sqrt{x} = 0$ f. $\left(x^{2/3} - 4\right)\left(x^{3/2} - 1\right) = 0$

8. Find the intersections, if any, between the pairs of graphs:

a. $f(x) = 2x^{2/3}$ *and* $g(x) = 4x$ b. $f(x) = x^{-2}$ *and* $g(x) = \dfrac{8}{\sqrt{x}}$

c. $f(x) = 2\cdot\sqrt[3]{x}$ *and* $g(x) = \sqrt{x}$

d. $f(x) = x + 2$ *and* $g(x) = 3\sqrt{x}$ (quadratic form)

Part I ANSWERS

1a. $\sqrt[4]{3}$ b. $\sqrt[3]{4^5}$ *or* $\left(\sqrt[3]{4}\right)^5$ c. $\sqrt[3]{x}$ d. $2\cdot\sqrt[4]{x^3}$ e. $\sqrt[3]{(2x)^4}$ *or* $\left(\sqrt[3]{2x}\right)^4$

2a. $14^{1/2}$ b. $5^{1/3}$ c. $7^{3/4}$ d. $7^{3/4}$ e. $(2x^3)^{1/4}$ *NOT* $(2x)^{3/4}$

3a. 2 b. 27 c. 4 d. -2 e. -4 f. 4 g. no real number

4a. $\dfrac{1}{\sqrt{x}}$ b. $\sqrt[3]{x+2}$ c. $\dfrac{3}{\sqrt[3]{x}}$ d. $\dfrac{1}{\sqrt[3]{3x}}$ e. $\dfrac{1}{\sqrt{x^3}} = \dfrac{1}{x\sqrt{x}}$ f. $\dfrac{1}{\sqrt{x^2+9}}$

5a. $\dfrac{1}{3}$ b. no real number c. $\dfrac{1}{4}$ d. $\dfrac{1}{9}$ e. $\dfrac{1}{2}$ f. 2 g. no real number

6a. 5 b. 8 c. $\dfrac{25}{9}$ d. $\dfrac{2}{3}$ e. $\dfrac{9}{4}$ f. $\dfrac{1}{4}$ g. 4 h. 4 i. 4 j. $\dfrac{1}{8}$ k. 1

7a. $3^{5/6}$ b. can't add: not like terms c. 3 d. $13^{1/4}$ e. $7^{1/6}$ f. $8^{3/5}$ g. $\dfrac{x^{1/3}}{3}$

Part II ANSWERS

1a. x^5 b. $x^{2/3}$ c. $x^{5/6}$ d. x^2 e. $9x^2$ 2a. false b. false c. false d. true e. true f. true g. false (its x^{18}) h. false i. true

3a. 1 b. -1 c. -1 d. 4 e. 25/4 f. 1/5 g. 27 h. 1/36 i. 1/7

4a. $\dfrac{1}{5x}$ b. $\dfrac{x^3y^4}{2}$ c. $\dfrac{-27y^3}{8x^6}$ d. $\dfrac{8}{x}$ e. $\dfrac{1}{x^2+2x+1}$ f. $\dfrac{9x^2}{100}$ g. $-10x^3$ h. $\dfrac{2y^2}{3x}$ i. $-27x^3$ j. 1

5a. $\pm\sqrt{6}$ b. 9, -7 c. -9 d. -4 e. 19 f. not real g. 5 h. ±¼ i. 5/9 j. 28 k. $2\pm2\sqrt{2}$ l. ±5 m. -2, 8 n. 17/4

6a. 1.51 b. ±2.06 c. 2.52 d. 4.62, -0.62

7a. 0, 9/25 b. 0, 8/27, -8/27 c. 1, 13/4 d. 3 (not 0) e. 0, $2/\sqrt{3}$ f. 8, -8, 1

8a. 0, 1/8 b. ¼ (-1/4 is extraneous) c. 0, 64 d. 1, 4

Unit 6 Handout #3: Properties of Rational Exponents

General Hints with Exponents

1. When multiplying terms:
   - if terms have same base → look to combine exponents
   - if terms have same exponent → look to combine bases
   - if term have same both → combine either
   - if terms have same neither → don't combine
2. Be wary of addition & subtraction when applying rules of exponents!
3. Test numbers when unsure.
4. Add or subtract like terms only.
5. Use fractional exponents instead of radicals when applying rules of exponents.
6. Use parentheses to break big problems into smaller problems.
7. Write exponents of "1" in places where you are unsure which exponent applies where.

Part I: Fractional Exponents. Do not evaluate, but write as one number to a given power. Acceptable answers are things like $5^{1.5}$ *or* $7^{-1/6}$. Negative exponents are OK for this part. You may use radical notation if you like. **Hint: I recommend that you rewrite all radicals as fractional exponents before you simplify.**

1. $\left(5^{1/2}\right)^3$

2. . $\dfrac{10}{\sqrt[3]{10}}$

3. $3^2 \cdot (\sqrt{3})^{-3}$

4. $x^{1/4} \cdot \sqrt{x}$

5. $\left(2\sqrt{x}\right)^4$

6. $\left(\sqrt{2x}\right)^4$

7. $\dfrac{4x^{1/2}}{8x^{-1}}$

8. $\dfrac{(4x)^{1/2}}{(8x)^{-1}}$

9. $\dfrac{6x}{\sqrt[5]{x}}$

10. $(\sqrt[3]{x})^{-6}$

Part II: Simplifying radicals. You should take all perfect-power factors out. So if you are taking a square root, make sure you have no perfect square factors underneath the radical sign. If you are taking a cube root, make sure you have no perfect cube factors underneath the radical sign. **Your final answers should be in radical form: no fractional exponents!**

11. $2\sqrt{27}$

12. $\sqrt{\dfrac{1}{72}}$

13. $\sqrt[3]{-40}$

14. $\sqrt[3]{16}$

15. $\sqrt{x^5}$

16. $2x\sqrt{x^3}$

17. $\sqrt[3]{8x^3}$

18. $\left(54x^5\right)^{1/3}$

19. $\left(\sqrt{100x^2}\right)^{-2}$

20. $\sqrt[3]{16x^7}$

Part III. Multiplying and dividing radicals. If two numbers (or terms) are raised to the same power, then you can combine the terms. Do this, and then simplify. **I recommend combining terms before simplifying individual terms.**

21. $\sqrt{2}\cdot\sqrt{8}$

22. $(3\sqrt{2})^2$

23. $\sqrt{\dfrac{5}{2}}\cdot\sqrt{\dfrac{5}{8}}$

24. $2\sqrt{3}\cdot 5\sqrt{12}$

25. $\dfrac{\sqrt{80}}{\sqrt{10}}$

26. $\dfrac{\sqrt[3]{16}}{\sqrt[3]{-2}}$

27. $2\sqrt[3]{9}\cdot 3\sqrt[3]{3}$

28. $5\sqrt[3]{7}\cdot 3\sqrt[3]{2}$

29. $3\sqrt[3]{4}\cdot 2\sqrt[3]{2}$

30. $\left(\sqrt[3]{7}\right)^6$

31. $2\sqrt{x}\cdot 6\sqrt{x}$

32. $\sqrt{2x}\cdot\sqrt{4x}$

33. $\sqrt[3]{4x^2}\cdot\sqrt[3]{2x^2}$

34. $\dfrac{\sqrt[3]{16x^4}}{\sqrt[3]{x}}$

35. $\left(2\sqrt{x}\right)^4$

36. $\left(-3x\sqrt{x}\right)^2$

Part IV: Adding and subtracting radicals. **<u>You can only add or subtract like term</u>**. Try to simplify each term individually and then combine them if they are like terms. You can multiply or divide terms that are not like. Some questions below may not simplify into like terms.

37. $2\sqrt{2}+3\sqrt{2}$

38. $\sqrt{20}-\sqrt{5}$

39. $\sqrt{48}-\sqrt{12}$

40. $2\sqrt[3]{3}+5\sqrt[3]{3}$

41. $2\sqrt{x}-5\sqrt{x}$

42. $\sqrt{12x}-\sqrt{27x}$

43. $\sqrt{2x}-5\sqrt{2x}$

44. $2x\sqrt{x}+\sqrt{9x^3}$

Part V: Combined Operations. These require using the distributive property as well as simplifying radicals; they may also involve FOIL-ing.

45. $\sqrt{2}(3\sqrt{2}-5)$ 46. $(5-2\sqrt{2})^2$ 47. $\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)$

48. $(1+\sqrt{2})(2-\sqrt{2})$ 49. $\left(2\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{12}\right)$ 50. $\sqrt{3x}\left(\sqrt{6}-2\sqrt{12x}\right)$

51. $\left(6-\sqrt{x}\right)\left(6+\sqrt{x}\right)$ 52. $\left(2\sqrt{x}-1\right)^2$

Answers

1. $5^{3/2}$ 2. $10^{2/3}$ 3. $3^{1/2}$ 4. $x^{3/4}$ 5. $16x^2$ 6. $4x^2$ 7. $0.5x^{1.5}$ *or* $\frac{x^{1.5}}{2}$

8. $16x^{3/2}$ 9. $6x^{4/5}$ 10. x^{-2} *or* $\frac{1}{x^2}$ 11. $6\sqrt{3}$ 12. $\frac{1}{6\sqrt{2}}$ or $\frac{\sqrt{2}}{12}$

13. $-2\sqrt[3]{5}$ 14. $2\sqrt[3]{2}$ 15. $x^2\sqrt{x}$ 16. $2x^2\sqrt{x}$ 17. $2x$ 18. $3x\cdot\sqrt[3]{2x^2}$

19. $\frac{1}{100x^2}$ 20. $2x^2\cdot\sqrt[3]{2x}$ 21. 4 22. 18 23. $\frac{5}{4}$ 24. 60 25. $2\sqrt{2}$

26. -2 27. 18 28. $15\sqrt[3]{14}$ 29. 12 30. 49 31. $12x$ 32. $2x\sqrt{2}$

33. $2x\cdot\sqrt[3]{x}$ 34. $2x\cdot\sqrt[3]{2}$ 35. $16x^2$ 36. $9x^3$ 37. $5\sqrt{2}$ 38. $\sqrt{5}$ 39. $2\sqrt{3}$

40. $7\sqrt[3]{3}$ 41. $-3\sqrt{x}$ 42. $-\sqrt{3x}$ 43. $-4\sqrt{2x}$ 44. $5x\sqrt{x}$ 45. $6-5\sqrt{2}$

46. $33-20\sqrt{2}$ 47. 2 48. $\sqrt{2}$ 49. $4-3\sqrt{15}$ 50. $3\sqrt{2x}-12x$

51. $36-x$ 52. $4x-4\sqrt{x}+1$

Unit 6 Handout #4: Harder Problems

1. Given that $10^{0.3}$ is very very close to 2 (check on your calculator if you like!), find the following without using your calculator. All should be do-able using the properties of exponents. There's one part where you can get different answers using different methods, demonstrating that $10^{0.3}$ is not 2.

a. $10^{0.6}$ b. $10^{-0.3}$ c. $10^{-0.9}$

d. $10^{1.3}$ e. $10^{2.3}$ f. $10^{-0.7}$

g. $10^{-0.1}$ h. $10^{0.2}$ i. $10^{-2.1}$ j. $\sqrt{10}$

2. Simplify the following. Your answers should have no negative exponents.

a. $\dfrac{2x^2y^{-2}}{(2x)^{-2}y}$ b. $\sqrt{x^{4/3}}$ c. $\left((2x)^{-1}+4\cdot(2x)^{-1}\right)^{-2}$

3. Determine whether each of the following statements is true or false.

a. $(x+4)^{1/2}=\sqrt{x}+\sqrt{4}$ b. $x^{1/2}+x^{3/2}=x^2$

c. $(x-5)^2(x-5)^4=(x-5)^6$ d. $(2x)^2\cdot(2x)^3=(4x^2)^5$

e. $\dfrac{(2x-1)^3}{(2x-1)^6} = \dfrac{1}{2}$

f. $(x+2)\sqrt{x+2} = (x+2)^{3/2}$

g. $3 \cdot 2^x = 6^x$

h. $\sqrt{x^{16}} = x^4$

i. $\dfrac{(x-3)}{\sqrt{x-3}} = \sqrt{x-3}$

j. $\left(3 \cdot 2^a\right)^2 = 9 \cdot 2^{2a}$

4. Answer the following questions without using your calculator.

a. If $\sqrt{x\sqrt{x}} = 8$ then what is x?

b. If $\sqrt[3]{x} = a$ then what is x^3?

c. If $x^{16} = 16$ then what is x^8?

d. If $x^{-8} = 8$ then what is x^{16}?

e. If $x^{-4} = 4$ then what is x^2?

Answer the following multiple-choice questions. **More than one answer may be correct: circle all answers that apply.** **They are hard—think carefully!** You may test with actual numbers if it helps.

5. $x^{-1/2}$ can also be written as:

a. x^2 b. $\sqrt{\frac{1}{x}}$ c. $\frac{x^{-1}}{x^2}$ d. $\frac{1}{\sqrt{x}}$ e. $\frac{-1}{x^2}$

6. $2^a \cdot 2^b$ is the same as:

a. 2^{ab} b. 4^{ab} c. $2^a + 2^b$ d. 2^{a+b} e. 4^{a+b}

7. $(x^2 - 4)^{1/2}$ is the same as:

a. $\frac{1}{\sqrt{x^2 - 4}}$ b. $\sqrt{x^2 - 4}$ c. $x - 2$ d. $(x^2 - 4)^{-2}$ e. $x + 2$

8. $\frac{7^x}{7}$ can also be written as:

a. 1^x b. x c. 7^x d. 7^{x-1} e. $\frac{1}{7} \cdot 7^x$

9. $\frac{1}{2 \cdot \sqrt[3]{x}}$ is the same as:

a. $2x^{1/3}$ b. $2^{-1}x^{-1/3}$ c. $(2x)^{-1/3}$ d. $2x^{-1/3}$ e. $0.5 \cdot x^{-1/3}$

10. $(x + y)^{-1/2}$ is the same as:

a. $\frac{1}{\sqrt{x} + \sqrt{y}}$ b. $\sqrt{\frac{1}{x} + \frac{1}{y}}$ c. $x^{-1/2} + y^{-1/2}$ d. $\frac{1}{\sqrt{x + y}}$ e. $(x + y)^2$

11. $\frac{8^{80}}{8^{40}}$ is the same as:

a. $\frac{80}{40}$ b. 8^{40} c. 8^2 d. 1^2 e. 1^{40}

12. $\frac{1}{-\sqrt{2x}}$ is the same as:

a. $(2x)^{-1/2}$ b. $(-2x)^{-1/2}$ c. $-2x^{-1/2}$ d. $-(2x)^{-1/2}$ e. $(2x)^{1/2}$

13. $\sqrt{25^{16}}$ is the same as:

a. 25^4 b. 25^8 c. 5^4 d. 5^8 e. 5^{16}

14. 4^{x-1} may also be written as:

a. $\dfrac{4^x}{4}$ b. 1^x c. $0.25 \cdot 4^x$ d. $\dfrac{1}{4^{1-x}}$ e. 2^{2x-1}

15. $(-2)^{-x}$ is the same as:

a. $\dfrac{1}{-2^x}$ b. $\dfrac{1}{(-2)^x}$ c. $\dfrac{-1}{2^x}$ d. 2^x e. $(-2)^{1/x}$

16. $6^{x^2} \cdot 6^x$ is the same as:

a. 6^{x^2+x} b. 6^{x^3} c. 36^{x^3} d. 6^{3x} e. none of these

17. $\dfrac{2^{2x}}{2^x}$ is the same as:

a. 2 b. 2^2 c. 2^x d. 1^x e. x

18. $3^x \cdot 3^x$ is the same as:

a. 3^{2x} b. 3^{x^2} c. 9^{2x} d. $2 \cdot 3^x$ e. 9^x

19. $\dfrac{1}{a^{-4/5}}$ is the same as:

a. $-a^{4/5}$ b. $a^{5/4}$ c. $a^{-5/4}$ d. $a^{4/5}$ e. $a^{-4/5}$

20. Write each as a linear combination of powers of x (Something like $2x^{3/2} + 4x^{-1/2}$)

a. $5x\sqrt{x} - \dfrac{3}{2\sqrt{x}} - \dfrac{1}{5x^2}$ b. $(x^{-1} + x^{1/2})(2x + 3x^{5/2})$

21. Rewrite the function $f(x) = 3^{2x-4}$ in the form $f(x) = a \cdot b^x$

22. Answer the following questions about $f(x) = x^2 + 4^x$.

a. What is $f(2)$?

b. What is $f(-1)$?

c. What is $f(-2)$?

d. What is $f(0)$?

e. What is $f(-0.5)$?

f. What is $f(1.5)$?

23. Answer the following questions about $f(x) = x^{1/2} + \left(\frac{1}{2}\right)^x$.

a. What is $f(0)$?

b. What is $f(4)$?

c. What is $f(1)$?

d. What is $f(-4)$?

24. A basic power function is in the form $f(x) = a \cdot x^b$. Without using your calculator, write the equation of the power function given by each set of two points. Plug the points in and solve the system for *a* and *b*. While we have not done the algebra for such equations, you'll be able to see the answers!

a. (1,-2) and (3,-54)

b. (4,32) and (9,108)

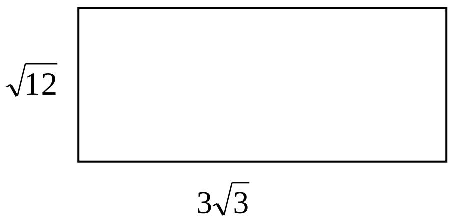
c. (2,-2) and (4,-8)

d. (4,3) and (9,2)

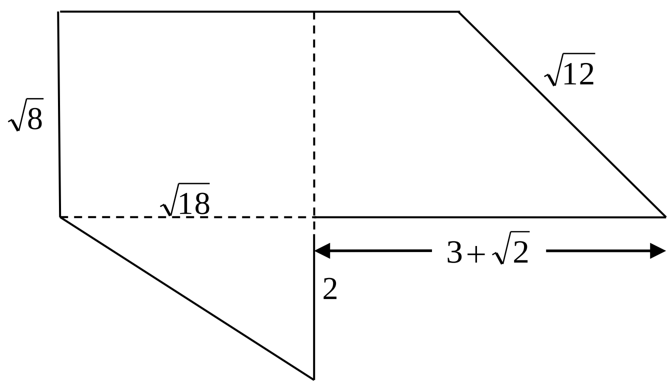
25. A piece of wire is $24+\sqrt{192}$ units long. It is cut into two equal pieces. One is folded to form the perimeter of a square; the other is bent to form a circle. What is area of each shape (in simplest radical form)? Using your calculator to approximate π, which object has a larger area?

26. The expression $\sqrt{2\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}}$ is equal to 2^a. What is a?

27. Given the following rectangle, find its area, perimeter, and the length of its diagonal. Simplify your answers fully.



28. Find the area and perimeter of the object below. The dashed lines are for reference purposes only and are not part of the perimeter. The diagram is not necessarily to scale.



29. The surface area of a cube is 108. Find the following:
    a. Its volume

    b. The total length of all of its edges

    c. The length of the diagonal of a face

    d. The length of the longest line segment that will fit inside the cube.

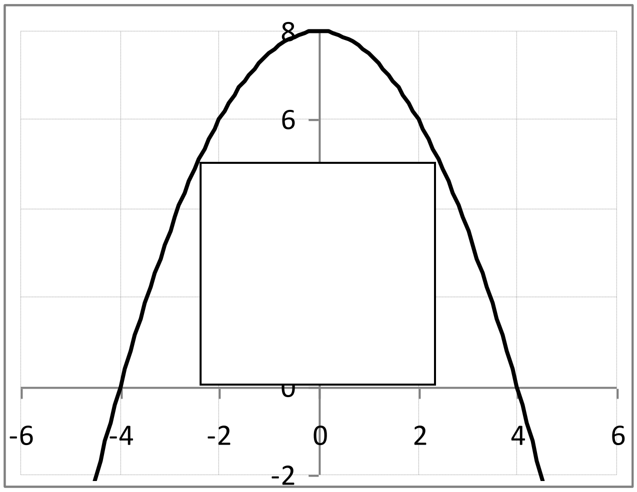
    e. The area of the largest circle that will fit on one of its faces.

    f. The area of a circle on one of the cube's faces leaving 1 unit from all edges.

    g. The area of an equilateral triangle where one side is a full edge of one of the cube's faces.

    h. The volume of the largest sphere that will fit inside the cube ($V = \frac{4}{3}\pi r^3$)

30. The graph below shows the function $f(x) = 8 - 0.5x^2$. A square is between the graph and the *x*-axis, with one side on the *x*-axis and the other two corners on the graph. What is the area of the square (in simplest radical form)?



Answers

1a. 4 b. ½ c. 1/8 d. 20 e. 200 f. 1/5 g. 0.8 h. 1.6 i. 0.008 or 1/128 j. 3.2

2a. $\frac{8x^4}{y^3}$ b. $x^{2/3}$ c. $\frac{4x^2}{25}$ 3. c, f, i, and j are true 4a. 16 b. a^9 c. 4 (not -4)!? d. 1/64 e. ½

5. b,d 6. d 7. b 8. d,e 9. b, e 10. d 11. b 12. d 13. b, e 14. a, c, d

15. b 16. a 17. c 18. a, e 19. d

20a. $5x^{3/2} - 1.5x^{-1/2} - 0.2x^{-2}$ b. $2 + 5x^{3/2} + 3x^3$ 21. $f(x) = \frac{1}{81} \cdot 9^x$

22a. 20 b. 5/4 c. 65/16 d. 1 e. ¾ f. 41/4 23a. 1 b. 33/16 c. 3/2 d. undefined

24a. $f(x) = -2x^3$ b. $f(x) = 4x^{3/2}$ c. $f(x) = -0.5x^2$ d. $f(x) = 6x^{-1/2}$

25. circle has larger area – most efficient shape (largest area for a given perimeter)

Square's area is $12 + 6\sqrt{3}$; circle's area is $\frac{48 + 24\sqrt{3}}{\pi}$ 26. 31/32

27. area=18; perim=$10\sqrt{3}$ and diag is $\sqrt{39}$ 28.perim is $7\sqrt{2} + \sqrt{22} + 6 + 2\sqrt{3}$; area is $16 + 7\sqrt{2}$

29a. $54\sqrt{2}$ b. $36\sqrt{2}$ c. 6 d. $3\sqrt{6}$ e. 4.5π f . $5.5\pi - 3\pi\sqrt{2}$ g. $\frac{9\sqrt{3}}{2}$ h. $9\pi\sqrt{2}$

30. side is $-2 + 2\sqrt{5}$ so area is $24 - 8\sqrt{5}$

Unit 7 Handout #1: Domains and Combining Functions

Part I: Function Domains
A function's domain is the set of possible input values. We can find the domain of a function graphically or algebraically:

Graphically: look at what *x* values correspond to points on the graph.
Algebraically: it is easiest to look for exceptions to the domain. In the real number system, the domain exceptions we will see are either from taking the **square root** (or any even root) **of a negative number** or **dividing by zero**.

Problems
1. Find the domain of the following functions.

a. $f(x)=\sqrt{x-5}$

b. $f(x)=(2x-7)^{1/2}$

c. $f(x)=\dfrac{7}{x-2}$

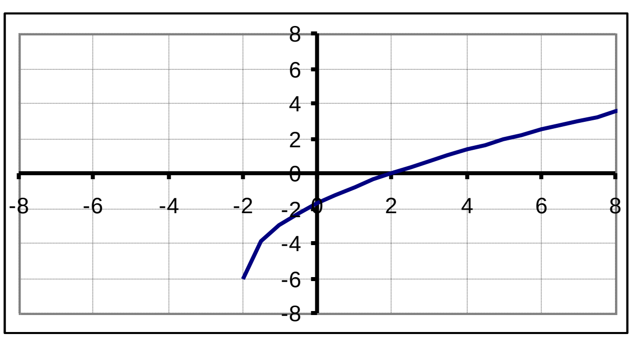
d. $f(x)=(x+3)^{-1}$

e. $f(x)=\dfrac{1}{x^2-4x}$

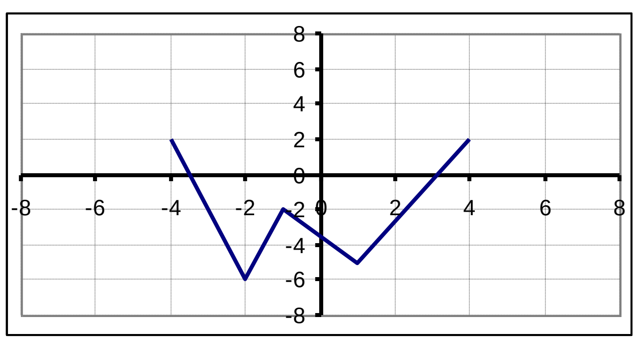
f. $f(x)=\sqrt{x^2-6x}$

2. Find the domain of the functions graphed below. Assume any part leaving the viewing window continues forever:

a.



b.



Part II: Combining Functions, Part I
You can combine functions by adding, subtracting, multiplying, and dividing them. The notation is as follows:

$(f+g)(x) = f(x)+g(x)$	adding functions is just adding outputs for each input
$(f-g)(x) = f(x)-g(x)$	subtracting functions is subtracting outputs for each inputs
$(f\cdot g)(x) = f(x)\cdot g(x)$	multiply the outputs for each inputs
$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$	divide the outputs for each input (as long as $g(x)\neq 0$)

Problems

3. Suppose $f(x)=x^2$ and $g(x)=x+6$. Find the following:

a. $f(3)+g(7)$

b. $(f+g)(6)$

c. $(f-g)(3)$

d. $(f\cdot g)(5)$

e. $\left(\frac{g}{f}\right)(4)$

f. $(f\cdot g)(x)$ and its zeros

g. $(f-g)(x)$ and its zeros

h. Suppose $h(x)=\frac{f(x)}{g(x)}$. What is the domain of $h(x)$?

4. Let $f(x)=\sqrt{2x}$ and $g(x)=\sqrt{8x}$. Answer the following questions. **Simplify all answers fully**.

a. What are the domains of $f(x)$ *and* $g(x)$?

b. What is $(f+g)(3)$?

c. What is $(f-g)(5)$?

d. What is $\left(\frac{g}{f}\right)(6)$?

e. What is the rule for $(f+g)(x)$?

f. What's the rule for $(f\cdot g)(x)$? What's the domain?

Part III. Expressions as inputs into functions

The function $f(x)=x^2$ is a function that squares its input to make it output. Here are some sample inputs and outputs

$$f(-5)=(-5)^2=25$$
$$f(x)=(x)^2=x^2$$
$$f(x+1)=(x+1)^2=x^2+2x+1$$
$$f(2x-5)=(2x-5)^2=4x^2-20x+25$$

Problems

5. For $f(x)=\sqrt{x}$, compute the following. Simplify your answers. No negative exponents.

a. $f(x^{16})$ b. $f(4x^2)$ c. $f(9x)$ d. $f(16x^{-2})$ e. $f(x^2+9)$

6. For $f(x)=x^2-2x$, compute the following (and simplify):

a. $f(x+2)$ b. $f(-3x)$ c. $f(2x^3)$ d. $f(x^{-2})$ e. $f(3x^{-1})$

7. For $f(x)=3x^{-2}$, compute the following (and simplify):

a. $f(-4)$ b. $f(\frac{3}{2})$ c. $f(4\sqrt{x})$ d. $f(x-5)$ e. $f(\frac{3}{\sqrt{x}})$

8. For the function $f(x)=x-3$, is there a difference between $f(x^2)$ *and* $\left(f(x)\right)^2$? Explain.

Part IV: Composite Functions

As in the example above, a composite function involves using the output of one function as the input of another function.

Notation: $g(f(x))$ means the output from f(x) is the input into g(x). Other notation is $(g \circ f)(x)$.

$(f \circ g)(x) = f(g(x))$ has the output of g(x) as the input of f(x).

Think of $g(f(x))$ *or* $(g \circ f)(x)$ as an assembly line, where the output from one part is the input to the next. **Evaluate from the inside out!**

$$x \longrightarrow \boxed{f} \longrightarrow f(x) \longrightarrow \boxed{g} \longrightarrow g(f(x))$$

Problems

9. Given $f(x) = x^2 - x$ and $g(x) = 2x - 3$ and $h(x) = \sqrt{x}$ find the following:

a. $(f \circ g)(-1)$

b. $(g \circ f)(4)$

c. $(f \circ g)(x)$

d. $(g \circ f)(x)$

e. $(g \circ g)(2)$

f. $(g \circ g)(x)$

g. $(f \circ h)(9)$

h. $(h \circ h)(4)$

i. $(f \circ h)(x)$

j. $(h \circ h)(x)$

10. Suppose $f(x) = x^2$ and $g(x) = x - 4$.

a. Find $(f \circ g)(5)$ *and* $(g \circ f)(5)$.

b. Find $(f \circ g)(x)$ *and* $(g \circ f)(x)$.

c. Solve the equation $f(g(x)) = 9$.

11. The radius of a circle starts at 4 cm and grows at 3 cm per minute.

a. Write a function $r(t)$ that relates the radius of the circle to time (in minutes).

b. Write a function $A(r)$ that relates the area of a circle to its radius.

c. Define the function $(A \circ r)(t)$.

d. What units are the input and output to the function $(A \circ r)(t)$? What meaning does this function have?

e. How long after the circle starts growing is its area equal to 625π?

12. Suppose $f(x) = 2x + 7$ and $g(x) = \dfrac{x-7}{2}$.

a. Find $f(g(x))$.

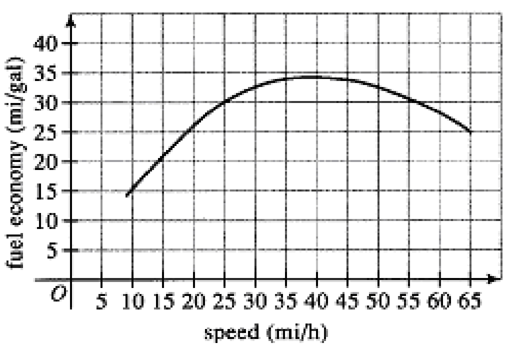
b. Find $g(f(x))$.

13. Functions f and g are defined by the graphs below. (adapted from *Advanced Mathematics* by R. Brown)

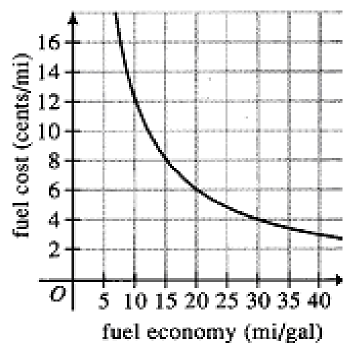
-Function f gives a car's fuel economy E as a function of the car's speed S. That is, $E = f(S)$.

-Function g gives the fuel cost per mile C as a function of fuel economy E. That is, $C = g(E)$.

Graph of f:



Graph of g:



a. If the car is driven at 55 mi/h, what is the fuel cost? Show how to get the answer, using the function names f and g.

b. If the fuel cost is 6 cents per mile then what speed is the car being driven?

c. If the fuel cost is to be kept at or below 4 cents per mile, at what speeds may the car be driven?

d. Based on the meanings of the variables, which composite function makes sense, $f \circ g$ or $g \circ f$? Explain why your choice makes sense, and the other choice doesn't.

14. The functions $F(x)$ *and* $M(x)$ are strange functions. Neither their inputs nor outputs are numbers. The function $F(x)$ gives the Father of person *x*, and the function $M(x)$ gives the mother of *x*.

a. Represent the maternal grandfather as a composition of the two functions.

b. Represent the paternal grandfather as a composition of the two functions.

c. Represent the paternal grandmother as a composition of the two functions.

ANSWERS

1a. x≥5 b. x≥3.5 c. x≠2 d. x≠-3 e. x≠0, 4 f. x≤0 or x≥6 2a. x≥-2 b. -4≤x≤4

3a. 22 b. 48 c. 0 d. 275 e. 5/8 f. x^3+6x^2 zeros are -6 & 0

g. x^2-x-6 zeros are 3, -2 h. x≠6

4a. x≥0for both b. $\sqrt{6}+\sqrt{24}=3\sqrt{6}$ c. $\sqrt{10}-\sqrt{40}=-\sqrt{10}$

d. 2 e. $3\sqrt{2x}$ f. $4x$ when x≥0

5a. x^8 b. 2x c. $3\sqrt{x}$ d. $\frac{4}{x}$ e. $\sqrt{x^2+9}$ which is NOT x+3!

6a. x^2+2x b. $9x^2+6x$ c. $4x^6-4x^3$ d. $\frac{1}{x^4}-\frac{2}{x^2}$ e. $\frac{9}{x^2}-\frac{6}{x}$

7a. 3/16 b. 4/3 c. $\frac{3}{16x}$ d. $\frac{3}{x^2-10x+25}$ e. $\frac{x}{3}$

8. $f(x^2)=x^2-3$ $[f(x)]^2=x^2-6x+9$

9a. 30 b. 21 c. $4x^2-14x+12$ d. $2x^2-2x-3$ e. -1

f. 4x-9 g. 6 h. $\sqrt{2}$ i. $x-\sqrt{x}$ j. $x^{1/4}$

10a. 1 and 21 b. $(x-4)^2$ *and* x^2-4 c. $(x-4)^2=9$ *so* 7 *or* 1

11a. $r(t)=4+3t$ b. $A(r)=\pi r^2$ c. $A(r(t))=A(4+3t)=\pi(4+3t)^2$

d. input time, output area: it shows the area at any time t e. $A(r(t))=\pi(4+3t)^2=625\pi$ t=7 min

12. both a and b are *x*

13a. $E(55)=30$ $C=g(30)=4\,cents/mile$ $C=g(f(55))$ b. 15 mph c. 25-55 mph d. $g\circ f$

14a. $(F\circ M)(x)$ b. $(F\circ F)(x)$ c. $(M\circ F)(x)$

Unit 7 Handout #2 Inverse Functions

1. Find the inverse of each of these functions. For parts a-c, verify your answers by finding $f(g(x))$ *and* $g(f(x))$

a. $f(x) = 4x - 11$

b. $f(x) = \frac{2}{3}x - 1$

c. $f(x) = 2(x-1)^3 + 4$

d. $f(x) = \frac{1}{2}\sqrt{x} - 5$

e. $f(x) = -\sqrt[5]{x-2} + 7$

f. $f(x) = 2(x-3)^2 - 6 \quad \textit{for } x \geq 3$

2. If $f^{-1}(3) = 7$ *and* $f^{-1}(7) = 4$, then what is the solution to $f(x) = 7$? Hint: think about inputs and outputs.

3. Draw a function whose inverse is not a function, if possible.

4. For the function $f(x)$, the domain is $x \geq -3$ and the range is $y \leq 4$. The y-intercept is 2 and the zero is 5. What does this tell us about the graph of the inverse function $f^{-1}(x)$?

5. Scientists studying alligators find that the weight (in pounds) is a function of the length (in feet) given by $W(L) = 0.2L^{3.1}$. (calculator OK)

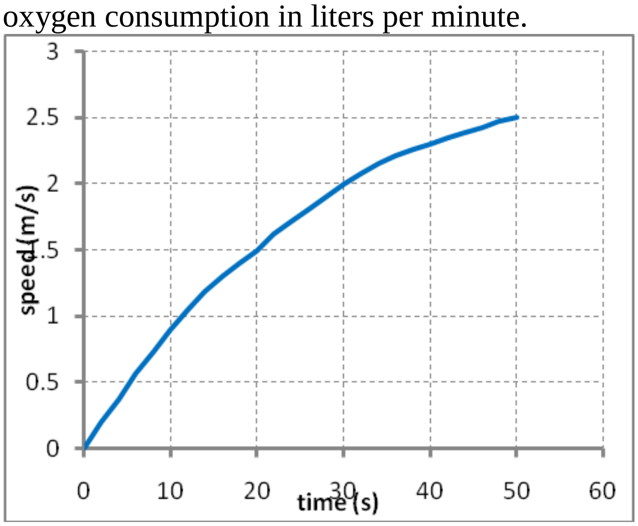
a. What is the meaning of $W^{-1}(L)$?

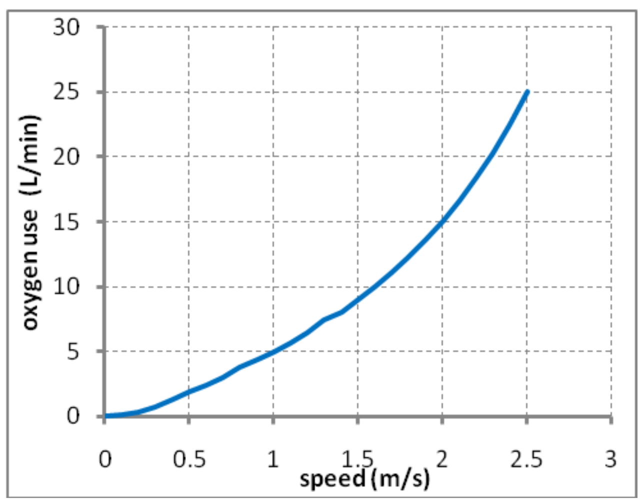
b. Find $W^{-1}(L)$.

c. What is $W(10)$ and what does it mean?

d. What is $W^{-1}(150)$ and what does it mean?

6. The graphs below show a swimmer's speed as a function of time and a swimmer's oxygen consumption as a function of speed. Time is measured in seconds, speed in meters per second, and oxygen consumption in liters per minute.





a. What is the speed after 20 seconds?

b. What is the oxygen consumption after 20 seconds?

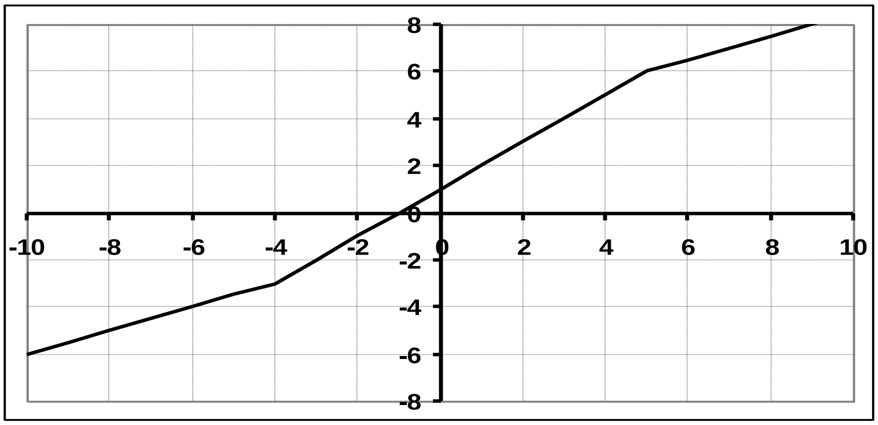
c. How many seconds have elapsed if the swimmer's oxygen consumption is below 15 L/min?

7. Give the meaning and units of the inverse function (assuming f is invertible).

a. $T = f(H)$ is the time to bake a cake at H degrees (Fahrenheit)

b. $N = f(t)$ total number of inches of snow having fallen in the first t days of January,

8. Use the graph of $f(x)$ below to evaluate each of the following expressions (if possible). The graph does NOT continue out of the window shown.



a. $f^{-1}(4)$ b. $(f \circ f^{-1})(-4)$ c. $(f^{-1} \circ f)(2)$ d. $(f^{-1} \circ f^{-1})(6)$

e. Draw a graph of $f^{-1}(x)$ on the grid above.

f. If $g(x) = 5 + \sqrt{1 + f(x)}$ then what are the domain and range of $g(x)$?

9. The function $b(t)$ gives the population of bluebirds in the park in year t. Describe what each of the following mean. Be sure to give units.

a. $b(2007) = 250$

b. $b^{-1}(180)$

c. $b^{-1}(b(2010))$ -- evaluate it and explain what it means

d. $b(2010) - b(2009) = 30$

10. The function $N(d) = 20 - \dfrac{90}{d+1}$ shows the number of 1000's of bacteria in a culture if the temperature is *d* degrees Celsius. The function $D(t) = 2t + 4$ shows the temperature (in degrees Celsius) at time t hours after noon.

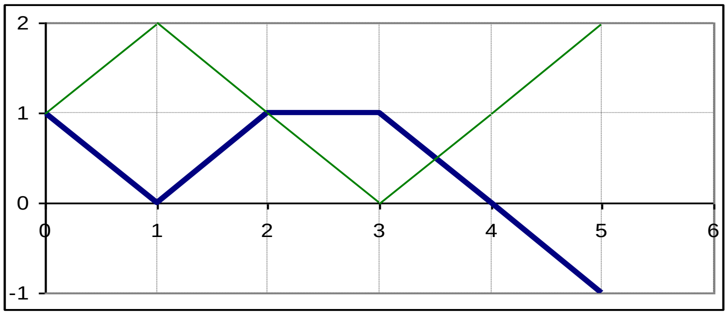
a. Which composition makes sense, $N \circ D \quad or \; D \circ N$? Explain briefly.

b. How many bacteria are in the culture after 4 hours? After 10 hours?

c. Write the rule for the composition that makes sense.

d. Using that rule, determine the time when the population is 14 thousand bacteria.

11. The graph below shows the functions $f(x)$ (darker) and $g(x)$. Use the graph to find the following. Some of these may be undefined. The entire domain of the functions are graphed here.



a. $(g \circ f)(2)$ b. $(f \circ f)(3)$ c. $(g \circ f)(5)$ d. $(f \circ g)(0)$

e. The zeros of $(f - g)(x)$ f. The zeros of $(f \cdot g)(x)$

g. The zeros of $(f \circ g)(x)$ *and* $(g \circ f)(x)$, if there are any. [hint: work backwards]

h. The zeros and domain of $(g / f)(x)$ i. The solution to $(f \cdot g)(x) < 0$

ANSWERS

1a. $f^{-1}(x) = \frac{x+11}{4}$ b. $f^{-1}(x) = \frac{3}{2}(x+1)$ c. $f^{-1}(x) = \sqrt[3]{\frac{x-4}{2}} + 1$

d. $f^{-1}(x) = (2x+10)^2$ e. $f^{-1}(x) = (-x+7)^5 + 2$ f. $f^{-1}(x) = \sqrt{\frac{x+6}{2}} + 3$

2. what input to f gives output of 7? Answer is 4.
3. anything that passes the vertical line test but not the horizontal line test
4. domain is $x \le 4$ range is $y \ge -3$ zero is 2 and y-int is at (0,5)
5a. input is weight, output is length (since W turns length into weight, inverse does opposite).
b. $W^{-1}(L) = (5L)^{1/3.1}$ *or* $(5L)^{0.323}$ c. $W(10) = 252$; 10 foot long gator weights about 252 lbs
d. $W^{-1}(150) = 8.46$; weight is 150 lbs then length is about 8.46 feet
6a. 1.5 mph b. ~9 L/min c. less than 30
7a. the temperature at which to bake the cake for it to take T time
b. The number of days in January in which it takes for N inches of snow to have fallen
8a.3 b. -4 c. 2 d. 4 f. $-2 \le x \le 9$ *and* $5 \le y \le 8$
9a. In 2007 there were 250 bluebirds in the park b. the year in which the population was 180 birds
c. b(2010) is the bluebird pop in 2010; b^{-1} of this is 2010, the year in which the population was what it was in 2010.
d. There were 30 more bluebirds in the park in 2010 than in 2009.
10a. $N \circ D$ since output of D is degrees which is input into N
b. 13,077 and 16,400 c. $(N \circ D)(t) = 20 - \frac{90}{2t+5}$ d. t=5
11a.2 b. 0 c. not defined d. 0 e. 0, 2, 3.5 f. 1, 3, 4 g. $(f \circ g)(x)$ is 0, 2, 4 $(g \circ f)(x)$ has none
h. domain is $0 \le x \le 5$ except for 1&4; zeros is 3 i. 4<x≤5

Unit 7 Handout #3: Graphs of Square-Root and Cube-Root Functions

1. Sketch a graph of each of these functions on the grids below. Label the coordinates of the vertex and of any one other point. Give the domain and range and find the y-intercept and zeros algebraically.

a. $f(x) = -2\sqrt{x} + 4$

b. $f(x) = \sqrt{x-3} - 2$

c. $f(x) = -0.5\sqrt{x+4}$

d. $f(x) = 2\sqrt{x+4} - 6$

e. $f(x) = -(x+1)^{1/2} - 2$

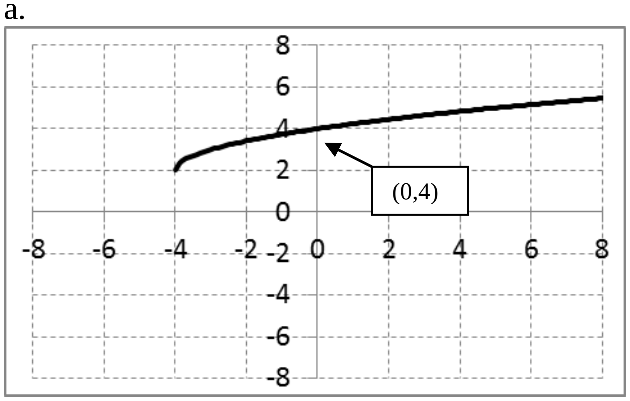
f. $f(x) = \sqrt{4-x} + 2$

2. Sketch a graph of $f(x) = x^3$; use this to graph its inverse $f(x) = \sqrt[3]{x}$. Then graph the following:

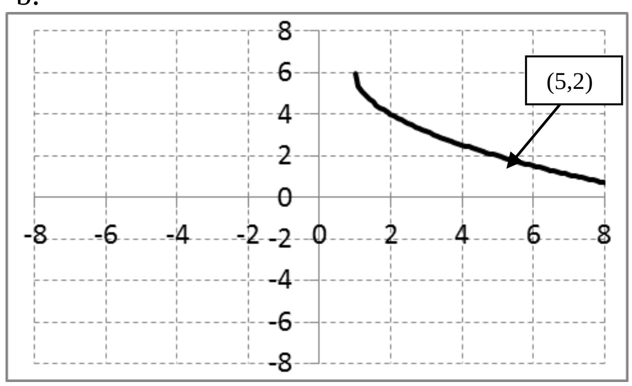
a. $f(x) = \sqrt[3]{x} - 3$ b. $f(x) = -\sqrt[3]{x-4}$ c. $2\sqrt[3]{x} - 2$

3. Write the equation of each function graphed below.

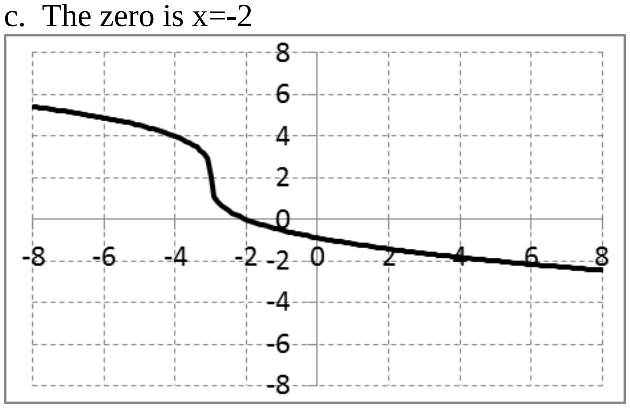
a.



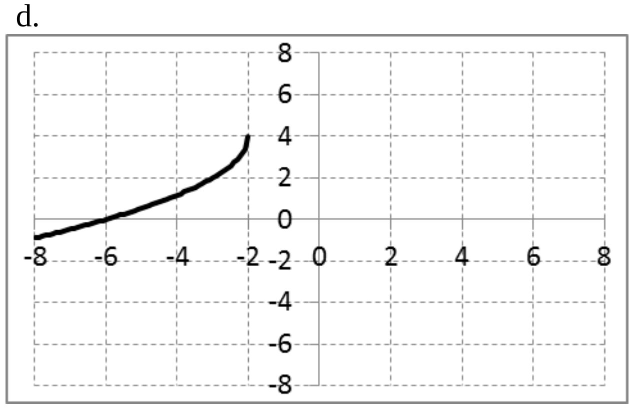
b.



c. The zero is x=-2



d.



4. Solve the following equations; watch for extraneous solutions.

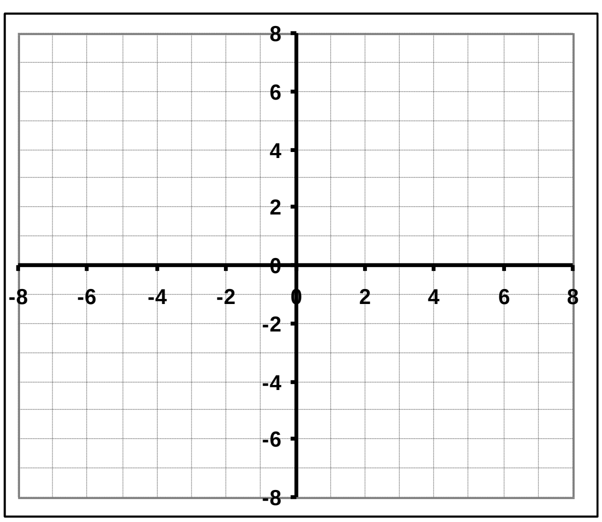
a. $2\sqrt{x+4}-3=-7$

b. $2\sqrt{x}=\sqrt{x+6}$

c. $\sqrt{x+5}=x-1$

d. $\sqrt{x+4}=-\frac{1}{3}x-2$

5. Sketch graphs of $f(x)=2\sqrt{x+3}-2$ *and* $g(x)=\frac{1}{2}x+1$. Then estimate the solutions to the equation $2\sqrt{x+3}-2=\frac{1}{2}x+1$. Finally, find the exact coordinates of the intersection(s) algebraically.



6. Write the equation of a square-root function whose domain is $x \geq -4$, whose range is $y \leq 5$, and whose y-intercept is 1 .

7. Solve the following equations. For ones with two radicals, it may be easiest to put one radical on each side, square both sides, isolate the radical and square both sides again.

a. $\sqrt{3x+13} = x+5$

b. $\sqrt[3]{2x+4} = 2(3-x)^{1/3}$

c. $\sqrt{\dfrac{x-4}{2}} = \sqrt{x}$

d. $x - \dfrac{1}{2} = \dfrac{-\sqrt{2-x}}{2}$

e. $\sqrt{x-5} = 2 - \sqrt{x}$

f. $\sqrt{2x+1} = 1 + \sqrt{2x}$

g. $\sqrt{x+3} = 4 - \sqrt{x}$

h. $\sqrt{x+1} + \sqrt{x-2} = 3$

ANSWERS: 1a. domain $x \geq 0$; range y≤4; y-int is (0,4) and zero is 4
b. x≥3; y≥-2; no y-int; zero is 7 c. x≥-4; y≤0; (0,-1); zero is -4 d. x≥-4; y≥-6; (0,-2); zero is 5
e. x≥-1; y≤-2; (0,-3); no zero (x=3 is an extraneous solution) f. x≤4; y≥2; (0,4); no zero

3a. $f(x) = \sqrt{x+4} + 2$ b. $f(x) = -2\sqrt{x-1} + 6$ c. $f(x) = -2\sqrt[3]{x+3} + 2$ d. $f(x) = -2\sqrt{-2-x} + 4$

4a. none: x=0 is extran b. 2 (square the 2 as well!) c. 4 (-1 is extran) d. none (both extran)

5. x=6, -2 and plug in to get (-2,0) and (6,4) 6. $f(x) = -2\sqrt{x+4} + 5$

7a. -3 or -4 b. 2 c. none d. -¼ e. 81/16 f. 0 g. 169/64 h. 3

Units 6 and 7 Practice Problems

1. Find the zeros and domains of the following functions.

a. $f(x) = -2\sqrt{x-3}+5$

b. $f(x) = \frac{1}{2}\sqrt[3]{x+2}+2$

c. $f(x) = \sqrt{x^2-2x-15}-3$

d. $f(x) = 2\sqrt{x^2-3x}-4$

2. Evaluate the following expressions or leave in simplest radical form. No negative exponents.

a. $\left(\frac{8}{27}\right)^{-1/3}$

b. $\frac{2^2 \cdot 3^{-1}}{3^{-2}}$

c. $\left(\sqrt[3]{11}\right)^6$

d. $\left(\sqrt[3]{\frac{-1}{8}}\right)^2$

e. $\frac{8-2\sqrt{12}}{4}$

f. $(-27)^{-2/3}$

g. $2\sqrt{5}\cdot 3\sqrt{10}$

h. $\sqrt{27}+\sqrt{12}-4\sqrt{3}$

i. $3\sqrt{2}\left(4\sqrt{8}-2\sqrt{6}\right)$

j. $\sqrt[3]{2}\left(3\cdot\sqrt[3]{4}\right)$

3. Express the following in simplest form. No negative exponents. Combine like terms when possible.

a. $\frac{x^2}{(-2x)^{-2}}$

b. $\left(\frac{-2x^4y}{xy^{-1}}\right)^3$

c. $x^{2/3}\cdot(3x^{-1/3})^2$

d. $(x+2y)^2$

e. $\frac{(-2x)^{-2}}{4x^{-3}}$

f. $\left(4x^2-(3x)^2\right)^2$

g. $2x\sqrt{x}+3x\sqrt{x}$

h. $\sqrt{2x}\cdot\sqrt{8x^2}$

i. $\sqrt{4x^8}$

j. $(5-2\sqrt{x})^2$

k. $2(x-1)^{-2}$

l. $\left(-2\sqrt{x^2}\right)^{-4}$

m. $\left(8x^6\right)^{-1/3}$

4. Find the domain of each function below:

a. $f(x) = \sqrt{2x-5}$

b. $f(x) = \sqrt{x^2-7x}$

c. $f(x) = \frac{1}{x^2+3x}$

d. $f(x) = \frac{5}{x^3-2x^2-15x}$

5. Solve the following equations. For the one with asterisks, you may use your calculator and should find all real solutions. For the others, no calculator and only one solution required.

a. $\frac{1}{2}(x+1)^{-2} = 8$

b. $2(3x-1)^{3/2} + 5 = 59$

c. $3(x+5)^{3/4} = 24$

d. $4x^{-2/3} + 2 = 18$

*e. $2(x-5)^3 = 38$

*f. $4(2x-1)^6 + 7 = 29$

6. If $f(x) = 2x - 1$, $g(x) = x^2 - 9$, $h(x) = \sqrt{2x}$, and $k(x) = 3x^{-2}$ then find the following:

a. $(f+g)(3)$

b. $f(x) - g(x)$

c. $\dfrac{f(0)}{g(0)}$

d. $(f \cdot g)(x)$ and its zeros

e. $g(x+3)$

f. $h(2x^4)$

g. the domain of $\left(\dfrac{h}{f}\right)(x)$

h. $g(2x^{-1/2})$

i. $k(3x)$

j. $f(g(3))$

k. $h(h(8))$

l. $(k \circ k)(2)$

m. $(k \circ g)(4)$

n. $(f \circ g)(x)$

o. $g(f(x))$

p. $(f \circ f)(x)$

7. Given that $f(x) = 2|x| - 8$ *and* $g(x) = x - 1$ find the following:

a. $f(g(-5))$

b. $f(g(x))$

c. What are the zeros of $f(g(x))$?

8. For the function $f(x) = 4x - 11$ find the inverse function $f^{-1}(x)$ and verify that it is the inverse.

9. If the points (2,4) and (-2,0) are on the graph of *f(x)* then what points must be on the graph of $f^{-1}(x)$?

10. State which one or ones of the functions below have inverse functions and which one or ones do not. A quick sketch may help.

a. $f(x) = 2x^2 - 3$

b. $g(x) = 2x - 4$

c. $h(x) = |x-3| + 4$

d. $k(x) = \frac{1}{3}(x+4)^2 - 3$

11. Find the inverse of the following functions and verify that they are inverses:

a. $f(x) = 2x^5 + 2$

b. $f(x) = \frac{2}{3}(x+1)^3$

c. $f(x) = 2\sqrt[3]{x-4} + 7$

12. Sketch the graph of each function below.

a. $f(x) = -2(x+1)^{1/2} + 3$ b. $f(x) = -2(x+1)^{1/3} + 3$ c. $f(x) = 3\sqrt{x-1} - 2$

13. Solve the following equations:

a. $3\sqrt{x-1} + 2 = 6$ b. $\sqrt{x+5} = x+3$ c. $-x = 3\sqrt{x} + 2$

Answers

1a. zero is 37/4 domain is $x \geq 3$ b. zero is -66 domain is all reals
c. zeros are 6, -4 domain is x≤-3 or x≥5 d. zeros are 4 and -1; domain is x≤0 or x≥3

2a. 3/2 b. 12 c.121 d. ¼ e. $2-\sqrt{3}$ f. 1/9 g. $6\sqrt{50} = 30\sqrt{2}$ h. $\sqrt{3}$
i. $48 - 12\sqrt{3}$ j. $3\sqrt[3]{8} = 6$

3a. $4x^4$ b. $-8x^9y^6$ c. 9 d. $x^2 + 4xy + 4y^2$ e. x/16 f. $25x^4$
g. $5x\sqrt{x}$ h. $4x\sqrt{x}$ i. $2x^4$ j. $25 - 20\sqrt{x} + 4x$ k. $\dfrac{2}{x^2 - 2x + 1}$ l. $\dfrac{1}{16x^4}$ m. $\dfrac{1}{2x^2}$

4a. x≥2.5 b. x≤0 or x≥7 c. all reals except 0 and -3 d. all reals except 0, 5, and -3

5a. -3/4 (or -5/4) b. 10/3 c. 11 d. 1/8 (or -1/8) e. 7.67 f. 1.164 or -0.164

6a. 5 b. $-x^2 + 2x + 8$ c. 1/9 d. $(x^2 - 9)(2x - 1)$ zeros are ½ and ±3
e. $x^2 + 6x$ f. $2x^2$ g. $x \geq 0$ *but not* 0.5 h. $\dfrac{4}{x} - 9$ i. $\dfrac{1}{3x^2}$
j. -1 k. $2\sqrt{2}$ l. 16/3 m. $\dfrac{3}{49}$ n. $2x^2 - 19$
o. $4x^2 - 4x - 8$ p. $4x - 3$

7a. 4 b. $2|x-1| - 8$ c. -3, 5

8. $f^{-1}(x) = \dfrac{x+11}{4}$ $f(f^{-1}(x)) = x = f^{-1}(f(x))$ 9. (4,2) and (0,-2)

10 b is the only one with an inverse function

11. *a.* $\sqrt[5]{\dfrac{x-2}{2}}$ *b.* $\sqrt[3]{\dfrac{3x}{2}} - 1$ *c.* $\left(\dfrac{x-7}{2}\right)^3 + 4$

13a. 25/9 b. -1 (-4 is extraneous) c. no solutions (1 and 4 are extraneous)

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Unit 8 Handout #1: Graphs of Exponential Functions

1. After you graduate from college you get offered a job as a junior executive at Acme Materials Company. They are a major supplier of things such as dynamite, sledge hammers, and rockets. Your initial hourly pay rate is $30 and they gave you two different options for what your raises will be. The raises will be given each year right before the end of the year.

Option #1: Each year you get a $10 raise in your hourly pay.
Option #2: Each year you get a 20% raise in your hourly pay (from its current level).

a. Who do you think Acme's largest client is?

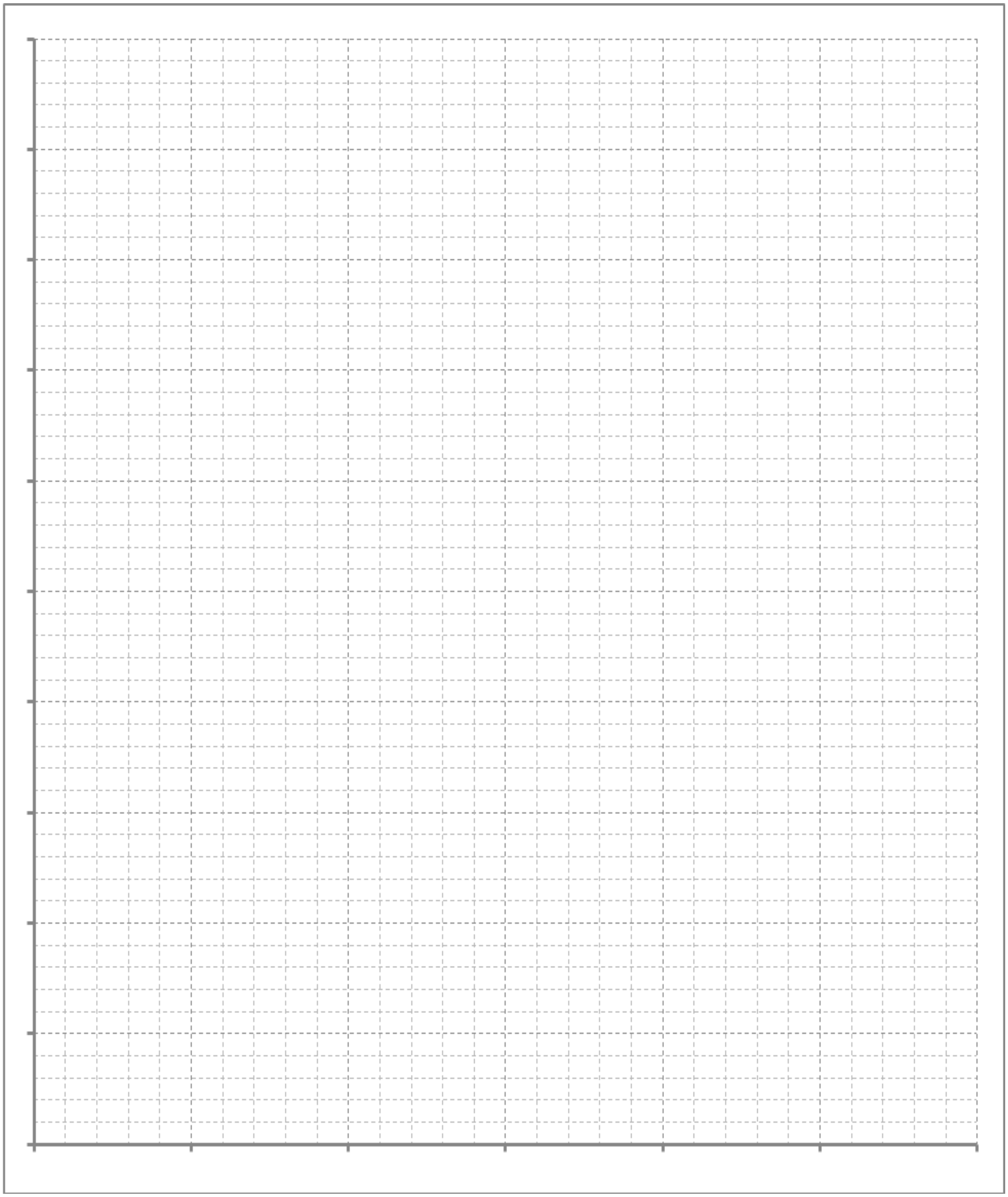
b. Before filling in the table below, which option of raises do you think is a better deal, and why?

c. Fill in the table of values below for what your hourly pay is at the end of each year. (Round!)

Year	Option 1	Option 2
0	30	30
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		

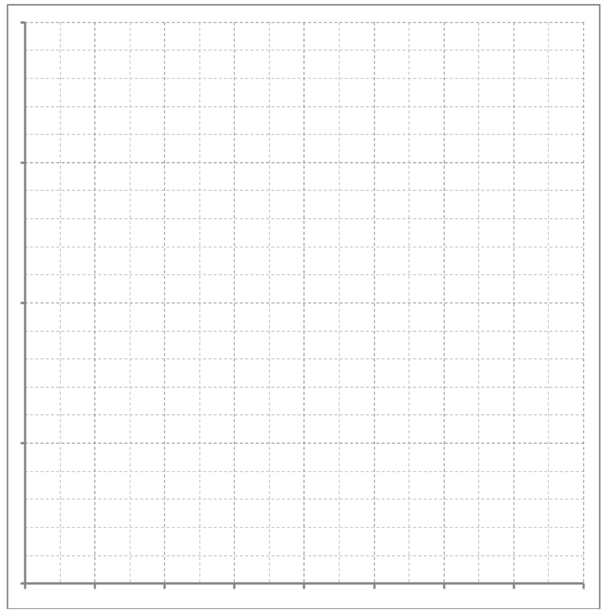
d. Plot these points on the grid on the next page paper. Plot both data series on the same graph. Choose your scale carefully.

e. Write the equation describing the pay (y) as a function of year (x) for each of the two pay options.



2. You start 100 feet away from a wall and every minute, you move halfway to the wall. Fill in the table below. Graph it and write an equation for distance from wall (output) as a function of time.

Time	Distance
0	100
1	50
2	
3	
4	
5	
6	
7	
8	
9	
10	



3. The populations, P, of six towns at time t (measured in years) are given by:

i. $P = 4000(1.08)^t$ ii. $P = 600(1.12)^t$ iii. $P = 2500(0.9)^t$ iv. $P = 1200(1.185)^t$

v. $P = 800(0.78)^t$ vi. $P = 2000(0.99)^t$

a. Which towns are growing in size?

b. Which town is growing the fastest, and what is the annual rate of growth of that town?

c. Which town in shrinking the fastest, and what is the annual percent “decay” rate for that town?

d. Which town has the largest initial population and which has the smallest?

e. If these growth rates continue, which town will eventually be the largest?

4. Match the stories in (a)-(e) with the formulas (i)-(v). Assume the constants $P_0,\ r,\ B,\ and\ A$ area all positive. In each case, state what the variables represent.

a. The percent of a lake's surface covered by algae, initially 30%, was halved each year since the passage of anti-pollution laws.

b. The amount of charge on a capacitor in an electric circuit decreases by 30% every second.

c. Polluted water is passed through a series of filters. Each filter removes 70% of the impurities in the water.

d. In 1920 the population of a town was 3000. Over the next many years it grew at the rate of 10 percent per year.

e. In 1920 the population of a town was 3000. Over the next many years it grew at a rate of 250 people per year.

i. $f(x) = P_0 + rx$ ii. $g(x) = P_0(1+r)^x$ iii. $h(x) = B(0.7)^x$ iv. $j(x) = B(0.3)^x$ v. $k(x) = A(2)^{-x}$

5. For each of the functions below sketch a rough graph. You should plot at least one or two positive and negative *x* values each.

a. $f(x) = 2^x$ *b.* $f(x) = 3^x$ *c.* $f(x) = 2^x + 3$

d. $f(x) = -2 \cdot 3^x$ (is this the same as -6^x ?) *e.* $f(x) = \left(\frac{1}{2}\right)^x$

6. What are the domain and range of each of the functions in question #5 above?

7. For each function below, think about what happens to the output as the input becomes a very large positive number and as the input becomes a very large negative number.

a. $f(x) = 2 \cdot 1.2^x$

b. $f(x) = 10 \cdot 0.7^x$

c. $f(x) = 10 \cdot 0.7^x + 20$

d. $f(x) = -3 \cdot 1.5^x + 7$

8. Jim thinks of himself as a good investor. He bought one share of stock for $100. The first year it lost 40% of its start-of-year value. The next year it gained 40% of its start-of-year value. Does this mean Jim ended with his $100 again? Do the calculations and explain your results.

9. Sketch a graph of each function below. I encourage you to graph exponential functions by plotting the y-intercept and reasoning through end behavior (as you did in question #7 above). Careful – not all of these are exponential functions!

a. $f(x) = -2 \cdot 0.8^x$

b. $f(x) = -3 \cdot 2^x + 8$

c. $f(x) = -3x^2 + 8$

d. $f(x) = 2 \cdot 1.5^{x+1} - 4$ e. $f(x) = -2x + 8$ f. $f(x) = 0.2 \cdot 0.6^{x} + 5$

10. With your calculator, graph the following two functions: $f(x) = 2 \cdot 3^{x}$ *and* $g(x) = 6 \cdot 3^{x-1}$. Be careful with parentheses. Describe what you see and explain why.

Answers

2. The equation is $100 \cdot 0.5^{x}$

3a. i, ii, iv b. iv; 18.5% per year c. v; 22% per year d. i is largest; ii is smallest e. iv

4a. v b. iii c. iv d. ii e. i

6. domains are all all real numbers. Ranges are: a. y>0 b. y>0 c. y>3 d. y<0 e. y>0

7a. as x→ ∞ y→∞ and as x→ -∞ y→0 b. as x→ ∞ y→0 and as x→ -∞ y→∞

c. as x→ ∞ y→20 and as x→ -∞ y→∞ d. as x→ ∞ y→-∞ and as x→ -∞ y→7

8. he ends up with $84 so he lost 16% of his original $100. He lost 40% of $100 ($40) but only gained back 40% of $60 ($24) so he was still $16 behind. 10. They are the same

Unit 8 Handout #2: Exponential Growth and Decay Models including Half-Life

1. The population of the US is about 300 million and has been increasing by about 1% per year.
   a. If this continues, what will the population in 20 years be?

   b. What was the population 15 years ago?

   c. What annual rate of increase is required for the population to reach 450 million in 25 years?

2. The population of Afghanistan is about 30 million. It has been increasing at 4.8% per year.
   a. If this continues, what will its population be in 15 years?

   b. What was the population 20 years ago?

   c. Graph the population functions of the US and Afghanistan on your calculator. Determine when the populations will be equal.

3. The value of a new sports car decreased from $40,000 to $20,000 in 7 years.
   a. What was the annual rate of decline in the car's value?

   b. Write an exponential function showing its value when it is *t* years old. (Assume the annual rate of decline stays constant).

   c. What will its value be when it is 10 years old?

   d. Graph the function in your calculator. Determine when its value is exactly $8000.

4. Two banks offer different interest rates on deposits. Bank A offers 20% per year, and Bank B offers 10% every six months.

a. If you put $5000 in Bank A, how much would you have after 10 years?

b. If you put $5000 in Bank B, how much would you have after 10 years? [Think # of periods]

5. The population of Uganda is now about 25 million and has been increasing by about 4% per year.

a. If this continues what will the population be in 15 years?

b. About what was the population 18 <u>*months*</u> ago?

6. An investment lost a total of half of its value in 8 years.

a. If the initial value is $1000, then what annual rate of decrease did it have?

b. If the initial value is $5000, then what annual rate of decrease did it have?

7. The value of an investment triples in 15 years. What is the annual rate of increase in value?

8. In 1950 the average Algebra II book had 412 pages. The average Algebra II book had 850 pages in 2010.

a. What was the annual percentage growth in the number of pages?

b. Write an equation describing the number of pages the average book had P(t) as a function of the number of years since 1950 (t). Assume that the annual growth rate has been constant.

c. Graph this on your calculator and determine about when the average was 600 pages.

d. What is the meaning and numerical value of $P(11) - P(10)$? How about $P(51) - P(50)$?

9. In Germany in the early 1920's there was a period of hyperinflation where prices were increasing by about 2.3% **per day**.

a. If something cost 50 Marks one day, how much would it be expected to cost 2 months later?

b. How much would that 50-Mark item cost one week earlier?

10. Germany's hyperinflation in 1923 amounted to about a 100% rise in prices per month.

a. If something cost 15 marks at one time, how much would it cost 8 months later?

b. If the price of some object grew at this rate, how many months would it take to increase in price from 10 to 320?

11. The amount of caffeine in one's body decreases by about 15% per hour as it gets metabolized. To stay awake for her late-night study session, Angela wants to keep the amount of caffeine in her bloodstream between 150 and 200 mg. She drinks two cups of coffee at 10 pm, resulting in her having 200 mg of caffeine in her system.

a. How much will she have two hours later, assuming she ingests no more?

b. When will her caffeine level hit 150 mg?

c. Her friend Amy drank a lot of coffee at 10 pm and, drinking no more, had a caffeine level of 130 mg at 3 am. How much coffee did she drink?

12. The population of bluebirds in one particular park is now 275 and is expected to grow at 4% per year. The population of crows in the same park is now 80 and is expected to grow at 12% per year.

a. Write a growth model for each, showing population as a function of time.

b. Graph both of these on your calculator. Determine when the population of crows will first exceed the population of bluebirds.

13. When we were kids, our math-genius neighbor named Victor offered my mom the following deal. He would wash our dinner dishes each day for 3 full weeks. The first day we would pay him one cent. His daily pay would double each day. Knowing Victor, we declined. If we accepted, how much would his pay have been on the 7^{th} day? The 14^{th} day? The last day?

14. A regular piece of notebook paper is about 0.005 inches thick. Every time you fold it in half, its thickness doubles.

a. How thick is it after 1, 2, and 3 folds?

b. Write an expression showing its thickness after x folds.

c. How thick is it after 20 folds?

d. How many miles thick is it after 30 folds? [One mile is 5280 feet].

e. The moon is about 384,000 miles away. Use your calculator to estimate how many folds would be theoretically required for it to reach the moon.

15. A population grows at a constant rate of 50% per year. In year five it was 1000 and in year seven it was 2250. Zora says that one way to write the function describing the population P in year t is $P = 1000 \cdot 1.5^{t-5}$.

a. Is she correct? Explain.

b. Yanice disagrees; she thinks the correct equation is $P = 2250 \cdot 1.5^{t-7}$. They argue for a while about who is right. Help them resolve their disagreement.

c. Xavier looks at the situation and determines that the relationship can be described as $P = A \cdot 1.5^{t}$. What is the numerical value of A?

d. The population model can be written as $P = C \cdot 1.5^{t-11}$. What is the numerical value of C and what does it represent in the context of the problem?

16. A new luxury automobile costs $50,000. Assume its value after seven years is $22,000.

a. Assume the function describing the car's value when it is *x* years old is linear. Write an equation for it (in slope-intercept form).

b. What is the slope and what does it mean? What is the intercept and what does it mean?

c. Assume instead that the value fell by a constant percentage rate each year. What was the percentage decline in value?

d. Write an exponential function describing the car's value when it is *x* years old.

e. Use each model to calculate the car's value when it is 3 years old and when it is 13 years old.

f. Which model do you think is better and why?

g. Use each model to determine when the car's value first hits $10,000. You will need your calculator for the exponential model.

h. Use each model to determine when the car is worthless.

17. The population of a small country is currently 2 million and is growing at the rate of 5% per year. The country currently produces enough food to feed 4 million people. Because of increases in population and farming techniques, the each year the food supply increases by enough to feed 0.3 million people.

a. Write a function *p(t)* showing the population (in millions) *t* years from now.

b. Write a function *f(t)* showing the food supply (for millions of people) *t* years from now.

c. Twenty years from now, what will the population be and what will the food supply be?

d. Graph both functions on your calculator. When will the population surpass the food supply?

e. What is the meaning of $f^{-1}(8)$? How about the meaning and numerical value of $f(p^{-1}(8))$?

18. The concentration of some compound in a person's bloodstream has a half life of 3 hours, meaning half of it disappears every 3 hours. An initial dose of 80 mg is administered.

a. How much is left after 3 hours? 6 hours? 7 hours? Think periodic change and number of periods—the period length does **not** have to be per **one** day/week/month/year.

b. What is the hourly rate of decline? [Hint: write an exponential growth model using any two points you know.]

19. One organism is put into a petri dish. It and its offspring each divide into 2 every minute (in other words, the population doubles every minute). Thirty minutes later, the entire petri dish is full. Use your calculator to answer the following questions:

a. How many organisms did it take to fill the petri dish?

b. When was the dish half full? Think before you compute!

c. How full was the dish after 15 minutes? (What percent full?)

d. When was the dish 10% full?

20. Write the following in the form $y = a \cdot b^x$. The only exponent should be *x*.

Example: $y = 4^{x-1} = 4^{-1} \cdot 4^x = 0.25 \cdot 4^x$ or $y = \frac{1}{4} \cdot 4^x$

a. $y = 3^{x-2}$

b. $y = 3^{2-x}$

c. $y = 2 \cdot 4^{2x+1}$

d. $y = -2 \cdot 5^{1-x}$

e. $y = 4^{0.5x-2}$

f. $y = \frac{1}{3} \cdot 8^{x/3}$

g. $y = 7 \cdot \left(\frac{1}{2}\right)^{-x+1}$

h. $y = -3 \cdot \left(\frac{2}{3}\right)^{-2x-3}$

ANSWERS

1a. 366.1 mill b. 258.4 mill c. 1.64%

2a.60.61 mill b. 11.75 million c. in 62.34 years

3a. 9.4% b. $V = 40000 \cdot 0.906^t$ c. $14,860 d. 16.25 years

4a. $30,959 b. $33,637

5a. 45.02 mill b. 23.57 mil c.

6a&b 8.3% → with constant percentage rate time to double does not depend on starting amount

7. 7.60%

8a. 1.2% b. $P(t) = 412 \cdot 1.012^t$ c. about 31.5 yrs (1981 or 1982)

d. 5.57; increase in avg size from 1960 to 1961; 8.98; increase in avg size from 2000 to 2001

9a. about 200 9b. 42.64

10a. 3840 b. 5 months

11a. 145 mg b. 1.77 hrs later, so about 11:46 c. 293 mg, so almost 3 cups

12a. $C(t) = 275 \cdot 1.04^t$ $B(t) = 80 \cdot 1.12^t$ b. ~17 yrs

13. 7th day is $0.01 \cdot 2^6 = \$0.64$ 14th day is $0.01 \cdot 2^{13} = \$81.92$ last day is $0.01 \cdot 2^{20} = \$10{,}485.76$

14a. 0.01, 0.02, and 0.04 inches b. $0.005(2)^x$ c. about 5243 inches d. about 84.73 miles

e. $384{,}000\ miles = 24{,}330{,}000{,}000\ inches$ so 42 or 43 folds

15a. yes; in year 5 it is 1000 and for each subsequent year, multiply by 1.5

b. both are correct; in year 5 it is 1000 and for each subsequent year, multiply by 1.5

c. 131.69 d. about 11391; it is the population in year 11

16a. $y = -4000x + 50000$ b. starts at $50,000 and falls by $4000 per year

c. -11.1% per year d. $y = 50000 \cdot 0.889^x$ e. 3 years: linear=38,000 exp=35,130

13 years: linear=-2000 exp=$10,832. f. I like exponential: value never becomes negative

g. linear 10 years; exp is 13.68 years h. linear: 12.5 years; exponential: never

17a. $p(t) = 2 \cdot 1.05^t$ b. $f(t) = 4 + 0.3t$ c. $p(20) = 5.31$ mill $f(20) = 10$ mill d. in 44.16 years

e. The time the food supply is 8 mil; the food when the pop is 8 million (enough for 12.52 million)

18a. 1 period so 40 mg; 2 periods so 20 mg; (7/3) periods so 15.874 b. falls by 20.63% per hour

19a. about 1.074 billion b. after 29 minutes (cool!) c. about 0.00305% full d. after 26.68 mins

20a. $y = \frac{1}{9} \cdot 3^x$ b. $y = 9 \cdot \left(\frac{1}{3}\right)^x$ c. $y = 8 \cdot 16^x$ d. $y = -10 \cdot 0.2^x$

e. $y = \frac{1}{16} \cdot 2^x$ f. $y = \frac{1}{3} \cdot 2^x$ g. $y = 3.5 \cdot 2^x$ h. $y = \frac{-81}{8} \cdot \left(\frac{9}{4}\right)^x$

Unit 8 Handout #3: More Exponential Growth and Decay

1. The doubling time refers to the amount of time it takes something to double in value.

a. Something doubles every 10 years. What is its annual growth rate?

b. If something doubles every 3 years, then what is its annual growth rate?

c. If something grows by 15% per year, what is its doubling time?

2. Write the equation of an exponential function in the form $f(x) = a \cdot b^x$ given that $f(0) = 10$ and $f(8) = 30.$ Hint: this is like saying a population is initially 10 and grows to 30 after 8 years with a constant percentage growth rate.

3. In each part below, write the equation of an exponential function in the form $f(x) = a \cdot b^x$ given two points. It may be easiest to think of a time line and find *b* first. Then plug in a point to find *a*.

a. $f(1) = 200 \;\; and \;\; f(4) = 1600$ (no calculator)

b. $f(4) = 20 \;\; and \;\; f(9) = 5$ (calc OK)

c. $f(2) = 100 \;\; and \;\; f(11) = 350$ (calc OK)

4. More challenging: An exponential function has a horizontal asymptote of $y = 11$. It goes through the points (0,40) and (5,30). What is its equation? Hint: you may want to think about shifting it down 11 units and doing what you have done above in the prior two questions, then shift it back up.

5. A yam's temperature is 70°F. It is placed in a 400° oven. After 20 minutes, its temperature is 130°. It will eventually become (exactly or very very very close to) 400°.Write a function showing the yam's temperature *t* minutes after being put in the oven. When is the yam's temperature 200°?

6. A dead body was found; its temperature was 93.2°. Two hours later, it had cooled to 90.5°. The temperature of the house where it was found was climate-controlled to be 66°. Assuming that its temperature over time can be modeled by the function $f(x) = a \cdot b^x + k$, what is $f(x)$? Then, assuming the temperature was 98.6° at the time of death, determine how long the person had been dead before the body was found.

7. Newton's Law of Cooling describes the way the temperature of objects adjusts to the ambient temperature over time. This relationship is an exponential function. Let $H(t) = 93(0.91)^t + 68$ describe the temperature of a beverage (in degrees F) t minutes after a Dunkin' Donuts employee hands it to you.

a. Is the beverage hot coffee or iced coffee? How can you tell by looking at the equation?

b. What is the asymptote of the graph of $H(t)$ and what does it mean in the context of this problem?

c. Sketch a rough graph of $H(t)$ by hand. Think reasonable domain!

d. Calculate the coordinates of the y-intercept of $H(t)$. What does it mean (in context)?

e. What is the range of $H(t)$? What meaning does it have in the context of the problem?

f. What is the value of $H(10)$ and what meaning does it have in context?

g. Exactly when does the temperature hit 90°? Solve with your calculator.

8. Answer the questions about the function $f(x) = -2 \cdot 0.8^x + 12$.

a. What is the equation of its asymptote?

b. What are the coordinates of its y-intercept?

c. Sketch a rough graph of $f(x)$.

d. What is the zero of $f(x)$? Calculator OK

e. Solve the equation $f(x) = 11$ (calculator necessary)

9. Write the equation of any exponential function where $f(0) = 20$ and as $x \to \infty$, $f(x) \to -4$.

10. Given $f(x)$ where $f(1)=20$ *and* $f(4)=160$, do the following without your calculator:
a. If the function *f* reflects constant percentage growth, write the equation for $f(x)$.

b. If the function *f* reflects a constant increase in number per period, write the equation for $f(x)$.

Answers

1a. $2=1\cdot(1+r)^{10}$ so r=7.18% b. 25.99% c. $2=1\cdot 1.15^t$ so t=4.96 years

2. b must be 1.147 so $f(x)=10\cdot 1.147^x$

3a. b is 2 so $f(x)=a\cdot 2^x$ and since f(1)=200 we know a=100 → so $f(x)=100\cdot 2^x$
b. $f(x)=60.58\cdot 0.758^x$ c. $f(x)=75.75\cdot 1.149^x$

4. shifting down 11 the points are (0,29) and (5,19) so this is $f(x)=29\cdot 0.919^x$ then shift up 11 to get $f(x)=29\cdot 0.919^x+11$

5. asymptote is 400… shifting down 400 units we get (0,-330) and (20,-270) which gives us $y=-330\cdot 0.990^t$ so temperature function is $f(t)=-330\cdot 0.99^t+400$.
It hits 200 when $200=-330\cdot 0.99^t+400$; using calc intersect I get 49.8 minutes

6. asymptote is 66 and points are (0,93.2) and (2,90.5)… function is $f(x)=27.2\cdot 0.949^x+66$
This is 98.6 when $98.6=27.2\cdot 0.949^x+66$ which gives x=-3.46 so the person died about 3½ hours before the body was found

7a. hot since it starts above 68; b. y=68; temperature drink eventually gets very close to
d. (0, 161); initial temp is 161° e. $68<H(t)\le 161$; temp is between 68 and 161 degrees
f. $H(10)=104.2$; this is coffee's temp after 10 minutes g. 15.29 mins

8a. y=12 b. (0,10) c. check on calc d. -8.03 e. 3.11

9. some examples: $f(x)=24\cdot 0.8^x-4$, $f(x)=24\cdot 0.2^x-4$

10a. exponential function $f(x)=10\cdot 2^x$ b. linear function $f(x)=20+\frac{140}{3}(x-1)$

Unit 8 Handout #4: Solving Exponential Equations by Finding a Common Base

Warm-ups

$a.\ 2^x = 8$

$b.\ 3^x = 9$

$c.\ 3^x = \frac{1}{9}$

$d.\ 2^x = \frac{1}{4}$

$e.\ 3 \cdot 2^x = 48$

$f.\ 4^x + 7 = 71$

$g.\ 2^{x+2} = 8$

$h.\ 2^x \cdot 2^{x-2} = \sqrt{2}$

Solve the following problems:

1. $2^x = 4$

2. $3 \cdot 2^x = 24$

3. $3^{x+3} = 9$

4. $2^x = \frac{1}{8}$

5. $2 \cdot 3^{2x-1} + 7 = 61$

6. $2^x = 4^{x+5}$

7. $3^x = 9^7$

8. $2^x = 2\sqrt{2}$

9. $5^x = \frac{1}{\sqrt[3]{5}}$

10. $4^x = 8$

11. $\left(\frac{1}{5}\right)^x = 25$

12. $2^{2x+1} \cdot 2^x = 16$

13. $6^{-x} = \dfrac{6}{\sqrt[5]{6}}$

14. $\left(\sqrt{2}\right)^x = 8$

15. $3^x = \left(\dfrac{1}{9}\right)^{4-x}$

16. $\left(2^{x+1}\right)^2 = \dfrac{1}{4}$

17. $4^{2x} = 8^{x-3}$

18. $\left(\dfrac{1}{6}\right)^{x-1} = 36$

19. $2^{x^2} \cdot 2^{3x} = 16$

20. $\dfrac{5^x}{5^2} = 5^{1/3}$

21. $\dfrac{10^{2x+3}}{10^{x-1}} = 100$

22. $7^{x^3} = 7^{x^2} \cdot 49^x$

23. $2^{x^2} = (2^x)^2$

24. $3^x = \sqrt[5]{27}$

25. $5^{-x} = \left(5\sqrt{5}\right)^3$

26. $6^{(x-1)^3} = \left(6^4\right)^2$

27. $\dfrac{3^{x^2}}{3^x} = 9$

Answers: 1. 2 2. 3 3. -1 4. -3 5. 2 6. -10 7. 14 8. 1.5 9. -1/3 10. 1.5 11. -2
12. 1 13.-4/5 14. 6 15. 8 16. -2 17. -9 18. -1 19. -4, 1 20. 7/3 21. -2 22. 0, 2, -1
23. 0, 2 24. 3/5 25. -4.5 26. 3 27. 2, -1

Unit 9 Handout #1: Introduction to Logarithms

1. Evaluate the following logarithms without your calculator by rewriting as exponential equations:

a. $\log_3 9$

b. $\log\frac{1}{10}$

c. $\log_6 6^5$

d. $\log_2 \frac{1}{8}$

e. $\log_3 \frac{1}{\sqrt{3}}$

f. $\log 1000$

g. $\log 10\sqrt{10}$

h. $\log_9 \frac{1}{27}$

i. $\log_{1/2} 4$

j. $\log_8 \frac{1}{2}$

k. $\log_{11} \sqrt[5]{11}$

l. $\log_4 \frac{1}{8}$

m. $\log_{49} \sqrt{7}$

n. $\log_6 \frac{6}{\sqrt[4]{6}}$

o. $\log_3 27^{11}$

p. $\ln e^3$

q. $\ln\frac{1}{\sqrt[3]{e}}$

r. $\ln e^{-31.67}$

2. Solve the following equations by rewriting the logarithm as an exponential equation. You may need to isolate the logarithm first.

a. $\log_3 x = -2$

b. $\log_4 (x+1) = 2$

c. $\log_5 (2x-1) + 3 = 4$

d. $\log(x-1) - 1 = 2$

e. $\log_4 (x-2) + 3 = 3$

f. $2\log_2 (x) + 2 = -4$

g. $\log_6 (2x) = \frac{1}{2}$

h. $\log(x^2 - 4) - 1 = 0$

i. $\log_7 \left(\sqrt{x-3}\right) = 1$

j. $2\log_4 (x+5) + 1 = 4$

k. $\log_{4/9} x = \frac{-1}{2}$

l. $2\log_{16} (3x+1) = -1$

3. Solve the following equations by rewriting as logs and then using the log button on your calculator. Use three decimal places precision.

a. $10^x = 43$ b. $3 \cdot 10^x + 1 = 19$ c. $2 \cdot 10^{3x-1} = 1824$

d. $e^x = 11$ e. $2e^x - 3 = 11$ f. $e^{3-x} = 0.25$

4. Try to evaluate the following logarithms.

a. $\log(-10)$ b. $\log_2(-4)$

5. What is the domain of $f(x) = \log x$? Why? Hint: look at previous problem and think of range of exponential functions.

6. If $f(x) = 5^x$ *and* $g(x) = \log_5 x$, then what is $g(f(x))$?

7. Find the inverse $f^{-1}(x)$ of each function below by isolating the term with the power and then writing it as a logarithm. Example: $f(x) = 4^{x-1} + 9$

$\rightarrow$ $x = 4^{y-1} + 9$ $\rightarrow$ $x - 9 = 4^{y-1}$ $\rightarrow$ $y - 1 = \log_4(x-9)$ $y = f^{-1}(x) = \log_4(x-9) + 1$

a. $f(x) = 10^x - 3$

b. $f(x) = 2 \cdot 3^x - 7$

c. $f(x) = 0.5 \cdot 1.3^{x-3}$

8. Answer the following questions about $f(x) = \log_3(x-1)$.

a. What is $f(4)$?

b. What is $f(10)$?

c. What is $f(4/3)$?

d. Does $f(x)$ have a y-intercept?

e. Find the zero of $f(x)$.

f. Where does the graph of $f(x)$ intersect $y = 4$?

g. Find the inverse function $f^{-1}(x)$.

9. Answer the following questions about $f(x) = \frac{1}{2}\log_3(x+3) + 1$.

a. What is the domain of $f(x)$?

b. What is the y-intercept of $f(x)$?

c. What is $f(6)$?

d. What is the zero of $f(x)$?

e. What are the coordinates where the graph of $f(x)$ intersects the graph of $g(x) = 3$?

f. The inverse of $f(x)$ is an exponential function; find it.

10. Use your calculator to evaluate the following. Look for patterns.

a. log 2 b. log 20 c. log 200 d. log ½ e. log 1/20 f. log 2 g. log 4 h. log 8

11. In each case below, you are told possible values of the **logarithm** of *p* (or some transformation of *p*). Without using your calculator, determine the possible values of *p*.

a. $1 < \log_2 p < 2$ b. $-1 < \log_{1/3} p < 0$ c. $-2 < \log_3(p-2) < 1$

12. Without using your calculator, you cannot know the exact value of the following logs. But you can know the part of the number to the left of the decimal place. Find them.

a. $\log 1721$ b. $\log 84$ c. $\log_5 100$ d. $\log_2 99$

ANSWERS

1a. 2 b. -1 c. 5 d. -3 e. -½ f. 3 g. 1.5 h. -1.5 i. -2 j. -1/3 k. 1/5 l. -1.5
m. ¼ n. ¾ o. 33 p. 3 q. -1/3 r. -31.67

2a. 1/9 b. 15 c. 3 d. 1001 e. 3 f. 1/8 g. $\sqrt{6}/2$ h. $\pm\sqrt{14}$ i. 52 j. 3 k. 3/2 l. -¼

3a. 1.633 b. 0.778 c. 1.320 d. 2.398 e. 1.946 f. 4.386

4. cannot be evaluated → because 10^x can never be a negative number 5. x>0

6. $g(f(x)) = x$ because f&g are inverses: $\log_5 5^x = w$ so $5^w = 5^x$ *and* $w = x$ so $\log_5 5^x = x$

7a. $f^{-1}(x) = \log(x+3)$ b. $f^{-1}(x) = \log_3(\frac{x+7}{2})$ c. $f^{-1}(x) = \log_{1.3}(2x) + 3$

8a. 1 b. 2 c. -1 d. no; can't take log of a neg # e. x=2 f. (82,4) g. $3^x + 1$

9a. x>-3 b. ½(1)+1=1.5 c. ½(2)+1=2 d. -26/9 e. (78,3) f. $3^{2x-2} - 3$

10a. 0.301 b. 1.301 c. 2.301 d. -0.301 e. -1.301 f. 0.301 g. 0.602 h. 0.903

11a. 2<p<4 b. 1<p<3 c. 19/9<p<5 12a. 3 b. 1 c. 2 d. 6

Unit 9 Handout #2: Graphing Log Functions

1. Sketch a graph of each function below. Be sure to give the domain and range, the equation of any asymptotes, and any zeros:

a. $f(x) = \log x$

b . $f(x) = 2\log_3(x-4)$

c. $f(x) = -\log(x+2)$

d. $f(x) = 0.5\log_5(x+1)-1$

e. $f(x) = \log_5(0.2x)+2$

f. $f(x) = \ln(x-3)+1$

g. $f(x) = \log_2(3 - x)$

h. $g(x) = \log(1/x)$

2. Find the domain of $f(x) = \log(x^2 - 8x + 15)$

3. Solve the following equation for *x*: $y = 2e^{2x} - 1$.

Answers

1a. domain is x>0; asymptote is x=0; range is all reals; zero is x=1
  b. domain is x>4; asymptote is x=4; range is all reals; zero is x=5
  c. domain is x>-2; asymptote is x=-2; range is all reals; zero is x=-1
  d. domain is x>-1; asymptote is x=-1; range is all reals; zero is x=24
  e. domain is x>0; asymptote is x=0; range is all reals; zero is x=1/5
  f. domain is x>3; asymptote is x=3; range is all reals; zero is x=3+1/e
  g. domain is x<3; asymptote is x=3; range is all reals; zero is x=2
  h. this is just –logx to domain is x>0; asymptote is x=0; range is all reals; zeros is x=1

2. $x < 3 \text{ or } x > 5$

3. $x = 0.5\ln((x+1)/2)$

Unit 9 Handout #3: The Laws of Logs

1. Evaluate the following by first condensing each into a single logarithm and then evaluating that log.

a. $\log 4 + \log 5 + \log 5$

b. $\log_3 2 - \log_3 18$

c. $\log_4 8 + \log_4 2$

d. $\log_8 20 - \log_8 5$

e. $\log 3 - \log 300$

f. $\log_9 6 - \log_9 2$

g. $\log_7 2 + \log_7 2 - \log_7 28$

h. $\log_5 4 - \log_5 2 - \log_5 10$

i. $\log_6 9 + \log_6 4$

j. $\log_6 48 - 3\log_6 2$

k. $\frac{1}{2}\log_4 9 - (\log_4 6 + \log_4 2)$

l. $2\log 5 + 0.5\log 16$

m. $\frac{1}{2}\log_5 16-(\log_5 10+\log_5 2)$

n. $\frac{1}{2}\log_6 3+0.5\log_6 12$

o. $2\log_{16} 6-\left(2\log_{16} 3+\frac{1}{2}\log_{16} 4\right)$

p. $\log_8 18-2(\log_8 24-2\log_8 2)$

2. Expand the following into log of a number, log of *x*, and log of *y*.

a. $\log(2xy)$

b. $\log\frac{8x}{y}$

c. $\log\frac{1}{xy}$

3. Solve the following equation by condensing all the logarithms into a single log and then using the definition of a logarithm to rewrite without the log

a. $\log_3 x-\log_3 2=-1$

b. $\log_6(x+1)+\log_6 3=1$

c. $\log x + \log 2 - \log 5 = 1$

d. $\log_3 2x + \log_3 5 = -2$

e. $\log_4 x + \log_4 (x-6) = 2$

f. $\log_2 (x+1) - \log_2 x = 1$

g. $\log_4 (x+1) = 2 + \log_4 (x-2)$

h. $\log_4 2x - (\log_4 3 + \log_4 5) = 0$

i. $\log_{16}(x-3) = 0.5 - \log_{16} x$

j. $0.5\log_2(x+8) = 1 + \log_2(0.5x+1)$

4. Answer the questions about the function $f(x) = -2 \cdot 0.9^x + 15$.

a. What is the equation of its asymptote?

b. What are the coordinates of its *y*-intercept?

c. Sketch a rough graph of $f(x)$.

d. What is the zero of $f(x)$? (use logs)

e. Solve the equation $f(x) = 14$.

f. What is $f^{-1}(x)$?

5. Answer the following questions about $f(x) = \frac{1}{2}\log_5(x+5)+1$.

a. What is the domain of $f(x)$?

b. What are the coordinates of the y-intercept of $f(x)$?

c. What is $f(20)$?

d. What is $f(-4)$?

e. What is the zero of $f(x)$?

f. Solve the equation $f(x) = 0.5$.

g. The inverse of $f(x)$ is an exponential function; find it.

ANSWERS

1a. 2 b. -2 c. 2 d. 2/3 e. -2 f. ½ g. -1 h. -1 i. 2
j. 1 k. -1 l. 2 m. -1 n. 1 o. ¼ p. -1/3

2. $\log 2 + \log x + \log y$ b. $\log 8 + \log x - \log y$ c. $\log 1 - \log x - \log y = -\log x - \log y$

3a. 2/3 b. 1 c. 25 d. 1/90 e. 8 (-2 is extran) f. 1 g. 33/15 h. 7.5 i. 4 (-1 is extran) j. 1

4a. y=15 b. (0,13) c. under asymptote—nears it on right d. -19.12 e. 6.58
f. $f^{-1}(x) = \log_{0.9}((15-x)/2)$

5a. x>-5 b. (0,1.5) c. 2 d. 1 e. -124/25 f. -24/5 g. $5^{2x-2} - 5$

Unit 9 Handout #4: Solving Exponential Equations with Logs

1. Solve the following equations (calculators OK but not using calc-intersect; use logs instead):

$a.\ 2^x = 7$ $b.\ 3^x = 15$ $c.\ e^{0.3x} = 45$

$d.\ 1.15^t = 2.13$ $e.\ 20(0.95)^x = 15$ $f.\ 75(0.5)^{x/6} = 20$

$g.\ 3 \cdot 1.23^x = 7$ $h.\ 100(0.71)^x = 6$ $i.\ 5^x = 100$

2. Solve the following analytically (means calculator OK but not graphically using calc-intersect)

a. If an investment grows at 20% per year, how long will it take to triple?

b. If bacteria grow at 5% per hour, then how long will it take for a population of 500 to reach 1875?

c. A population of amoeba grows from 50 to 500 over a 30-hour period. What rate of hourly compounding do they grow at?

3. It turns out that $\log_3 7 \approx 1.77$ (check it by raising 3 to the 1.77^{th} power!). You should know what $\log_3 3$ is. Use these two things and the laws of logs to evaluate the following without your calculator.

Example: $\log_3 \frac{7}{3} = \log_3 7 - \log_3 3 = 1.77 - 1 = 0.77$. This checks, as 3 to the 0.77^{th} power is 7/3 (2.33)

a. $\log_3 (7^{-2})$

d. $\log_3 49$

b. $\log_3 \frac{1}{7}$

e. $\log_3 \frac{9}{7}$

c. $\log_3 21$

f. challenge: $\log_3 \sqrt[10]{\frac{98}{54}}$

4. If log 6 is about 0.8 then evaluate the following without your calculator.

$a.$ $\log 60$

$b.$ $\log \frac{1}{36}$

$c.$ $\log 3 + \log 12$

$d.$ $\log 360$

$e.$ $\log(1/600)$

5. If $\log 2 \approx 0.3$ and $\log 3 \approx 0.5$ then approximate these without your calculator.

a. $\log 18$ b. $\log \frac{3}{4}$ c. $\log 600$ d. $\log 0.4$

e. $\log 15$ f. $\log \frac{4}{30}$ g. $\log \sqrt{6}$ h. $\log \sqrt[5]{24}$

6. Evaluate the following (no calculator)

a. $\log_6 12 - \frac{1}{2}\log_6 4$ b. $2\log_4 3 - 2\log_4 6$ c. $3\log 2 + \log 25 - \log 2$

d. $\log_9 12 - 2\log_9 2$ e. $5^{\log_5 11}$ f. $6^{\log_6 4}$

7. Solve the following equations using logarithms:

a. $10 \cdot 1.15^x = 20 \cdot (1.04)^x$

b. $800(0.91)^x = 300(1.07)^x$

c. $20 \cdot 2^x = 18 \cdot 2.4^{x-1}$ (deal with the -1 first!)

d. $50 \cdot e^{0.1x} = 80 \cdot 1.12^x$

8. The population of orioles in the park is initially 200 and grows by 8 percent per year. The population of bluebirds is initially 100 and grows by 11 percent per year. Use logarithms to find the time that the populations are equal.

9. The population of one type of bacteria is initially 500 and grows by 8% per hour. A different type of bacteria has an initially population of 1400 but shrinks by 8% per hour. When are the two populations equal, and what are the populations at this time?

10. Ernie joined Sesame Street at the start of 1960, earning \$500 per year. Bert joined in 1965 earning only \$400 per year. Ernie's salary has been rising by 11 percent per year since then and Bert's has risen at 15 percent per year.

a. When did Ernie's salary first hit \$1000? (answer as a fractional/decimal year is ok)

b. What was Ernie's salary at the time Bert's salary hit \$700 per year. (Notationally you can think of this as $E(B^{-1}(700))$.)

c. When were the two stars' salaries equal?

d. Can you use logs to determine when Bert's salary was exactly \$600 more than Ernie's? Explain.

e. When was Bert's salary twice Ernie's?

11. Solve the following equations using logarithms. Remember to isolate the exponential term first:

a. $\dfrac{5}{1+4\cdot 2^{x}}=2$ b. $\dfrac{200}{1+5\cdot e^{-0.2x}}=50$ c. $\dfrac{20}{1+5\cdot 1.12^{x}}=2$

12. The population of deer in a park t years from now is $P(t)=\dfrac{1000}{1+19e^{-0.2t}}$.

a. What is the current population?

b. What will the population eventually approach? (no calculator—reason it out)

c. What is $P(1)-P(0)$ and what does it mean? (calc OK)

d. Analytically determine when, if ever, the population will reach 750.

13. A power function can be written in the form of $f(x)=a\cdot x^b$ where a and b are constants.

a. Are power functions exponential functions? Explain.

b. For power function $f(x)$, $f(5)=20$ *and* $f(7)=40$. Find the equation of $f(x)$. You will probably need logarithms.

c. Find the equation of a power function $g(x)$ where $g(2)=10$ *and* $g(10)=2$. No calculator allowed!

d. Algebraically, find any/all points of intersection between the functions $f(x)$ *and* $g(x)$ described above.

14. In Mathletics, Wayne Wilson approximates the likelihood (p) of a professional football kicker making a field goal of w yards as $\ln\left(\frac{p}{1-p}\right) = 7.05 - 0.134w$.

a. What is the likelihood of making a 40-yard field goal?

b. If the likelihood of making a field goal is 50%, how long is the field goal?

c. Challenge: solve for p in terms of w.

15. It turns out that, if the probability of an event occurring (like a coin coming up tails) is p, then the probability that it occurs at least one time in n attempts is equal to $1-(1-p)^n$. Since the probability of a coin coming up tails is 0.5, the probability that at least one tail comes up in 6 flips is $1-(1-0.5)^6$ or 63/64.

a. Suppose the probability of a machine part failing is 1% (ie 0.01). If there are 12 such parts in a machine, what is the probability that at least one fails?

b. How many such parts are needed in a machine before the probability that one fails first exceeds 40%? Use logarithms.

<u>Answers</u>

1a. 2.81 b. 2.46 c. 12.69 d. 5.41 e. 5.61 f. 11.44
g. 4.09 h. 8.21 i. 2.86 2a. 6.03 years b. 27.09 hours c. 7.98%
3a. -3.54 b. -1.77 c. 2.77 d. 3.54 e. 0.23 f. 0.054
4a. 1.8 b. –1.6 c. 1.6 d. 2.6 e. -2.8
5a. 1.3 b. –0.1 c. 2.8 d. –0.4 e. 1.2 f. –0.9 g. 0.4 h. 0.28
6a. 1 b. -1 c. 2 d. ½ e. 11 f. 4 7a. 6.89 b. 6.06 c. 5.38 d. -35.26
8. 25.3 years 9. After 6.42 hours both populations are about 819.6 (What is 0.6 of a bacteria?)
10a. after 6.64 years (mid-late 1966) b. $1279 c. after 26.04 years so in 1986 d. no
e. 45.6 years so in 2005 11a. -1.415 b. 2.554 c. 5.187
12a. 50 b. 1000 c. 10.4; the population increase from this year to next year d. after 20.22 years
13a. no; the variable is in the base, not the exponent. b. $f(x) = 0.726 \cdot x^{2.06}$
c. $g(x) = 20 \cdot x^{-1}$ d. (2.96, 6.77) 14a. 84.4 b. 52.6 yds c. $p = \frac{e^{7.04-0.134w}}{1+e^{7.04-0.134w}}$
15a. $1-0.99^{12} \approx 11.4\%$ b. $1-0.99^n = 0.4$ so n=50.82 or when 51 parts are there.

Unit 9 Handout #5: Practice with Logs
Work on Separate Paper

1. Evaluate the following (no calculator)

a. $\log_6 12 - \frac{1}{2}\log_6 4$

b. $2\log_4 3 - 2\log_4 6$

c. $3\log 2 + \log 25 - \log 2$

d. $\log_9 12 - 2\log_9 2$

e. $5^{\log_5 11}$

f. $6^{\log_6 4}$

2. For each part below, condense the expression below into a single logarithm and then evaluate it.

e. $\frac{1}{2}\log_5 16 - (\log_5 10 + \log_5 2)$

f. $\frac{1}{2}\log_6 3 + 0.5\log_6 12$

g. $2\log_{16} 6 - \left(2\log_{16} 3 + \frac{1}{2}\log_{16} 4\right)$

h. $\log_8 18 - 2(\log_8 24 - 2\log_8 2)$

3. Condense each of the following into one logarithm:

a. $2\log x + 3\log 3$

b. $\frac{1}{3}\log 8 + \frac{2}{3}\log 27$

c. $2\ln(xy) - \ln(2x)$

d. $2\log 3x - (\log 9 + \log 6x)$

e. $2\log(9x) + \log x - 3\log(2x)$

f. $2\log(3x) - 0.5\log y + \log 4x$

4. Condense the logs on one side of the equation until to get something in the form $\log_a b = c$. Then rewrite as an exponential equation and solve it. Note: extraneous solutions occur when an answer would cause you to take the log of zero or a negative number in the original equation:

a. $\log 4 + 2\log x = 2$

b. $0.5\log_4 x - \log_4 5 = 1$

c. $\log_8(2x^2) = 2$

d. $\log_6 x + \log_6(x+5) = 2$

e. $\frac{1}{2}\log_4(x-4) + \log_4 2 = 2$

f. $2\log_5(x-2) - \log_5 3 = 2$

5. Expand the following logs: Write in terms of $\log x$, $\log y$, *and* $\log z$ (and log of a number):

Example: $\log 2x^2 y$ - $\log 2 + \log x^2 + \log y = \log 2 + 2\log x + \log y$

a. $\log\frac{3}{x^3}$

b. $\log(2x^2 \cdot \sqrt{y})$

c. $\log\left(\frac{7}{\sqrt[3]{x}}\right)$

d. $\log\left(\frac{3\sqrt{y}}{5x^2}\right)$

6. Solve the following equations. You should only need your calculators for the **'ed ones. Don't forget to check for extraneous solutions. Look at the first one: given that it is log(a)=log(b), you do NOT need to put both logs on one side and condense it; you can just set a=b. This only works if there is one logarithm on each side with the same base and no numbers outside the logarithm—so it will not work for question e below.

a. $\log x = \log 15$

b. $\log_{1/2} x - 2\log_{1/2} 3 = 2$

c. $\log_3 2x - \log_3 5 = 2\log_3 8$

d. $2\log_3 3x = 3$

e. $3\log x = 2 + \log 2$

f. $\log_5 2x = \log_5 7 + 2\log_5 3$

g. $2\ln x = \ln 16$

h. $2^x - 1 = 7$

i. ** $2^x - 1 = 17$

j. ** $2^{x-5} = 17$

k. $2\ln 2 + \ln x^2 = \ln 9$

l. $\frac{1}{3}\log(x-3) = \log 6 - \log 3$

m. ** $5 \cdot 1.09^x = 7.8$

n. $2\log x - 3\log 2 = 2 - \log 5$

o. ** $1.18^x - 3 = 2.5$

p. $2\log_5 3x - 3\log_5 2 = 2\log_5 3$

q. $\log_4 2x + 2\log_4 3 = -\log_4 5$

r. ** $(x+2)^5 = 37$

7-11. Solve the following word problems. Calculators are OK, but **you should not use calc-intersect**; use logarithms instead (we call this "solving analytically.")

7. The population of some town is 12,500. It has been growing at 4% per year and expected to continue to grow at this rate.

a. What was its population 6 years ago?

b. What do you expect its population to be in 15 years?

c. When will its population reach 19,000?

d. When was its population 7,500?

e. How many years will it take its population to triple?

f. Instead of growing at 4% per year going forward, it grew faster. Its population reached 17,000 in 6 years. What was its annual growth rate?

8. How long will it take an investment to double in value if it increases by 15% per year?

9. A population of bacteria doubles every 6 days.

a. What is the daily percentage growth rate?

b. Using this rate, how long would it take the population to grow from 50 to 600?

10. The value of a car is best modeled by an exponential function because it falls by approximately the same percentage amount each year. A new car is worth $25,000. Seven years later it is worth $8,800.

a. By what percentage did the car's value fall each year? (percentage rounded to one decimal place)

b. How old is the car when its value is $2000?

11. A population of bacteria declines such that its half-life is 21 days (ie, it loses half of its value every 21 days).

a. How long does it take to fall from 100 to 30? Estimate before you calculate.

b. What is the daily percentage decline in the population?

c. A different population falls from 200 to 20 in 60 days. What it its half life? Hint: you may want to find the daily percentage decline first.

12. Find the y-intercepts (if any) and zeros of these functions analytically (meaning not graphically). You may need your calculators to evaluate logarithms.

a. $f(x) = 3 \cdot 2^x - 7$

b. $f(x) = 2 \cdot 5^{x-2} - 12$

c. $f(x) = 2\log_9(x-4) + 1$

d. $f(x) = 2\log_7(x+7) - 4$

13. A sweet potato is put in a hot oven. The potato's temperature (in degrees Fahrenheit) after t minutes is given by the function $P(t) = -240 \cdot 0.97^t + 300$.

a. What is the significance of the 300 in the equation?

b. Sketch a very rough graph of the potato's temperature over time. What is a reasonable domain?

c. What is the potato's temperature initially?

d. When does the potato's temperature first hit 200°?

e. What is the meaning of $P^{-1}(260)$ and what is its value?

14. An exponential function has a horizontal asymptote of the x-axis. Additionally, $f(4) = 50$ and $f(7) = 20$.

a. Write an equation for $f(x)$.

b. Find $f(5)$ *and* $f^{-1}(5)$ (note: for the second part DO NOT find the inverse function first).

c. Find the inverse function $f^{-1}(x)$.

15. Lemonade stand. Cindy has a gallon (128 fluid ounces) of lemonade in a pitcher. But sales are slow, so she decides to drink some. Not wanting her parents to know, she takes 4 ounces of lemonade out and replaces it with four ounces of water, then mixes it.

a. If the lemonade originally was "100% strength", then what percent strength is the lemonade after Cindy does this?

b. Cindy continues to do this. What percent strength is the lemonade after she does it 4 times?

c. How many times can Cindy do this before the lemonade is less than half its original strength?

ANSWERS

1a. 1 b. -1 c. 2 d. ½ e. 11 f. 4

2a. 1 b. -1 c. 2 d. ½ e. -1 f. 1 g. 1/4 h. -1/3

3a. $\log 27x^2$ b. $\log 18$ c. $\ln\left(\frac{xy^2}{2}\right)$ d. $\log\frac{x}{6}$ e. $\log\frac{81}{8}$ f. $\log\frac{36x^3}{\sqrt{y}}$

4a. 5 (-5 is extran) b. 400 c. $\sqrt{32}$ *or* $4\sqrt{2}$ (negative is extran) d. 4 (-9 is extran)
e. 68 f. $2+5\sqrt{3}$ $(2-5\sqrt{3}$ *is extraneous*)

5a. $\log 3 - 3\log x$ b. $\log 2 + 2\log x + 0.5\log y$ c. $\log 7 - \frac{1}{3}\log x$ d. $\log 3 + 0.5\log y - \log 5 - 2\log x$

6a. 15 b. 9/4 c. 160 d. $\sqrt{3}$ e. $\sqrt[3]{200}$ f. 31.5
g. 4 (not -4) h. 3 i. 4.17 j. 9.09
k. ±3/2 (since we are squaring before taking logs, -3/2 is OK)
l. 11 m. 5.16 n. $4\sqrt{10}$ o. 10.30 p. $2\sqrt{2}$ q. 1/90 r. 0.059

7a. 9879 b. 22512 c. 10.68 yrs d. 13.02 yrs ago e. 28.01 f. 5.26%

8. 4.96 yrs 9a. 12.25% b. 21.5 days 10a. 13.9% b. 16.9 years

11a. about 36.5 days (you should have know it was a little less than 42 days)
b. 3.25% c. 18.06 days

12a. zero is 1.22 and y-int is -4 b. zero is 3.11 and y-int is -298/25
c. zero is $4\frac{1}{3}$ and no yint (not in domain) d. zero is 42 and y-int is -2

13a. the horizontal asymptote is the oven's temperature. b. t≥0
c. $P(0) = 60$ degrees d. after 28.74 minutes e. when the temp hits 260; 58.8 minutes

14a. $f(x) = 50 \cdot 0.737^{x-4}$ or $f(x) = 169.65 \cdot 0.737^x$ b. $f(5) = 36.9$
$f^{-1}(5)$ *means* $169.65 \cdot 0.737^x = 5$ *so* $x = 11.55$ c. $f^{-1}(x) = \log_{0.737}(x/169.65)$

15a. 124/128 = 96.9% b. $\left(\frac{124}{128}\right)^4 \approx 88.1\%$ c. $\left(\frac{124}{128}\right)^n = 0.5$ so n = 21 times; on 22nd it gets < 50%

Unit 9 Handout #6: More Log Equations

1. Solve the following equations. Note: in these equations, all terms are logs (with the same base), so instead of condensing into $\log(blah) = 0$ and then rewriting as an exponential equation, we can just write it as $\log(a) = \log(b)$ and then equate a and b. Be sure to check for extraneous solutions.

a. $\log x = \log 7$

b. $\log x = -\log 7$

c. $2\log x - \log 3 = \log 24 - 0.5\log 4$

d. $0.5\ln(2x) = 2\ln 3$

e. $(2/3)\log x + \log 2 = \log 18$

f. $\log(2 - x) = \log(x + 6)$

g. $2\log x - \log(x + 4) = 0.5\log(4)$

2. Solve each of these equations for y in terms of the other variables. Hint: start solving them like any other log equation, where you either make it log(a)=log(b) or log(a) = #.

a. $\log x = 1 + 0.5\log y$

b. $\log_4 y - \log_4 3 = \log_4(x) + 1$

c. $2\log_3 y = 1 + 2x$

d. $3^{y-4} = 4x - 1$

3. Solve the following equations using factoring and logs. At least one has a "hidden quadratic". You may leave answers in log form rather than converting to a decimal)

a. $x^2 e^{-x} - 2xe^{-x} = 0$

b. $e^{2x} - 3e^{x} = 4$

c. $x^2 \ln x - 5\ln x = 0$

d. $x^2 e^{-x} - 2x^2 e^{2x} = 0$

e. $4^x - 5 \cdot 2^x + 4 = 0$

f. $\dfrac{e^x - 5e^{-x}}{2} = 2$

Answers

1a. 7 b. 1/7 c. 6 (-6 is extran) d. 40.5 e. 27 f. -2 g. 4 (-2 is extran)

2a. $y = x^2/100$ b. $y = 12x$ c. $y = \sqrt{3^{1+2x}}$ *or* $y = 3^{0.5+x}$ d. $y = 4 + \log_3(4x - 1)$

3a. 0, 2 (e^{-x} is never zero) b. $\ln 4$ (the other answer, ln(-1) is not valid)

c. 1, $\sqrt{5}$ (but not -$\sqrt{5}$) d. 0, $\ln(0.5)/3$ e. 2, 0 (since $4^x = 2^{2x}$) f. ln5

Units 8-9 Practice Problems
Work on Separate Paper

1. Sketch a graph of the following functions. Indicate the range, domain, and asymptote of each. (no calculator)

a. $f(x) = 3\cdot 2^x - 1$ *b.* $f(x) = -2\cdot\left(\frac{1}{2}\right)^x + 3$ *c.* $f(x) = -3\cdot 4^x - 2$

d. $f(x) = 2\log_4(x-3)$ e. $f(x) = -0.5\log(x+4) - 2$

2. Find the zero and y-intercept of each function below:

a. $f(x) = -\frac{3}{2}\cdot\left(\frac{1}{2}\right)^x + 6$ *b.* $f(x) = 2\log_2(x+4) - 6$

3. Solve the following exponential equations without using a calculator:

a. $4\cdot 3^{x+1} = 108$ b. $2^{x+4} = 16^2$ c. $\left(\frac{1}{4}\right)^x = 8$

d. $3^{2x} = \left(\frac{1}{9}\right)^{x-3}$ e. $3\cdot 4^{3x} - 6 = 42$ f. $2^2\cdot 2^x = 8^{3-2x}$

g. $3^x\cdot 9^{x-2} = \frac{1}{27}$ h. $\frac{2^{x^2}}{2^{3x}} - 14 = 2$

4. Evaluate the following logarithms without calculator

a. $\log_5 125$ b. $\log_3\frac{1}{9}$ c. $\log 5 + \log 20$ d. $\log_{1/2} 4$

e. $\log_4\frac{1}{2}$ f. $2\log_6 2 + \log_6 9$ g. $\log_3\left(27\sqrt{3}\right)$ h. $2\log_8 2 - \log_8 3 + \log_8 12$

i. $\ln e^{11}$ j. $\ln\frac{1}{\sqrt[5]{e}}$ k. $0.5\log_3 16 - 2\log_3 6$ l. $\frac{2}{3}\log_5 8 - (\log_5 10 + \log_5 2)$

5. Condense each of the following expressions into a single log:

a. $\log 5 + 2\log 3$

b. $\log 4 + 2\log 3 - 3\log 2$

c. $2\log 5 - 5\log 2$

d. $2\log_7 2x + \log_7 y - 2\log_7 3$

e. $2\ln 3 - 0.5\ln x$

f. $2(\log 6 - \log 2) + 3(\log 8 - \log 4)$

g. $\frac{1}{3}\log_5 27 + \frac{2}{3}\log_5 8 - \frac{3}{2}\log_5 9$

6. Expand the expression until everything is written in terms of $\log x$, $\log y$, *and* $\log z$:

a. $\log x^2$ b. $\log(yz)^2$ c. $\log\sqrt[5]{x^3}$

7. Answer the following questions about $f(x) = \log_3(x+4) - 2$.

a. What is $f(-1)$?

b. What is $f(-3)$?

c. What is the domain of $f(x)$?

d. Find the zero of $f(x)$.

e. What is the solution to $f(x) = -3$?

f. What is the solution to $f(x) = 1$

g. Find the inverse function $f^{-1}(x)$.

8. Find the inverse of $f(x) = 2 \cdot 4^x - 6$

9. Solve the following equations (calc OK for d and i only):

a. $\log_4 x - \log_4 2 = 2$

b. $2\log x - \log 3 = 2$

c. $\log_4 x - 2 = 1$

d. $4^x + 1 = 11$

e. $2\log x - 1 = \log 2x + 1$

f. $3 \cdot 2^{2x} + 3 = 27$

g. $\log_8(x-5) = \log_8(2x-9)$

h. $\log_5(3x+1) = 2$

i. $10^x + 4 = 16$

j. $\log_3 x + \log_3(x-2) = 1$

k. $\left(\frac{1}{4}\right)^{x-1} = 8$

l. $1 - 2\log_3 2x = -3$

m. $4\log_7 3x = 8$

n. $\log x + 2\log 2 = \log(2x+4) - 3\log 2$

o. $\log_x 8 = 3$

p. $\log 2 + 3\log(x-1) = 2$

q. $\log(x+7) = 2\log(x+1)$

10. Solve with logs (calculator OK):

a. $10^x = 91$

b. $2e^{2x-1} + 3 = 97$

c. $20 \cdot 1.7^x = 80$

11. Solve the following word problems:

a. In 1985 Americans ate an average of 250 apples per year each. This number has fallen at 1% per year. How many apples per person did Americans eat in 2002?

b. Based on problem (a) above, in what year was the annual per-person consumption equal to 225?

c. If the population of bacteria in a given culture rose from 50 to 150 in 22 hours then what was the hourly growth rate?

d. An antique table is now worth $5000. If its value increased by 9% per year over the past 10 years, then what was its value 10 years ago?

e. How much will that table be worth seven years from now (assuming the rate of growth remains 9%)?

f. Money left is some bank grows at 5% per year. How many years will it take your money to double in value?

12. Write the equation of each function described below:

a. An exponential function with an asymptote of the x-axis where $f(0) = 10 \textit{ and } f(5) = 40$.

b. An exponential function with an asymptote of the x-axis where $f(3) = 90 \textit{ and } f(7) = 30$.

13. Given that $\log 2 \approx 0.3$, find the following without your calculator:

a. $\log\sqrt{2}$

b. $\log 25$

c. $\log\frac{1}{200}$

d. $\log\frac{5}{8}$

ANSWERS

1a. Dom: is all reals; Range is y>-1; asymptote is y=-1
 b. Dom is reals; range is y<3; asymptote is y=3
 c. domain is all reals; range is y<-2; asymptote is y=-2
 d. domain is x>3; asymptote is x=3; range is all reals
 e. domain is x>-4; asymptote is x=-4; range is all reals

2a. y-intercept is (0,4.5); zero is –2 b. y-intercept is (0,-2); zero is 4

3a. 2 b. 4 c. -3/2 d. 3/2 e. 2/3 f. 1 g. 1/3 h. 4 or –1

4a. 3 b. -2 c. 2 d. -2 e. -1/2 f. 2 g. 3.5 h. 4/3 i. 11 j. -1/5 k. -2 l. -1

$5a.\ \log 45 \quad b.\ \log\frac{9}{2} \quad c.\ \log\frac{25}{32} \quad d.\ \log_7\frac{4x^2y}{9} \quad e.\ \ln\frac{9}{\sqrt{x}} \quad f.\ \log 72 \quad g.\ \log_5\frac{12}{27}=\log_5\frac{4}{9}$

$6a.\ 2\log x \quad b.\ 2\log y+2\log z \quad c.\ \frac{3}{5}\log x$

7a. -1 b. -2 c. x>-4 d. 5 e. -11/3 f. 23 g. $y=3^{x+2}-4$

8 $f^{-1}(x)=\log_4(\frac{x+6}{2})$

$9a.\ 32 \quad b.\ \sqrt{300}\ or\ 10\sqrt{3} \quad c.\ 64 \quad d.\ 1.66 \quad e.\ 200 \quad f.\ \frac{3}{2} \quad g.\ no\ solutions \quad h.\ 8$

$i.\ 1.08 \quad j.\ 3 \quad k.\ -\frac{1}{2} \quad l.\ \frac{9}{2} \quad m.\ \frac{49}{3} \quad n.\ \frac{2}{15} \quad o.\ 2 \quad p.\ \sqrt[3]{50}+1\ or\ 4.68 \quad q.\ 2$

10a. 1.96 b. 2.425 c. 2.61

$11a.\ 211 \quad b.\ 10.5\ years\ or\ 1995 \quad c.\ 5.1\% \quad d.\ \$2112 \quad e.\ \$9140 \quad f.\ 14.2\ years$

12a. $f(x)=10\cdot 1.320^x$ b . $f(x)=205\cdot 0.760^x$

13a. 0.15 b. 1.4 c. -2.3 d. note that 5/8=10/16 so log10-4log2 = -0.2

Unit 10 Handout #1: Fraction Review

Key ideas with fractions:

1. To multiply fractions, multiply numerators and denominators. You may simplify and/or "cross-cancel" first.

2. Dividing by a fraction is the same as multiplying by its reciprocal. (Why???)

3. When you add or subtract fractions, you must first get a common denominator.

1. Do the following fraction review problems:

a. $\frac{28}{8}$

b. $\frac{5+10}{5}$

c. $\frac{9-1}{3}$

d. $\frac{2}{3}\cdot\frac{1}{4}$

e. $\frac{3}{4}\cdot\frac{8}{9}$

f. $\frac{5}{7}\cdot 14$

g. $\left(\frac{-2}{3}\right)^2\cdot 4^{-1}$

h. $\frac{6}{11}\div 3$

i. $4\div\frac{3}{5}$

j. $\frac{2/5}{4/3}$

k. $2-\frac{1}{3}$

l. $\frac{2}{3}-\frac{1}{4}$

2. What is $\frac{4+7}{2}$? Can you cancel the 4 and 2 to get $\frac{2+7}{1}$? Why or why not?

3. Mom gives her three children \$15 and a box of 30 cookies to share evenly. What is each child's share? What does this tell us about the correct way to simplify $\frac{15d+30c}{3}$?

4. What is $3^{-1}-4^{-1}+5^{-1}$?

Answers

1a. $\frac{7}{2}$ b. 3 c. $\frac{8}{3}$ d. $\frac{2}{12}=\frac{1}{6}$ e. $\frac{2}{3}$ f. 10 g. $\frac{1}{9}$ h. $\frac{2}{11}$ i. $\frac{20}{3}$ j. $\frac{3}{10}$ k. $\frac{5}{3}$ l. $\frac{5}{12}$

2. $\frac{11}{2}\neq 9$, only cancel if it is part of all terms (or if you have factored in out)

3. \$5 and 10 cookies; $\frac{15d+30c}{3}=\frac{15d}{3}+\frac{30c}{3}=5d+10c$ 4. $\frac{17}{60}$

Unit 10 Handout #2: Rational Expressions and Equations

Examples

1. Multiply and divide rational expressions:

-Cancel terms: “factors” or “whole terms only”

-Multiply numerators; multiply denominators.

-To divide, multiply by the reciprocal.

Example #1

$$\frac{x^3-4x}{x^2+x-2}\cdot\frac{3x-3}{6x}=\frac{x(x+2)(x-2)}{(x+2)(x-1)}\cdot\frac{3(x-1)}{6x}=\frac{x-2}{2}$$

Example #2

$$\frac{x^2-3x}{x^2+1}\div(x^2-9)=\frac{x(x-3)}{x^2+1}\cdot\frac{1}{(x+3)(x-3)}=\frac{x}{(x^2+1)(x+3)}$$

2. Add and subtract rational expressions:

-Find the lowest common denominator: look for common factors

Example #3

$\frac{3}{x+1}-\frac{5x}{x+2}$ the LCD is $(x+1)(x+2)$ so $\frac{x+2}{x+2}\cdot\frac{3}{x+1}-\frac{5x}{x+2}\cdot\frac{x+1}{x+1}=$

$$\frac{3x+6}{(x+2)(x+1)}-\frac{5x(x+1)}{(x+2)(x+1)}$$

Which is $\frac{3x+6-5x(x+1)}{(x+2)(x+1)}$ or $\frac{-5x^2-2x+6}{(x+2)(x+1)}$

Example #4

$\frac{3x-1}{x^2-x}+\frac{x+2}{x^2-1}$ → factor denominators to find LCD : $\frac{3x-1}{x(x-1)}+\frac{x+2}{(x-1)(x+1)}$

So the LCD is $x(x-1)(x+1)$ (like the LCD of 18 and 24 is 3*6 and 4*6 so 3*4*6 = 72)

$$\frac{x+1}{x+1}\cdot\frac{3x-1}{x(x-1)}+\frac{x+2}{(x-1)(x+1)}\cdot\frac{x}{x}=\frac{3x^2+2x-1}{x(x-1)(x+1)}+\frac{x^2+2x}{x(x-1)(x+1)}=\frac{4x^2+4x-1}{x(x-1)(x+1)}$$

3. **Solving Rational Equations**

-Can cross multiply if there is one fraction on each side.

-Can multiply through by the LCD to eliminate all fractions

-Watch for extraneous solutions by checking that the all terms in the original equation are defined for the solutions you find.

Example #5

Solve: $\dfrac{x}{x+8}=\dfrac{2}{x+2}$

Cross multiply: $x(x+2)=2(x+8)$ so $x^2+2x=2x+16$ and $x^2=16$ so $x=\pm 4$

Both answers check.

Example #6

Solve: $\dfrac{5}{x-1}-1=\dfrac{8}{x}$

Multiply all terms by $(x)(x-1)$ so $\dfrac{x(x-1)}{1}\cdot\dfrac{5}{x-1}-\dfrac{x(x-1)}{1}\cdot 1=\dfrac{x(x-1)}{1}\cdot\dfrac{8}{x}$

Now simplify each terms: all denominators should disappear if you've done it right!

$5x-(x^2-x)=8x-8$ so $0=x^2+2x-8$ and $(x+4)(x-2)=0$ so $x=2\ or\ -4$

Both answers check.

Example #7

Solve: $\dfrac{3x}{(x-1)^3}=\dfrac{7}{(x-1)^2}$

If you cross-multiply you end up with a cubic which is ugly. Instead multiply all terms on both sides by the LCD which is $(x-1)^3$.

You get $3x=7(x-1)$ so $x=7/4$, which checks.

Example #8

Solve: $\dfrac{1}{x-6}+\dfrac{x}{x-2}=\dfrac{4}{x^2-8x+12}$

Multiply both sides by the LCD of $(x-6)(x-2)$ to get $(x-2)+x(x-6)=4$

Which simplifies to $x^2-5x-6=0$ *or* $(x-6)(x+1)=0$.

So *x* appears to be 6 or -1. But you cannot plug 6 into the original equation since that would involve dividing be zero so *x*=6 is extraneous and the only solution is *x*=-1.

Note: the only extraneous solutions will be ones that cause you to divide by zero. It should never be the case that you plug a solution in and the two sides are both defined but unequal (unless there is an absolute value or a radical involved somewhere.)

Key ideas with fractions:
1. To multiply fractions, multiply numerators and denominators. You may simplify and/or "cross-cancel" first.
2. Dividing by a fraction is the same as multiplying by its reciprocal. (Why???)
3. When you add or subtract fractions, you must first get a common denominator.

RATIONAL EXPRESSIONS: Simplify the following, if possible. If unsure, plug in numbers to test!

1. $\dfrac{2x+4}{6}$

2. $\dfrac{x^2-1}{2x-2}$

3. $\dfrac{x^2+1}{x^2-1}$

4. $\dfrac{3x-3}{x^2+4x-5}$

5. $\dfrac{x^2+5x}{x^2}$

6. $\dfrac{x^2+5x}{x^2+5}$

7. $\dfrac{x^4-x^2}{x^3-5x^2-6x}$

8. $\dfrac{2x^2y}{3y^2}\cdot\dfrac{9x^3y^2}{18x}$

9. $\dfrac{16x^3+8x}{4}\cdot(2x^2+1)^{-1}$

10. $\dfrac{9-x^2}{x^2-3x}$ hint: factor -1 out of numerator.

11. $\dfrac{x^2-1}{x^2-4x+3}\cdot\left(x^3-5x^2+6x\right)$

12. $\dfrac{4x^2y}{6xy^2} \div \left(\dfrac{2x^2y^2}{3y}\right)^{-1}$

13. $\dfrac{\dfrac{6x}{3x-12}}{\dfrac{x^2-2x}{x^2-6x+8}}$

14. $\dfrac{4x^2-8x-12}{-x^2+1} \div \left(2x^2-18\right)$

15. $\dfrac{2x-1}{x+3} \cdot (x^2-9) \div \dfrac{2x^2-7x+3}{17x}$

16. $\dfrac{(x+2)^{-2} \cdot (x^3-3x^2-10x)}{25x-x^3}$

17. Find the least common denominator for each problem below. You DO NOT have to do the addition.

a. $\frac{7}{100}+\frac{3}{20}$

b. $\frac{8}{15}+\frac{5}{21}$

c. $\frac{4}{9}+\frac{5}{12}$

d. $\frac{5}{x}+\frac{6}{x+1}$

e. $\frac{5}{2x}+\frac{3}{7x^2}$

f. $\frac{5}{x^2+x}+\frac{2x}{x^2-1}$

Add or subtract the following. Your answers should be single fractions, reduced if possible.

18. $\frac{5}{3x}-\frac{2}{3x}$

19. $\frac{6}{5x}-\frac{3}{x}$

20. $\frac{x+1}{x^2}-\frac{2}{x}$

21. $\frac{2}{x-3}+\frac{3}{x-2}$

22. $\frac{x+1}{x^2-4}-\frac{3}{x+2}$

23. $\frac{x-1}{x}-\frac{x}{x-1}$

24. $\frac{3x}{x^2-9}+\frac{4}{x^2+3x}$

25. $\frac{2x-1}{x-3}-\frac{4x+2}{x+3}$

26. $\frac{x}{x-3}-\frac{x-2}{x^2-2x-3}$

Solve the following equations. You can either multiply by the LCD or cross-multiply, if you have one fraction on each side.

27. $\frac{2}{3}x - 1 = \frac{3x+2}{5}$

28. $\frac{x-1}{x+2} = \frac{x-2}{x+3}$

29. $\frac{2x-1}{x+4} = -2$

30. $\frac{2x-1}{x+3} = \frac{x+1}{x-3}$

Answers

1. $\frac{x+2}{3}$ 2. $\frac{x+1}{2}$ 3. cannot simplify 4. $\frac{3}{x+5}$ 5. $\frac{x+5}{x}$

6. cannot simplify 7. $\frac{x(x-1)}{x-6}$ 8. $\frac{x^4 y}{3}$ 9. $2x$ 10. $\frac{-(x+3)}{x}$

11. $x(x-2)(x+1)$ 12. $\frac{4x^3}{9}$ 13. 2 14. $\frac{-2}{(x-1)(x+3)}$ 15. $17x$

16. $\frac{-1}{(x+5)(x+2)}$ 17a. 100 b. 105 c. 36 d. $x(x+1)$ e. $14x^2$

17f. $x(x+1)(x-1)$ 18. $\frac{1}{x}$ 19. $\frac{-9}{5x}$ 20. $\frac{-x+1}{x^2}$ 21. $\frac{5x-13}{(x-2)(x-3)}$ 22. $\frac{-2x+7}{x^2-4}$

23. $\frac{-2x+1}{x(x-1)}$ 24. $\frac{3x^2+4x-12}{x(x+3)(x-3)}$ 25. $\frac{-2x^2+15x+3}{(x-3)(x+3)}$ 26. $\frac{x^2+2}{(x-3)(x+1)}$ 27. 21

28. -0.5 29.-7/4 30. 0 or 11

Unit 10 Handout #3: Rational Equations and Expressions

Part I: Simplify the following rational expressions. Your answers should be single fractions, fully reduced, with no negative exponents.

1. $\dfrac{x^3-4x}{x^2-3x+2}\cdot\dfrac{2x^2-2}{x^2+4x+4}$

2. $\dfrac{2x^2-32}{6x^2+24x}\div\dfrac{x^2-5x+4}{3x^2}$

3. $\dfrac{5}{3}-\dfrac{x-1}{3x+6}$

4. $\dfrac{x+1}{x^2-1}-3$

5. $(x+2)^{-2}+(x+2)^{-1}$

6. $x+x^{-1}+x^{-2}$

7. $x\cdot x^{-1}\cdot x^{-2}$

8. $\dfrac{x+4}{x^2-4}-\dfrac{3}{x+2}$

9. $\dfrac{x+x^{-1}}{2x}$

10. $\dfrac{\frac{2}{x}-3}{1+\frac{5}{2x}}$

11. $\dfrac{3}{2x^2+6x}-\dfrac{1}{2}+\dfrac{x-1}{x+3}$

12. $(x+1)\left(\dfrac{1}{x^2-1}+\dfrac{2}{x+1}\right)$

13. $\dfrac{\frac{1}{x}-\frac{1}{2}}{x-2}$

14. $\dfrac{\frac{1}{x+4}-\frac{1}{6}}{x-2}$

Part II: Find all solutions to the following equations. Be sure to check for extraneous solutions.

1. $\frac{3}{x}+\frac{1}{3}=1$

2. $\frac{2}{x+1}-3=\frac{1}{2}$

3. $\frac{3}{x}+1=\frac{-9}{x-12}$

4. $\frac{x+3}{x+1}=\frac{4}{x-2}$

5. $\frac{2}{x+1}+1=\frac{3}{x-2}$

6. $\frac{5x}{x-2}-7=\frac{10}{x-2}$

7. $\frac{x+5}{x-3}+1=\frac{30}{x^2-9}$

8. $\frac{10}{x}-\frac{6}{x+1}=1$

9. $\dfrac{2x-3}{x-1}=\dfrac{x+1}{x+2}$

10. $2x-\dfrac{x}{3}=\dfrac{5}{x}$

11. $\dfrac{20+0.5x}{30+x}=0.6$

12. $\dfrac{3x}{x^2-1}=\dfrac{5}{x+1}$

ANSWERS

Part I: 1. $\dfrac{2x(x+1)}{(x+2)}$ 2. $\dfrac{x}{x-1}$ 3. $\dfrac{4x+11}{3(x+2)}$ 4. $\dfrac{-3x+4}{x-1}$ 5. $\dfrac{x+3}{(x+2)^2}$

6. $\dfrac{x^3+x+1}{x^2}$ 7. $\dfrac{1}{x^2}$ 8. $\dfrac{-2x+10}{(x+2)(x-2)}$ 9. $\dfrac{x^2+1}{2x^2}$ 10. $\dfrac{4-6x}{2x+5}$

11. $\dfrac{x^2-5x+3}{2x(x+3)}$ 12. $\dfrac{2x-1}{x-1}$ 13. $\dfrac{-1}{2x}$ 14. $\dfrac{-1}{6(x+4)}$

Part II: 1. 4.5 2. -3/7 3. 6, -6 4. 5, -2 5. $1\pm\sqrt{10}$ 6. no solns (2 is extran) 7. 2, -6

8. 5, -2 9. $\dfrac{-1\pm\sqrt{21}}{2}$ 10. $\pm\sqrt{3}$ 11. 20 12. 2.5

Unit 10 Handout #4: Rationals Practice and Word Problems

1. Simplify fully, if possible.

a. $\dfrac{x^2-5x+6}{x^2-4}\cdot\dfrac{2x+4}{2x^2-18}$

b. $\left(x^2-3x-4\right)\left[\dfrac{1}{x+1}-\dfrac{x}{x-4}\right]$

c. $\dfrac{x+4}{x^2-4}-\dfrac{3}{x+2}$

d. $\dfrac{5}{3x^2}-\dfrac{4x}{x+1}$

2. Simplify fully, if possible. Your answers should be single fractions, fully reduced:

a. $\dfrac{3x-9}{x^3-9x}\cdot(x^3+3x^2)$

b. $\left(\dfrac{6}{x+4}-\dfrac{5x-1}{x^2+2x-8}\right)\cdot\dfrac{x^2-16}{x^2-12x+11}$

c. $\left(\dfrac{x}{2}-3+\dfrac{6}{2x-2}\right)\div\dfrac{x^3-3x^2-4x}{4x^2-4}$

3. Solve the following equations. Be sure to check for extraneous solutions.

a. $\dfrac{3}{x-2}-\dfrac{1}{2}=5$

b. $\dfrac{10}{x^2-2x}+\dfrac{4}{x}=\dfrac{5}{x-2}$

c. $\dfrac{2}{3x}+\dfrac{2}{3}=\dfrac{8}{x+6}$

d. $\dfrac{3x}{x^2-4}=5x^{-1}$

e. $\dfrac{-2}{x-1}=\dfrac{x-8}{x+1}$

f. $\dfrac{3x+6}{x^2-4}=\dfrac{x+1}{x-2}$

4. These four equations all look the same. Solve them and see what happens. Be sure to check for extraneous solutions.

a. $\dfrac{x}{x-2}+\dfrac{1}{x+4}=\dfrac{12}{x^2+2x-8}$

b. $\dfrac{x}{x-2}-\dfrac{2}{x+4}=\dfrac{19}{x^2+2x-8}$

c. $\dfrac{x}{x-2}-\dfrac{2}{x+4}=\dfrac{12}{x^2+2x-8}$

d. $\dfrac{1}{x-2}+\dfrac{2}{x+4}=\dfrac{3x}{x^2+2x-8}$

5. You want to buy some books on line. Each book costs \$9, and the total shipping cost (no matter how many books) is \$15.

a. What is the average cost of each book if you buy 3 books? 5 books?

b. Write a function $c(x)$ showing the average cost when you buy x books.

c. How many books do you need to buy for the average cost to be \$9.50?

d. What is the horizontal asymptote of $c(x)$, and what meaning does it have in this problem?

6. A baseball or softball player's **batting average** is defined as the ratio of total hits to total times at bat. Lindsay got 10 hits in her first 20 times at bat.

a. If she gets 18 hits in her next 30 times at bat then what is her batting average?

b. How many hits does she need in her next 30 at bats to move her batting average to 0.600? [this is instead of the 18 hits in (a) above.]

7. Young's Rule is a formula that physicians use to determine the dosage levels of medicine for children based on adult dosage levels. The child's dose can be modeled by $c(t) = \dfrac{ta}{t+12}$ where c is the child's dose (in milligrams), a is the adult's dose (in milligrams), and t is the age of the child (in years). For a certain medicine, a = 100.

a. What is the dose for an 8-year old?

b. If the dose is 60 mg, how old is the child?

c. What is an equation of the horizontal asymptote and what does the asymptote represent in the context of the problem?

8. A different batter started the season with 15 hits in 40 at-bats.

a. If he gets hits in all of his next 12 at-bats, then what will his average be?

b. If he gets hits in all of his next x at bats, then what will his average be? Call this function $A(x)$.

c. How many at-bats in a row must he get hits in so that his average for the season will be 0.800?

d. What is the asymptote on the graph of $A(t)$, and what does it mean?

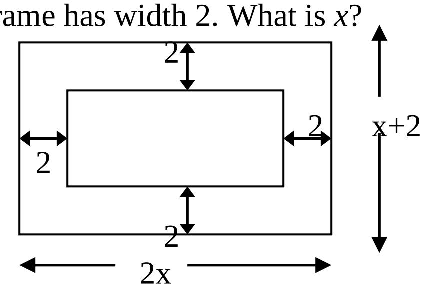
9. A positive number minus 3 times its reciprocal is –0.5. What is the number?

10. Shaq starts the basketball season by making 62.5% of his first 80 free-throw attempts.

a. How many in a row must he make to bring his free-throw percentage (successes divided by attempts) up to 70%?

b. Instead, if he hits 80% of his next x free throws, then what must x be to make his season free-throw percentage 70%?

11. In the diagram below, the ratio of the area of the large rectangle to that of the small rectangle is 3:1. The frame has width 2. What is x?



12. Twenty friends go out for pizza and split the check evenly. Each person's share is $x.

a. What is the total cost of the pizza?

b. If five more people join them (and they don't order any more pizza) then the cost falls by $1 per person. What is x?

13. A bunch of people shared the $240 daily cost of a math tutor equally. If three more people came, then the cost per person would be $4 less. How many people were there originally?

14. Natalie runs 3 miles at x mph and then increases her speed by 2 mph and runs another 4 miles. If her total time is 1 hour, then what was her original pace? [hints: rate*time=distance]

15. Bob can rake his yard in 3 hours (working alone). His son can do it in 5 hours (working alone). How long will it take them to do it together? Think of the fraction of the job they each do per hour.

16. Pipe A fills a tank in 3 hours and pipe B fills it in 2 hours. How long will it take to fill it together?

17. Sally can complete a job alone in 4 hours. Working together with Christine, it takes only 1.5 hours. How long would it take Christine to do the job herself?

18. Howie can grade the math placement exams alone in 7 hours. With Mark helping, it takes only 5 hours. How long would it take Mark to grade them alone?

19. A boat travels 10 miles per hour when there is no current. How long will it take the boat to go 20 miles, 10 upriver and 10 back downriver if the current is 2 mph? Hint: what are the boat's speeds relative to the shore in each direction?

20. A boat travels 15 km per hour when there is no current. It took a trip 18 km upriver, stopped for 1.5 hours, and then returned. If the total time was 4 hours, then what was the speed of the current?

21. If a train increased its speed over a 300 mile trip by 10 miles per hour, it would save one hour. What was its original speed?

22. If you drive half-way to your destination at 20 mph and average 30 mph for the entire trip, how fast do you need to go for the second half? The answer is not 40 mph!

ANSWERS

1a. $\frac{1}{x+3}$ b. $-x^2-4$ c. $\frac{-2x+10}{(x+2)(x-2)}$ d. $\frac{-12x^3+5x+5}{3x^2(x+1)}$ 2a. $3x$ b. $\frac{x-4}{(x-2)(x-1)}$ c. $\frac{2(x-3)}{x}$

3a. 28/11 b. no solutions (0&2 are extran) c. 2, 3 d. $\pm\sqrt{10}$ e. 2, 5 f. no solutions

4a. -7 (2 is extran) b. 3, -5 c. none (-4, 2 are extran) d. it is always true (except 2 & -4)

5a. $14, $12 b. $c(x)=\frac{9x+15}{x}$ c. $9.5=\frac{9x+15}{x}$ so x=30 d. y=9; as orders get big, avg cost → $9

6a. $\frac{hits}{at\_bats}=\frac{10+18}{20+30}=\frac{28}{50}=0.56$ b. $\frac{10+x}{20+30}=0.6 \;\; so \;\; x=20$

7a. $c(8)=800/20=40mg$ b. 18 years old c. y=100; as child gets older, dose → adult level of 100

8a. $\frac{15+12}{40+12}=0.519$ b. $A(x)=\frac{15+x}{40+x}$ c. $\frac{15+x}{40+x}=0.8 \;\; so \;\; x=85$ d. 1; if he keeps on getting hits, his average will get closer and closer to 1.000 (but will never get there since he had 25 misses).

9. $x-\frac{3}{x}=-\frac{1}{2}$ so x=1.5 10a. 20 b. 60

11. 6 (1 makes no sense) 12a. 20x b. 5 13. 12 14. 6

15. 15/8 hours 16. 6/5 hours 17. (1/4 + 1/x)(1.5)=1 so x=12/5 or 2.4 hours 18. 17.5 hours (yuk!)

19. upriver goes 8 mph and downriver goes 12 mph so 5/4 hr to go upriver and 5/6 to go down, making total time 25/12 or 2 hours & 6 mins. Fighting the current hurts more than going with it helps!

20. 3 km per hour 21. 50 mph 22. 60mph

Units 1-2 Challenge Problems

1. The coordinates of point A are (h,k). The midpoint of segment AB is (5,3) and the midpoint of segment (B,C) is (-1,8). What ate the coordinates of point C in terms of h and k?

2. A lattice point is a point in the coordinate plane whose coordinates are both integers. A line segment connects (1,1) with (100,1000). How many lattice points does it contain (including the endpoints)?

3. Triangle ABC has vertices (0, 0), (11, 60), and (91, 0); respectively. The line $y = kx$ cuts the triangle into two triangles of equal area. Find k.

4. This exercise will have you prove that, if two lines are perpendicular, then their slopes are negative reciprocals of each other.

   a. Graph the lines $y = ax \text{ and } y = bx$ where a>0 and b<0. Then graph the line $x = 1$.

   b. Label the following points: C is the origin; D is where $x = 1$ intersects $y = ax$; E is where $x = 1$ intersects $y = bx$; F is where $x = 1$ intersects the x-axis. Find the coordinates of these four points, some of them may be in terms of a and b. Be careful with point E!

   c. Find the lengths of segments CD, CE, and DE. Again, your answers may be in terms of a and b.

   d. If the two slanted lines are perpendicular, then the Pythagorean Theorem will apply in triangle CDE. What does this tell you about the relationship between a and b?

5. Stephanie has a new book (a paper copy—not electronic!). Before reading the book, the pages fit compactly together in the nice, new-book kind of way. As she reads the book, the pages she has read tend to crumple slightly, making them not fit together as tightly as the unread pages. In fact, Stephanie estimates that this effect makes the pages she has read about 20% thicker than those she has not. So a paperback book that is 2 inches thick before reading will be 2.4 inches thick when finished.

   a. Stephanie has read one third of the pages of a book. How far is the bookmark in? In other words, looking at the book from its side, what percentage of the height of the book is the bookmark?

   b. One day, glancing at her book from the side, Stephanie estimates that her bookmark is as far from the front cover than the back cover. She knows that, even though it appears she is halfway done with her book, she is not—due to the crumpling effect. How much of her book has she read?

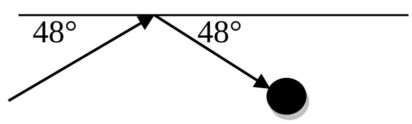
   c. If p represents the percentage of the pages Stephanie has read, then, in terms of p, how far is the bookmark in—what percentage of the height of the book is the bookmark?

   d. Instead, if b represents how far the bookmark is (in terms of inches, not pages), then what percent of the pages has Stephanie read? You may want to try some numbers here.

6. A 3-4-5 right triangle is places on the coordinate plane such that its hypotenuse is along the x-axis and the vertex containing the right angle is on the positive y-axis. Its longer leg is in the first quadrant, and its shorter one is in the second quadrant. Without using your calculator, find the coordinates of its vertices.

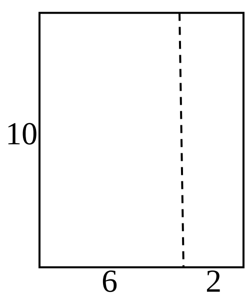
7. Elise is riding up a long escalator. She walks quickly and takes 60 steps before she gets to the top. Her brother Alex takes steps 3/5th as often as she does. Alex takes 40 steps to get to the top. How long is the escalator? (how many steps are showing at any point in time). From *Exeter Book 2*.

8. Follow the bouncing ball: When a ball reflects off a wall, the angle at which it comes in is the same as the angle at which it leaves (note that if the ball has some spin on it, this will not be true). For example:



a. Suppose a ball bounces off a horizontal line as above, where its slope is positive before bouncing. If its slope before bouncing is *m*, then what is its slope after bouncing?

b. The rectangular box below is 8 units wide and 10 units high. A ball starts in the lower left-hand corner with a slope of 2 and bounces around the box for a while. (Assume the ball is small enough relative to the box that nothing funny happens around corners.) There is a vertical blue stripe painted on the box's floor 6 units from the left (dashed line) that does not affect the ball's path. How high above the bottom of the box is the ball the first three times it crosses the blue stripe?



c. A ball starting in the lower-left hand corner bounces off the top wall and then hits the bottom of the box at the blue stripe. What was its initial slope?

d. (continuation of *c*) Now assume it made three bounces off the top and/or bottom walls (top-bottom-top) before first hitting the bottom at the blue stripe. What was its initial slope? How about five or seven bounces off a horizontal wall before first hitting the bottom at the blue stripe?

e. A ball leaves the lower left-hand corner with a slope of 0.012. How many other times does it hit the left wall before first hitting the top?

f. A different ball starts at the bottom of the box at the blue stripe, has a positive slope initially, and, after hitting the right, top, then left walls, returns to its starting place. In fact, it will continue to bounce around hitting the walls at the same four places forever (this repeating of its path is called a "fixed orbit"). What was its initial slope?

g. (continuation of *f*) There are other initial positive slopes the ball may have to give it a fixed orbit where it returns to the bottom wall only at the blue stripe. These other fixed orbits will hit some walls more than once. Give some other initial slopes that result fixed orbits.

h. A ball leaves the lower left-hand corner with a slope of 2, hits the top and right walls, and then hits the bottom. How far did it travel? Simplify the radicals.

i. A ball leaves the lower left-hand corner, hits the top and right walls and then stops at the bottom. If the total distance the ball traveled was 24 units, then what was its initial slope?

Unit 3 Challenge Problems

1. Sketch a graph of each relation below. You may want to start with points where the graphs intersect the coordinate axes.

a. $x = |y|$ b. $|x| = |y|$ c. $|x| + |y| = 12$

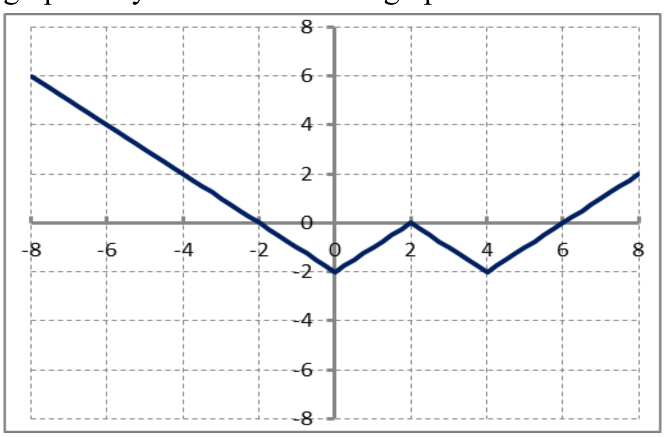
d. $2|x| + |y| = 12$ e. $|x-2| + |y| = 12$ f. $2|x| + |y| < 12$

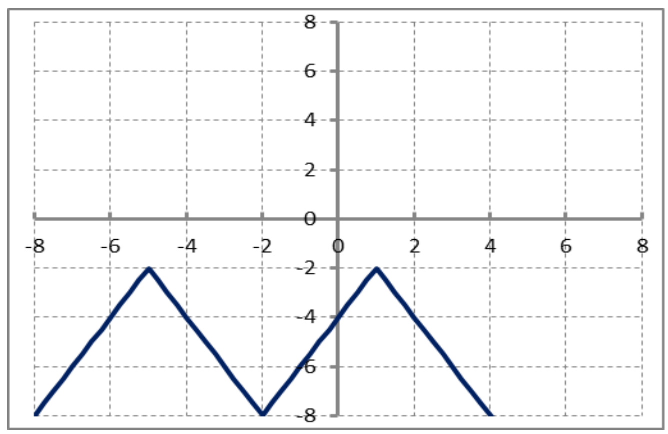
g. $f(x) = \dfrac{|x|}{x}$ h. $|y| = 1 + |x|$

2. What is area enclosed by the graph of the relation in question 2f above?

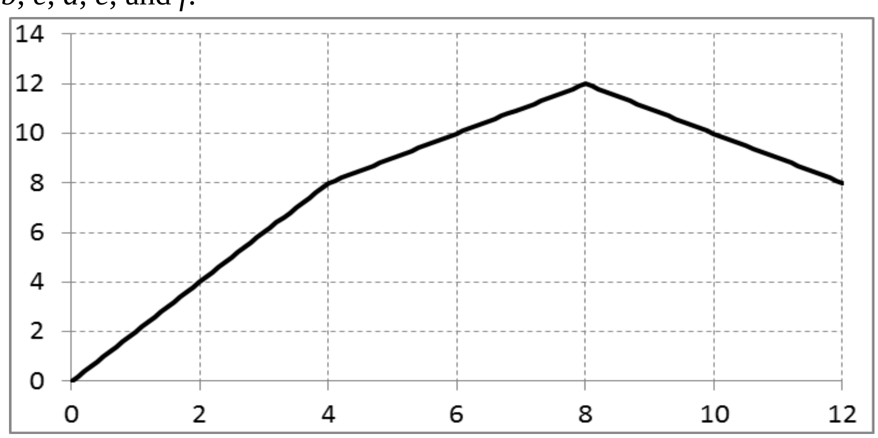
3. For different values of *a*, *b*, and *c*, the function $f(x) = \left||x| + b\right| + c$ may have anywhere from zero to four unique zeros. Give one set of parameters (*b*,*c*) causing it to have each different number of zeros.

4. The equations graphed below each have two absolute values in them. Can you determine them? You may try to attack it with algebra, or you may think about what the absolute value function does graphically and "unfold" this graph.





5. The function graphed below can be written in the form $f(x) = a|x-b| + c|x-d| + ex + f$. Find *a*, *b*, *c*, *d*, *e*, and *f*.



Unit 4 Challenge Problems

1. The zeros of a quadratic function are *p* and *q*. It also goes through the point (1,*k*) where *k* is not zero. What is $f(2)$, in terms of *p*, *q*, and *k*?

2. An isosceles triangle has sides 15, 15, and 18. A rectangle is placed inside of the triangle, with the base on the triangle's longest side and vertices on the two congruent sides.
   a. If the base of the rectange has width 6, what is its area?
   b. Of all rectangles as described in the problem, what is the largest possible area?

3. Given the line $y = mx + b$ where $m<0$ and $b>0$, find the area of the largest rectangle that fits under the line in the first quadrant. One corner of the rectangle is the origin and the opposite corner is on the line.

4. Find the area largest rectangle that will fit inside an equilateral triangle of side *s*, where one side of the rectangle lies on one side of the triangle. Hint: put it on the coordinate plane?

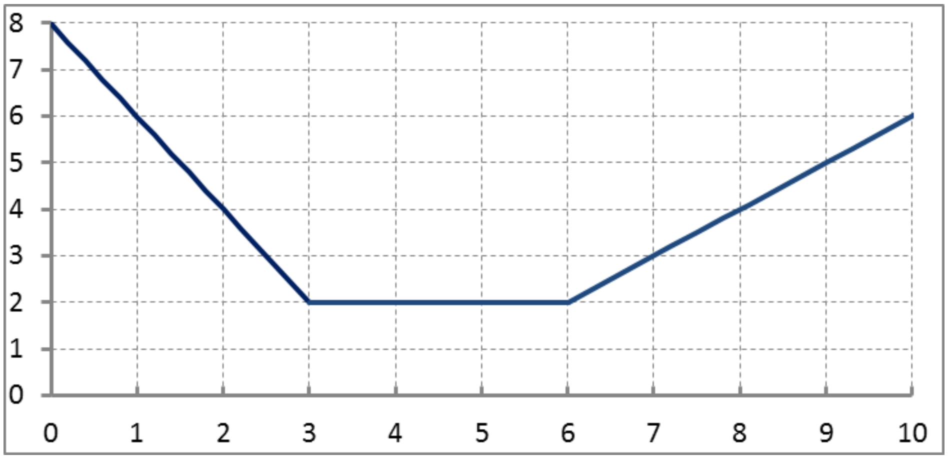
5. Given a triangle ABC (0,0), (12,0) and (3,9). You want to cut the triangle into two pieces of equal area with a line.
   a. If the line is vertical, then what is its equation? (Don't expect an integer answer!)
   b. If the line is horizontal, then what is its equation?
   c. If the line has a slope of 2, then what is its equation?
   d. If the line goes through the origin, then what is its equation?

6. Given points A(1,7) and B(7,4). Triangle ABC is a right triangle where B is the right angle.
   a. What is the equation of the line on which point C lies?
   b. If ABC is an isosceles right triangle then there are two possible places for point C. What are their coordinates?
   c. Instead, if ABC is a 30-60-90 triangle where C is 30 degrees and B is the right angle, then what can the x-coordinate of point C be?

7. The Greeks believed that rectangles where the ratio of length to width was the same as the ratio of length plus width to length were the most asthaetically pleasing. They called this ratio the golden ratio. What is it equal to?

8. For the parabola $f(x) = a(x-h)^2 + k$, show that if $f(h+w) = v$ then $f(h-w) = v$ also. What is the significance of this for the graph of $f(x)$?

9. The graph below is $f(x)$.



a. What is the area of the trapezoidal region below the graph of $f(x)$ and above the x-axis on the interval $0 \leq x \leq 3$ (meaning between x=0 and x=3)? Hint: one way to express the area of a trapezoid is the average of the two parallel bases times the height.

b. The area of the trapezoidal region above the x-axis and below the graph of $f(x)$ on the interval $0 \leq x \leq k$ is 10. What is the value of k?

c. For some k where $0<k<1$, the area of the region above the x-axis and below the graph of $f(x)$ between $x=k$ and $x=k+2$ is 8.5. What is k? In other words, some trapezoid with a base of 2 has an area of 8.5. what is the x value of its left end?

d. The area below the curve and above the x-axis from x=5 to some k (where k>5) equal to 9. What is k?

e. The area under the curve and above the x-axis from some k to k+3 is equal to 9 for 2 different values of k.

i. between what two consecutive integers are each of them?

ii. exactly what are they?

Unit 5 Challenge Problems

1. Given that $f(x) = ax^7 + bx^5 + cx^3 - 6$ and $f(5) = 18$, what is $f(-5)$?

2. The function $f(x) = x^4 + ax^3 + bx^2 + cx + 2$ has a triple root at x=-1. What is a?

3. Give the equation of any polynomial function such that the solution to $f(x) > 3$ is $x < -4 \; or \; 1 < x < 2$.

4. If $a + b = 5 \;\; and \;\; a^2 + 3ab + 2b^2 = 40$ then what is the value of $2a + 4b$?

5. Given that $a + b = 3 \;\; and \;\; a^2 + b^2 = 6$, find $a^3 + b^3$

6. A bathtub is a rectangular prism (like most cardboard boxes), in that its base is a rectangle and its sides extend upwards at right angles to its base and to each other. Suzanne's bathtub's base is 170 cm by 80 cm, and Suzanne fills it to a height of 40 cm.

a. For some reason, someone puts a solid cube 60 cm on each side in the tub after Suzanne has filled the tub to the desired level and it sinks to the bottom. What is the height of the water?

b. Instead, if the cube's side were 42 cm what would the height of the water be?

c. What would the side of the cube have to be such that the new water level was exactly as high as the top of the cube? Your equation will be a cubic function and you may use your calculator to try to solve it.

7. For the cubic $f(x) = a(x - h)^3 + k$, show that if $f(h + w) = k + v$ then $f(h - w) = k - v$. What is the significance of this for the graph of $f(x)$?

8. The function $f(x)$ is a 6th degree polynomial function. All coefficients are either +1 or -1, and the constant term is +1. Given that $f(2) = 27$ find $f(x)$.

Unit 6 Challenge Problems

1. A line with a positive slope makes a 60° angle with the *x*-axis. The line goes through the point $(4\sqrt{2},2)$. Find the zero and y-intercept of the line. Hint: find the slope first.

2. Show that $(5+\sqrt{3})^2 = 28+10\sqrt{3}$. Thus $\sqrt{28+10\sqrt{3}} = 5+\sqrt{3}$. Now find the following. Hint: look for the form $a+b\sqrt{c}$.

a. $\sqrt{7-4\sqrt{3}}$ b. $\sqrt{43+30\sqrt{2}}$ c. $\sqrt{29-12\sqrt{5}}$

d. harder one: (slightly different form) $\sqrt{11+4\sqrt{6}}$

3. Simplify: $\left(\sqrt{2}+\sqrt{3}-\sqrt{5}\right)\left(\sqrt{2}+\sqrt{3}+\sqrt{5}\right)$. Hint: think about a good place to put some parentheses in!

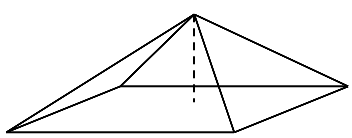
4. Simplify $\sqrt{5+2\sqrt{6}}-\sqrt{5-2\sqrt{6}}$.

5. Solve $x^{2/3}+x^{1/3}=6$.

6. Andrew was confused with the laws of exponents and believed that $\left(2^a\right)^b = 2^a \cdot 2^b$. He was using this "rule" to work through a problem and somehow got the right answer anyway! The value of *b* was 3; what was the value of *a*?

7. The equation $x^{x\sqrt{x}} = \left(x\sqrt{x}\right)^x$ has two positive solutions. The obvious one is *x*=1. What is the other one?

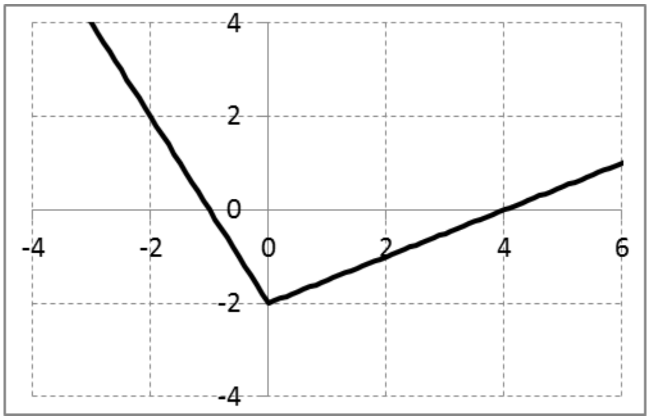
8. A pyramid has a square base and all eight edges have length *x*. Given that its volume is one third times its base times its height, find the volume and the surface area.



9. Solve $\sqrt{x+4}-6\cdot\sqrt[4]{x+4}+8=0$

Unit 7 Challenge Problems

1. The function $f(x)$ is defined by the graph below. It is NOT defined outside of the interval shown. $g(x) = -2\sqrt{f(x)} + 3$. Find the domain and range of $g(x)$.

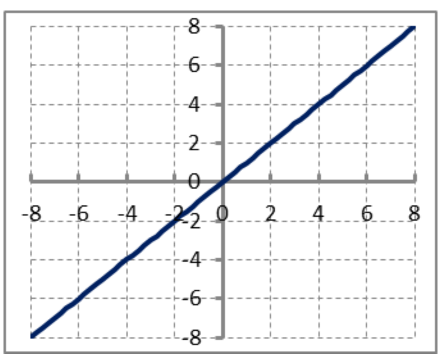


2. The domain of $f(x)$ is $-3 \le x \le 4$ and the range of $f(x)$ is $1 \le f(x) \le 7$. Find the domain and range of each of these:

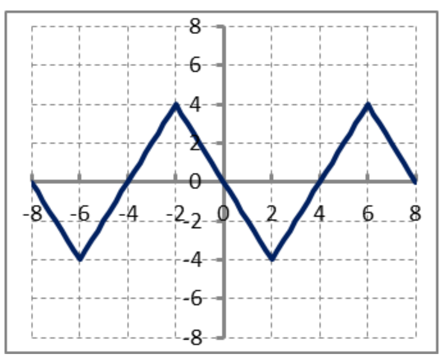
a. $f(-x)+2$ b. $-3f(x+3)-1$ c. $0.5f(2x)$

3. Given the graphs of $f(x),\ g(x), and\ h(x)$ below, sketch graphs of the functions described. Hints: you may want to focus on zeros, end behavior, and other interesting points such as turning points.

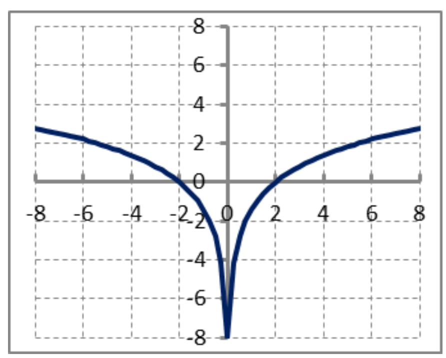
$f(x)$



$g(x)$



$h(x)$



a. $(f+g)(x)$ b. $(f \cdot g)(x)$ c. $(f \cdot h)(x)$ d. $(h \circ h)(x)$

e. $(f-h)(x)$ f. $\dfrac{1}{f(x)}$ g. $\dfrac{1}{g(x)}$

4. For the function $f(x)$, $f(x)+2f(5-x)=x$ for all real numbers x. What is the value of $f(1)$?

5. Given that $f(x^2)=4x \cdot f(x+2)+3$, find the value of $f(4)$.

6. What is the domain of $f(x)=\sqrt{\sqrt{x}-x}$?

Unit 8 Challenge Problems

1. Given that $2^x = 24$ and $24^y = 32$, what is the value of xy?

2. Given that $f(x)$ is an exponential function with a horizontal asymptote of the x-axis and $f(0) = 1$ and $f(a) = b$, find the following in terms of *a* and/or *b*:
a. $f(5a)$ b. $f(7)$

3. Solve the following:
a. $4^x - 4^{x-1} = 48$ b $2^{2x-1} - 3 \cdot 2^{x-1} + 1 = 0$

4. Verify that an exponential function with a vertical asymptote of $y = 0$ going through the points (a,b) and (c,d) can be written as both of the following: (to do this, show that they both go through both points). Then explain why these formulas work and what the different pieces represent.

$$f(x) = d \cdot \left(\frac{d}{b}\right)^{\frac{x-c}{c-a}} \quad \text{and} \quad f(x) = b \cdot \left(\frac{d}{b}\right)^{\frac{x-a}{c-a}}$$

5. You deposit \$100,000 in a bank that promises 12% interest per year (a great rate these days!).
   a. How much money do you have after 10 years?
   b. Another bank promises 6% every 6 months. Is this the same, better, or worse? Compute your balance after 10 years.
   c. Why is 12% per year different than 6% every six months? Hint: look over the course of one year only.
   d. How much better is it to get 1% per month for ten years?

6. Milk in the coffee: You have 20 ounces of 170° coffee and have to walk 15 minutes to your studio on a 70° day. You have one ounce of half-and-half that is at 70°. When you add the half-and-half to the coffee, the coffee's temperature becomes the weighted average of the temperatures, or $\dfrac{20 \cdot coffee + 1 \cdot halfnhalf}{21}$. Your goal is to have the coffee as warm as possible when you get to your studio. The coffee cools according to the following function, where *t* is measured in minutes and F is the temperature in degrees Fahrenheit: $F(t) = I(0.95)^t + 70$ where I is the initial temperature minus 70 (such that F(0) is the current temperature. Is it better to add the half-and-half now or to add it when you get to your studio?

Unit 9 Challenge Problems

1. If $x = \log_5 b$ and $5^x + 5^{x-2} = 20$, then what is *b*?

2. If $x = \log_5 b$ and $5^x + 5^{2x} = 20$, then what is *b*?

3. Add up the first 99 terms of the series below without your calculator.

$$\log\frac{2}{1} + \log\frac{3}{2} + \log\frac{4}{3} + \log\frac{5}{4} + ...$$

4. What is the value of $5^{2\log 7} \cdot 16^{0.5\log 7}$ (no calculators allowed!)?

5. Mary is trying to find the common logarithms of all integers between 1 and 10. Without a calculator she cannot do it exactly, but she can get pretty close. Here's what she does:

a. Mary knows $\log 1$ and $\log 10$ pretty easily. What are they?

b. For $\log 2$ Mary does the following. Since $2^{10} = 1024$ she can say that $2^{10} \approx 1000$, it must be the case that $10\log 2 \approx \log 1000$. Finish up to approximate $\log 2$.

c. With $\log 2$, Mary can also approximate $\log 4$, $\log 8$, and $\log 5$. Do so.

d. Next Mary decides to approximate $\log 3$. She uses the fact that $3^4 = 81 \approx 80$. She can get $\log 80$ from what she already did. Use this to approximate $\log 3$.

e. Finish up. You need $\log 6,\ \log 7,\ \mathit{and}\ \log 9.$

6. Write the function $f(x) = 3 \cdot 4^{x-2}$ in the form $f(x) = a \cdot e^{bx}$ without using your calculator:

7. Solve the following:

a. $\log_4(\log_3(\log_2 x)) = 0$ b. $\log_4(\log_3(\log_2 x)) = 1$

8. If $1 < v < w$, then what is the sign of $\log_w(\log_w(v))$?

Unit 10 Challenge Problems

1. When a right triangle of area 3 square units is rotated 360 º about its shortest leg, the cone that results has a volume of 30 cubic units. What is the volume, in cubic units, of the cone that results when the same right triangle is rotated about its longer leg? The volume of a cone whose circular base has radius r and whose height is h is $V = \frac{1}{3}\pi r^2 h$.

2. If $a+b=4$ and $a^2+b^2=13$ then what is $\frac{1}{a}+\frac{1}{b}$? Note: you do not need to separately solve for *a* and *b* to answer this question, though you can.

3. Triangle ABC has vertices A(-4,5), B (-1,8), and C(*w*,5). The *y*-axis cuts triangle ABC into two pieces with different shapes but the equal areas. What is *w*?

4. Triangle ABC has vertices A(0,3), B(8,3), and C(2,6). A line through the origin cuts ABC into two pieces of equal area. What is the equation of this line? Hint: call it $y = mx$ and find the coordinates of the vertices of the smaller triangle in terms of *m*.

5. Brenda and Sally run in opposite directions on a circular track, starting at diametrically opposite points. They first meet after Brenda has run 100 meters. They next meet after Sally has run 150 meters past their first meeting point. Each girl runs at a constant speed (but not necessarily the same speed). What is the length of the track?

Preparing for the Accelerated Algebra 2 Final Exam

The final will last two hours.

The format
There will be a brief calculator section followed by a much longer no-calculator section. I will tell you the relative importance of the two sections before the exam. You will initially have both sections in front of you. I recommend that you do the calculator section first and then turn it in before addressing the no-calculator section.

What to expect
The exam should be fairly reflective of the material we covered this semester. The level of difficulty of the questions will, on average, be somewhat lower than the questions on the unit tests. It will focus primarily on the fundamental techniques and procedures; there will be very few (if any) questions requiring you to think outside the box and apply the material in novel and creative ways. We covered too much material this semester to expect you to be able to do much of this, and two hours is too short for me to assess both your understanding of the basics and ability to problem-solve creatively. The difficulty of the questions will be more like the quizzes than the unit tests. Put another way, it may be like the unit tests minus most of the “curveballs”.

How to study
Look over your old tests and quizzes. Then look at the review problems for each test and quiz in your workbooks. I am attaching five review sheets of additional practice questions.

Help from me
We will talk about a review session (optional) on the afternoon or evening before the exam. I will also be around quite a bit between now and the final exam.

Final Exam Review Part 1: Functions

Summary Notes

A **function** defines a relationship or mapping between an input variable and an output variable. For a relation to be a function, no input may have more than one output. Graphically, this is represented by the **vertical line test**. A function may be represented by an input-output table, an algebraic rule, or a graph. The set of inputs of a function is called the **domain**, the set of outputs is called the **range**. The **zeros** of a function are all inputs that cause the output to be zero. Graphically, this is an *x*-intercept (although a function may have imaginary/complex zeros that do not show up as *x*-intercepts).

Domains can be found graphically or algebraically. Algebraically, with the functions we have studied this year, the only **domain exceptions** are inputs caused one to: divide by zero, take the square-root of a negative number, or take the logarithm of a negative number or zero.

Functions can be put together in various ways to build more functions. We can add, subtract, multiply, or divide function outputs. Notationally, these are represented by $(f+g)(x)$, $(f-g)(x)$, $(f\cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ respectively. Another way to combine functions is composition. In a **composite function,** the output from one function becomes the input into another function. A composite function is denoted by $f(g(x))$ *or* $(f\circ g)(x)$.

Inverse Functions: An inverse function switches the input and output. A function has an inverse function if it passes the horizontal-line test. You can find the inverse function (denoted by $f^{-1}(x)$) by replacing $f(x)$ with *y*, switching the *x*'s and *y*'s and then solving for *y*.

Questions

1. Use the graph of the function $f(x)$ below to answer the following questions:

a. What is the domain of $f(x)$?

b. What is the range of $f(x)$?

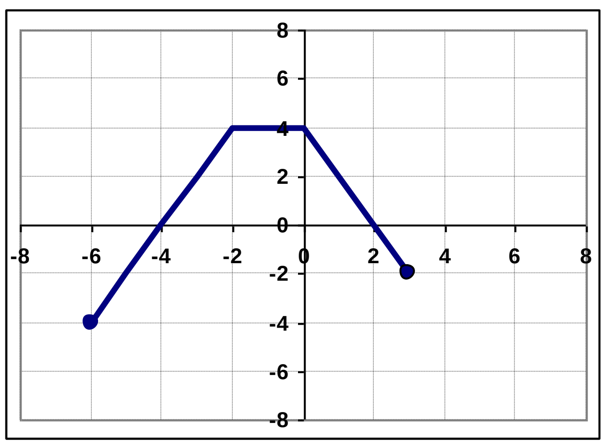
c. What are the zeros of $f(x)$?

d. What is $f(-2)$?

e. What are the solutions to $f(x)=2$?

f. Write an inequality showing the solution to $f(x)>2$.

g. Write an inequality showing the solution to $f(x)<0$.



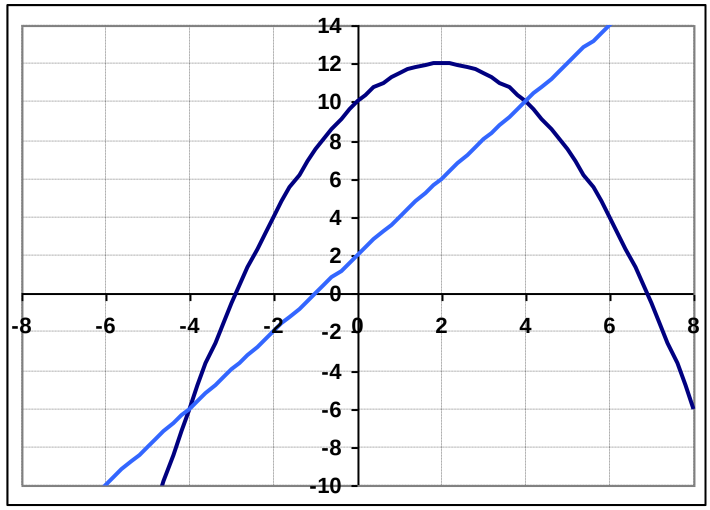
2. After reading for t minutes one night, Sally is on page $P(t)$ of her book.

a. What is the meaning of $P(20)=60$?

b. What is the meaning of $P^{-1}(40)$?

3. The parabola $f(x)$ and line $g(x)$ are graphed below. **The domains of the functions are all real numbers, only a portion of their graphs are shown.** Use their graphs to answer the following. You SHOULD NOT write equations of the functions to answer these questions.

a. What is the range of $f(x)$?
b. What is the range of $g(x)$?
c. What are the zeros of $f(x)$ and $g(x)$?
d. Estimate the solutions to $f(x) = -2$.
e. What is $g(-6)$?
f. What is $f(6)$?
g. What are the solutions to $f(x) = g(x)$?
h. What is the solution to $g(x) \le 0$?
i. What is the solution to $f(x) \le g(x)$?
j. What is $g(f(-3))$?
k. What are the equations of $f(x)$ and $g(x)$?
l. What is $g^{-1}(6)$?



4. Find the domain of the following functions algebraically. Hint: look for exceptions: "you can't ____"

a. $f(x) = \sqrt{x+5} - 1$ b. $f(x) = \dfrac{5}{2x-8}$ c. $f(x) = \dfrac{2x-1}{x^2+6x}$

d. $f(x) = -2\sqrt{2x-7} + 3$ e. $f(x) = -0.5\log_2(x+1) - 7$ f. $f(x) = \sqrt[4]{7x - x^2}$

5. Find the real zeros of the following functions algebraically. One will require the quadratic formula (or completing the square).

a. $f(x) = \dfrac{2}{3}x - 7$ b. $f(x) = 3(x-2)(x+7)$ c. $f(x) = \sqrt{x-3} - 4$

d. $f(x) = \dfrac{3}{x} - 6$ e. $f(x) = x^2 - 6x + 3$ f. $f(x) = 3(x-1)^2 - 12$

g. $f(x) = -3 \cdot 2^x + 24$ h. $f(x) = 2\log_7(x+1) + 2$ i. $f(x) = \dfrac{1}{2}|x-5| - 2$

j. $f(x) = \dfrac{-2}{x-1} + 3$ k. $f(x) = -2\sqrt[3]{x-1} + 3$ l. $f(x) = -2\sqrt{x+4} - 1$

m. $f(x) = -2(x+1)^{-2/3} + 8$ n. $f(x) = \log_3 x - \log_3(x-3) - 1$ o. $f(x) = x^3 - 2x^2 - 8x$

p. $f(x) = x^3 + x^2 - 9x - 9$ (hint: x=-3 is a zero)

6. Find the inverse, $f^{-1}(x)$, of each of the following functions.

a. $f(x)=3x-2$ b. $f(x)=(x-4)^3+2$

c. $g(x)=2\cdot\sqrt[5]{x+3}-7$ d. $f(x)=6^x-2$ e. $f(x)=\log_5(x-3)+4$

7. Given that $f(x)=2x-1$ and $g(x)=x^2-5$, find the following:

a. $(f-g)(x)$ b. $(f+g)(-2)$ c. $(f\cdot g)(0)$

d. $(f\circ g)(3)$ e. $g(f(x))$ f. $f(f(x))$

g. $g(i)$ h. $g(1+i)$

8. Given that $f(x)=2x^2$, $g(x)=2x^{-1}$, and $h(x)=\sqrt{6x}$, find the following. Simplify all answers (no negative exponents and all radicals should be in simplest radical form).

a. $h(8)$ b. $g(1/3)$ c. $f(g(3))$

d. $(f\circ g)(x)$ e. $(g\circ f)(x)$ f. $(f\cdot g)(x)$

g. $h(f(x))$ h. $h(3)+h(12)$ i. $h(3)\cdot h(12)$

Answers

1a. $-6\le x\le 3$ b. $-4\le y\le 4$ c. -4, 2 d. 4 e. -3, 1 f. $-3<x<1$

g. $x<-4$ *or* $x>2$ (or, to take into account the domain: $-6\le x<-4$ *or* $2<x\le 3$)

2a. twenty minutes after starting she is on page 60 b. the time she has been reading to get to page 40

3a. $y\le 12$ b. all reals c. $f(x)$: -3, 7 $g(x)$: -1 d. about -3.3, 7.3 e. -10 f. 4

g. x=-4, 4 h. $x\le -1$ i. $x\le -4$ *or* $x\ge 4$ j. this is $g(0)$ which is 2

k. $g(x)=2x+2$ $f(x)=-0.5(x-2)^2+12$ l. 2

4a. $x\ge -5$ b. all reals $x\ne 4$ c. all reals $x\ne 0,-6$ d. $x\ge 3.5$ e. x>-1 g. $0\le x\le 7$

5a. 10.5 b. 2, -7 c. 19 d. ½ e. $3\pm\sqrt{6}$ f. -1, 3 g. 3 h. -6/7

i. 9, 1 j. 5/3 k. 35/8 l. none (extraneous) m. -7/8,-9/8 n. 4.5 o. 0,4,-2 p. 3, -3, -1

6a. $f^{-1}(x)=\dfrac{x+2}{3}$ b. $f^{-1}(x)=\sqrt[3]{x-2}+4$ c. $g^{-1}(x)=\left(\dfrac{x+7}{2}\right)^5-3$ d. $f^{-1}(x)=\log_6(x+2)$

e. $f^{-1}(x)=5^{x-4}+3$

7a. $-x^2+2x+4$ b. -6 c. 5 d. 7 e. $4x^2-4x-4$ f. $4x-3$ g. -6 h. $-5+2i$

8a. $4\sqrt{3}$ b. 6 c. $\dfrac{8}{9}$ d. $\dfrac{8}{x^2}$ e. $\dfrac{1}{x^2}$ f. $4x$ g. $2x\sqrt{3}$ h. $9\sqrt{2}$ i. 36

Final Exam Review Part 2: Simplifying and Evaluating Expressions

Expressions can be simplified in various ways but <u>they cannot be solved</u>! They can only be multiplied by expressions numerically equivalent to 1. This may be to find the common denominator or to rationalize.

Key things to keep in mind:
1. Is it addition, subtraction, multiplication, or division? Obviously, different rules apply for each….
2. Do a really have to FOIL? Foil binomials or trinomials…. only!
3. With exponents: if base is same, combine exponents. If exponent is same, combine bases (ie, $\sqrt{3}\cdot\sqrt{5} = 3^{1/2}\cdot 5^{1/2} = 15^{1/2}$ *or* $\sqrt{15}$)
4. You ***<u>can't</u>*** "distribute" exponents when there is addition or subtraction involved!
5. Complex numbers: i is $\sqrt{-1}$, not -1. **If it were -1 we wouldn't need the *i*!** All complex numbers can be written in the form $a+bi$; when you see an i^2, remember that it is equal to -1.

1. Simplify the following. All like terms should be combined, radicals should be in simplest radical form, and your answers should be written without exponents when possible. Be careful in deciding when to FOIL!

a. $16^{3/2}$ b. $2\sqrt{6}\left(3\sqrt{2}-2\sqrt{3}\right)$ c. $4^{-3/2}$

d. $\left(\dfrac{8}{27}\right)^{-2/3}$ e. $\sqrt{32}-\sqrt{18}$ f. $(-2\sqrt{5})^2$

g. $\left(3+2\sqrt{2}\right)^2$ h. $(3^2+4^2)^{1/2}$ i. $\sqrt[3]{4}\cdot 3\sqrt[3]{4}$

2. Simplify the following. All like terms should be combined and radicals should be in simplest radical form. Your answers should have no negative exponents.

a. $\dfrac{2x^2\cdot 3x^{-1}}{12x^{-3}}$ b. $\left(3\sqrt{2x^3}\right)^2$ c. $3(x+4)^{-2}$

d. $\left(\dfrac{-2x^3}{3}\right)^2$ e. $(x+2y)^2-[x^2+(2y)^2]$ f. $(2x^3y^2)^{-2}\cdot(4xy^2)^2$

g. $3\sqrt{2x}\cdot 4\sqrt{8x}$ h. $(3-2\sqrt{x})^2$

3. Factor the following completely (all factors should be real and rational):

a. $3x^3-12x$ b. $-3x^2+6x-3$ c. $-x^3+4x^2+21x$

d. $2x^2-9x-5$ e. x^4-10x^2+9 f. $-2x^5-4x^3+48x$

g. x^3-5x^2+4 given that (x-1) is one factor

4. Simplify the following complex numbers: write in the form $a+bi$ (remember, when dividing complex numbers, multiply numerator and denominator by the denominator's conjugate).

a. $(3-2i)-2(5-3i)$ b. $2(3-2i)+i(-2+4i)$ c. $(2-i)(4+3i)$

d. $\sqrt{-27}-\sqrt{-12}$ e. $(2-3i)^2$

5. Perform the divisions below. Use long division (or "fiddling") to find the quotients

a. $\dfrac{x^3-2x^2+4x-3}{x-1}$ b. $\dfrac{2x^3-5x+6}{x+2}$ c. $\dfrac{x^3-8}{x-2}$

6. Simplify the following rational expressions fully. Be careful to determine which require multiplication/division and which involve addition and subtraction! You need common denominators for the latter type but not the former.

a. $\dfrac{2}{x}-\dfrac{3}{x+1}$ b. $\dfrac{x^2-3x-4}{2x^2-32}\cdot(x+1)^{-2}$ c. $\dfrac{3}{x-2}+\dfrac{x-1}{x^2-4}$

d. $\dfrac{x^3-2x^2+x}{4x-4}\div\dfrac{x^2-3x+2}{2x^2-4x}$ e. $\dfrac{\frac{x}{2}-\frac{3}{x}}{1+\frac{x-2}{2x}}$ f. $\dfrac{2}{3}-\dfrac{x-2}{3x-3}$

7. Condense into a single logarithm and then evaluate.

a. $\log_{12}3+2\log_{12}2$ b. $2\log_6 3-\log_6 54$ c. $\log_8 28-\log_8 7$

d. $\log_2 2\sqrt{2}+\log_2\sqrt{2}$ e. $2\log 5-\log 25$ f. $\dfrac{1}{2}\log_3 36-\log_3 2$

8. Condense each of the following into a single logarithm.

a. $\dfrac{1}{3}(\log a+\log b)+\log c$ b. $2\log 2a-\log 3$ c. $\log 6-(\log 3+3\log 2)$

Answers

1a. 64 b. $12\sqrt{3}-12\sqrt{2}$ c. 1/8 d. 9/4 e. $\sqrt{2}$ f. 20 g. $17+12\sqrt{2}$ h. 5 i. $6\sqrt[3]{2}$

2a. $\dfrac{x^4}{2}$ b. $18x^3$ c. $\dfrac{3}{x^2+8x+16}$ d. $\dfrac{4x^6}{9}$ e. $4xy$ f. $\dfrac{4}{x^4}$ g. $48x$ h. $9-12\sqrt{x}+4x$

3a. $3x(x+2)(x-2)$ b. $-3(x-1)(x-1)$ c. $-x(x-7)(x+3)$ d. $(2x+1)(x-5)$

e. $(x+1)(x-1)(x+3)(x-3)$ f. $-2x(x^2+6)(x+2)(x-2)$ g. $(x-1)(x^2-4x-4)$

4a. $-7+4i$ b. $2-6i$ c. $11+2i$ d. $i\sqrt{3}$ e. $-5-12i$

5a. x^2-x+3 b. $2x^2-4x+3$ c. x^2+2x+4

6a. $\dfrac{-x+2}{x(x+1)}$ b. $\dfrac{1}{2(x+4)(x+1)}$ c. $\dfrac{4x+5}{(x+2)(x-2)}$ d. $\dfrac{x^2}{2}$ e. $\dfrac{x^2-6}{3x-2}$ f. $\dfrac{x}{3(x-1)}$

7a. 1 b. -1 c. 2/3 d. 2 e. 0 f. 1

8a. $\log\left(c\cdot\sqrt[3]{ab}\right)$ b. $\log\dfrac{4a^2}{3}$ c. $\log\dfrac{1}{4}$

Final Exam Review Part 3: Solving Equations and Inequalities

Notes: There are many types of equations we've solved this year, but many/most of them are basically the same. Here are a few types of equations that you need to handle algebraically:

1. Most equations with only one x in them. Solve these by "isolate and undo". The equations below look very different but all fall into this camp, since they have only one x in them.

   $3x-1=12$ $\quad$ $5(x-2)^2+7=18$ $\quad$ $3|x-2|+5=31$ $\quad$ $-2\log(x-3)-1=-3$

   Solve these by isolating the expression with the x in it (by adding, subtracting, multiplying, or dividing) and then undoing the operation by doing the opposite. To undo a square, take the square root of both sides; to undo a cube root, cube both sides; to undo an absolute value, take the positive or negative of the other side. To undo any power, take both sides to the reciprocal power. To undo a logarithm, write as an exponential equation.

2. Polynomial equations with more than one x in them. These can be solved by factoring or by using the quadratic formula. Be sure to set one side to zero first!

3. Other types of equations can usually be manipulated into one of these types by doing thinks to "get rid" of the other mathematical operations. These include equations such as:

   $\sqrt{x-2}+1=x-5$ $\quad$ $2^x\cdot 4^{x^2}=16$ $\quad$ $\log_2 x+\log_2(x-2)=3$ $\quad$ $\dfrac{3}{x}-\dfrac{x+1}{x-5}=11$

   -In the first one, isolate the radical and square both sides.
   -In the second one, write each side as 2 to some power and set the exponents equal to each other.
   -In the third one, condense the logs and rewrite as an exponential equation.
   -In the fourth one, multiply through by the LCD of $x(x-5)$

   In each case you will have a quadratic or linear equation that can be solved by factoring or the quad formula.

4. Equations where x is in the exponent (exponential equations). If you cannot find a common base and solve as another type of equation, you need to take the log of both sides.

   $3^x=31$

<u>**Common themes**</u>
-remove fractions first (even if they have x's) by multiplying by the LCD
-combine like terms
-look for multiple solutions (undoing absolute values, square roots, 4^{th} roots)
-check for extraneous solutions
Domain issues: root of negative, divide by zero, log of zero or negative

<u>**When permissible**</u> you can usually find the real solutions with <u>**calc-intersect**</u> (when graphing both sides) or <u>**calc-zero**</u> (when one side of the equation is zero).

<u>**Solving equations also comes up in these types of questions:**</u>
-finding zeros or *x*-intercepts (remember, x-intercepts are real zeros)
-solving inequalities: for most of them, solve the related equation and then check which parts of the number line it is true on (between and around points).
-finding intersections of two graphs: you typically will need the y-coordinates here as well

Problems

1. Find all real solutions to the following equations using "isolate and undo". Be sure to check for extraneous solutions and watch for multiple solutions (by when undoing even powers and absolute values). You should only need your calculator for ones marked with *.

a. $2x-7=11$

b. $3(x+2)^2-5=67$

c. $|x-3|+1=5$

d. $\log_3(x-3)-3=-1$

e. $\frac{2}{3}x-1=x-\frac{3}{5}$

f. $2|x-2|+5=3$

g. $x^{2/3}-7=2$

h. $x^2-7=13$

i. $3(x+1)^{1/3}-2=3$

j*. $10(1+x)^{12}=45$

k. $2\sqrt{2x-1}=6$

l. $2(x+4)^{-1}=6$

m. $\frac{3}{x-1}+2=1$

n. $2-3\sqrt{x}=7$

o. $3(x+6)^3+8=5$

p. $2\log_4(x-1)+1=5$

q.* $20x^{11}+5=143$

r. $3(2x-1)^{3/2}+1=25$

s. $\log_{2/3}x=2$

t. $2\log_7(x-1)+1=5$

2. Find all real and imaginary/complex solutions to the following equations by factoring or using the quadratic formula.

a. $2x^2-3x=x+30$

b. $2(x+3)^2-10(x+2)=x^2+4x+6$

c. $(x-2)(x+3)=6$

d. $x^2+x+7=0$

e. $x^3+6x^2=3x$

f. $-x^2-6x=13$

g. $(x^2+3)^2=3x^2+49$

3. Find all real solutions to the following equations. You will first have to perform operations to make these look like polynomial equations that you can solve by isolating, factoring, or using the quadratic formula.

a. $\frac{x+3}{x}=\frac{x+1}{x-1}$

b. $\log(x-3)=\log(2x-5)$

c. $3^{2x-1}=\frac{1}{9}$

d. $\log_3 x-\log_3 4=-1$

e. $\sqrt[3]{6x^2}=2x$

f. $\frac{3}{x}+1=\frac{-9}{x-12}$

g. $\frac{1}{2}\log_3(x-2)-\log_3 4=1$

h. $4^x=2\sqrt{2}$

i. $\frac{x+1}{x-3}+1=\frac{8}{x-3}$

j. $\log_4 x+\log_4(x+2)=\log_4 3$

k. $4^{3x}=16^{x+2}$

l. $\sqrt{x+5}=x-1$

4. Solve the following equations with logs (calculators OK)

a. $7^x = 31$ b. $2 \cdot 3^x - 7 = 81$ c. $10^x = 0.18$

5. Solve the following inequalities. I recommend that you solve the related equation first and then either "sign test" or reason through the graph to determine which segments of the number line the inequality holds for.

a. $|x-4| < 3$ b. $(x-5)(2x+7) \ge 0$ c. $-x^2 + 9x < 0$

d. $7 - 2|x+1| \le -3$ e. $x^4 - 12x^2 \ge x^3$ f. $-2(x+1)(x-3)(x+4)^3 < 0$

6. Algebraically find the intersection between each pair of curves below (if they meet). Your answers should include the x and the y coordinates. No calculators allowed.

a. $2x + 5y = 8$ *and* $x - 2y = 3$ b. $y = x^2 - 3x + 5$ *and* $y = -7x + 5$

c. $y = -(x-2)^2 + 13$ and $y = x^2 + 6x - 15$ d. $y = 3$ *and* $y = 0.5(x+2)^{3/2} - 1$

e. $y = \dfrac{8}{x-1} + 3$ *and* $y = x$ f. $y = x^3 - 7x + 18$ *and* $y = 6(x-1)^2$ give that they meet at (4,54)

g. $y = 9x^2$ *and* $y = (3x)^{-1}$ h. $y = \log_4(x+1)$ *and* $y = \log_4(10-2x)$

Answers

1a. 9 b. $-2 \pm 2\sqrt{6}$ c. 7, -1 d. 12 e. -6/5 f. none g. 27, -27 h. $\pm 2\sqrt{5}$ i. 98/27 j. 0.134, -2.134 k. 5 l. -11/3 m. -2 n. none o. -7 p. 17 q. 1.192 r. 2.5 s. 4/9 t. 50

2a. 5, -3 b. 4, -2 c. 3, -4 d. $\dfrac{-1 \pm 3i\sqrt{3}}{2}$ e. 0, $-3 \pm 2\sqrt{3}$ f. $-3 \pm 2i$ g. $\pm 2i\sqrt{2}, \pm\sqrt{5}$

3a. 3 b. none (x=2 is extraneous) c. -½ d. 4/3 e. 0, ¾ f. 6, -6 g. 146 h. ¾ i. 5 j. 1 (-3 is extraneous) k. 4 l. x=4 (-1 is extraneous)

4a. 1.765 b. 3.445 c. -0.745

5a. $1 < x < 7$ b. $x \ge 5$ *or* $x \le -3.5$ c. $x < 0$ *or* $x > 9$ d. $x \ge 4$ *or* $x \le -6$ e. $x \le -3$ *or* $x = 0$ *or* $x \ge 4$ f. $-4 < x < -1$ *or* $x > 3$

6a. (31/9, 2/9) b. (0,5) and (-4,33) c. (3,12) and (-4,-23) d. (2,3) e. (5,5) and (-1,-1) f. (4,54) and (3,24) and (-1,24) g. (1/3, 1) h. (3,1)

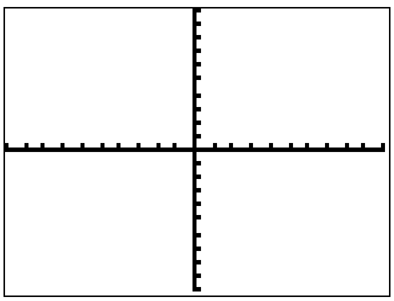
Final Exam Review Part 4: Graphs

When solving equations we made the distinction between equations with one *x* (where we "isolate and undo") and those with multiple *x*-terms (where we look to factor). A similar distinction can be made with graphing. Graphs with just one *x*-term we can think of in terms of "parent functions" and transformations (parts I and II below). When functions have multiple *x*-terms, they typically need to be factored-- we then focus on zeros and end behavior (part III below).

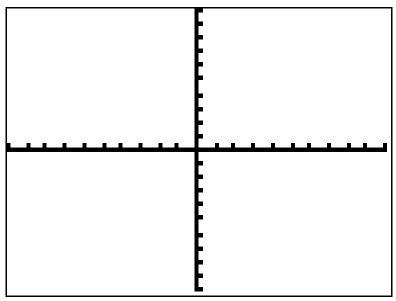
Part I: Graphs of "Parent Functions"

Sketch a graph of each of the "parent" functions below. Here are some suggestions:
-Y-intercept and zeros
-Domain
-End behavior—what happens to $f(x)$ as x gets to be a big positive or negative number?
-Plot points

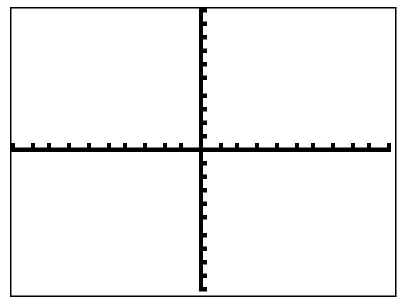
$f(x) = x$



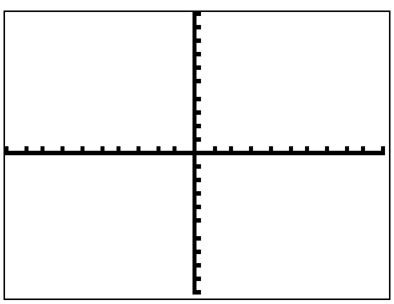
$f(x) = |x|$



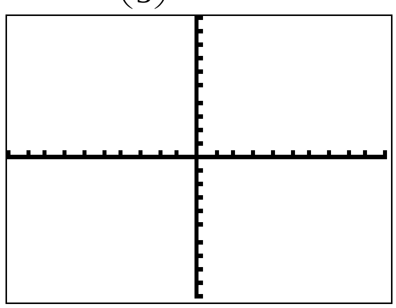
$f(x) = x^2$



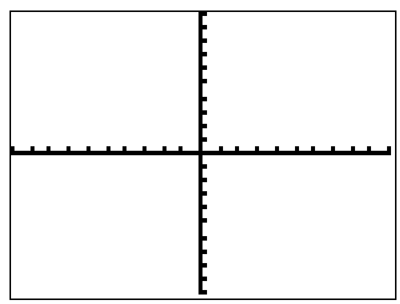
$f(x) = 2^x$



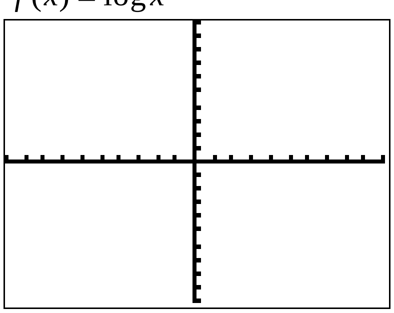
$f(x) = \left(\frac{1}{3}\right)^x$



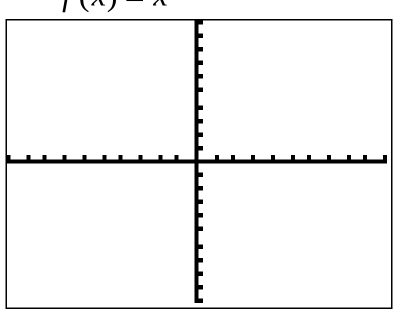
$f(x) = \sqrt{x}$



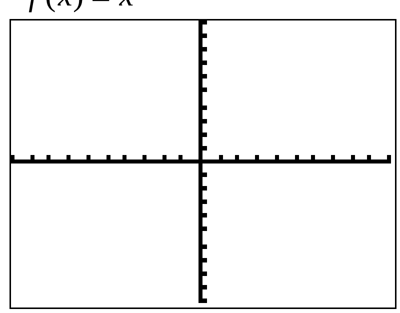
$f(x) = \log x$



$f(x) = x^{1/3}$



$f(x) = x^3$



Part II: Graphing with Transformations

Sketch a rough graph of each of these functions below. Use some combination of the following to help:

-Shape of parent function.

-Domain

-End behavior

-*a* determines "slope" or steepness and orientation (upside down or not)

-*h* determines horizontal positioning → **important point is where we are doing something to zero** (squaring it, taking absolute value of it, multiplying by it)

-*k* determines vertical positioning → is it shifted up or down and by how much

-Plotting a few points (the *y*-intercept is usually a good one)

a. $f(x) = -\frac{1}{2}x + 4$

b. $f(x) = \frac{1}{2}x^2 - 4$

c. $f(x) = \frac{1}{2} \cdot 2^x + 4$

d. $f(x) = -2|x-4|$

e. $f(x) = \frac{1}{2}(x+4)^2 - 1$

f. $f(x) = -(x+5)^2 + 3$

g. $f(x) = 5 \cdot \left(\frac{2}{3}\right)^x - 2$

h. $f(x) = -1 \cdot 3^x + 6$

i. $f(x) = \frac{1}{2}|x+2| - 3$

j. $f(x) = 2 \cdot \sqrt{x-3} + 2$

k. $f(x) = -0.5 \cdot \sqrt{x+4}$

l. $f(x) = -\log_2(x-3)$

m. $f(x) = \log(x+4) - 2$

n. $f(x) = 2 \cdot \sqrt[3]{x+4} - 3$

Part III: Graphing Parabolas or Polynomial Functions in Standard or Factored ("Intercept") Form

Sketch a graph of each. For parabolas, find the coordinates of the vertex and two other points (no necessarily zeros). For polynomial functions of degree 3 or higher, get the zeros and end behavior right and think about when the graph is above and below the x-axis.

a. $y = -x^2 + 6x + 3$

b. $f(x) = 2x^2 + 8x + 11$

c. $f(x) = -x^2 - 4x + 12$

d. $f(x) = -3(x-1)(x+5)$

e. $f(x) = 0.5x(2x-5)$

f. $f(x) = (x+2)(x-5)$

g. $f(x) = -0.5(x+2)^2(x-1)(x-3)$

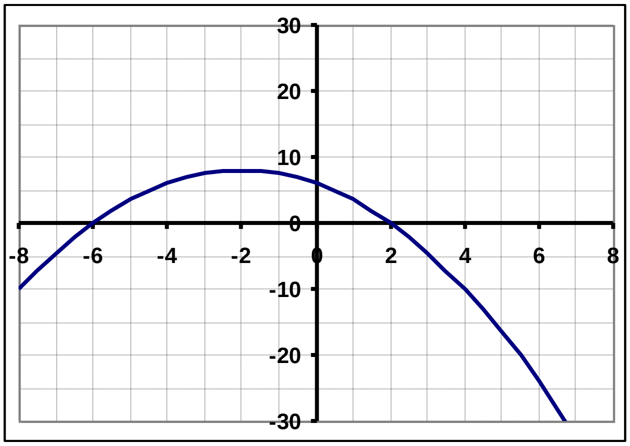
h. $f(x) = x^4 + 3x^3$

i. $f(x) = -2x^4 + 6x^3 + 20x^2$

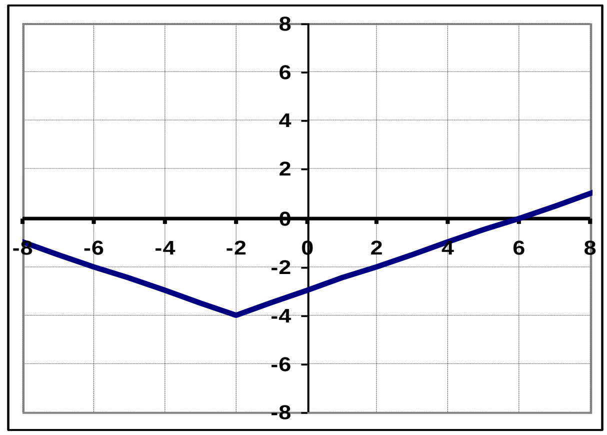
Part IV: Writing Equations of Graphs

Write the equation of the following
a. Line going through (4,5) and (-2, 8).
b. Horizontal and vertical lines through (4,-2).
c. Parabola with a vertex of (2,-4) going through the point (4,8).
d. Absolute value function with vertex (-3,1) going through (5,-7)

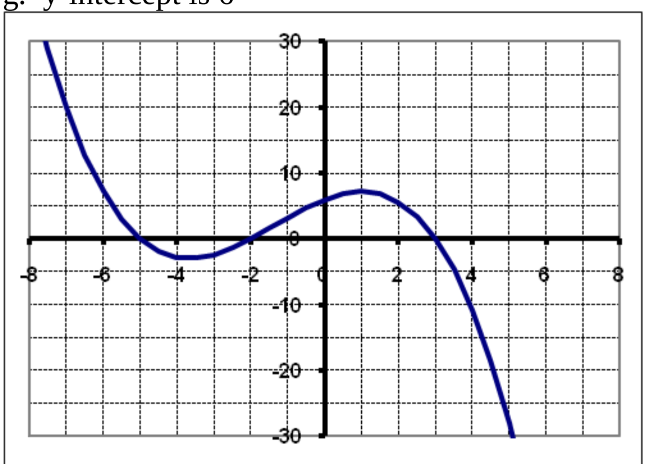
e. Goes through (4,-10) vertex unknown



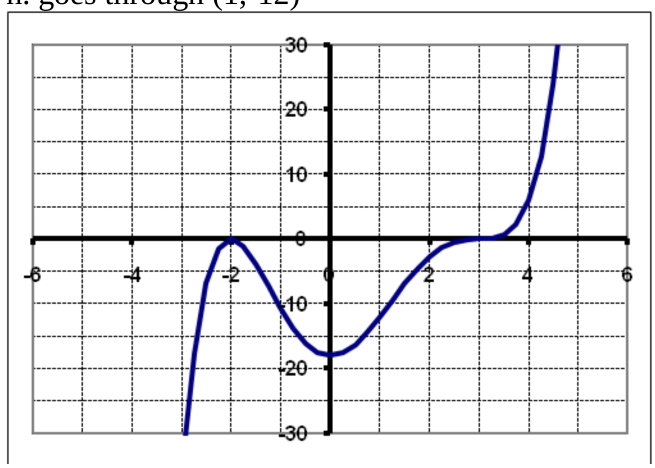
f. Goes through (2,-2)



g. y-intercept is 6



h. goes through (1,-12)



i. Exponential function with x-axis as horizontal asymptote going through the points (0,20) and (2,5). (no calculator)

j. Exponential function of the form $f(x) = a \cdot b^x$ going through the points (2,30) and (5,12).

k. Any log function with a zero of 3 and a domain of x>-1

l. Any exponential function with a y-intercept of 6 where, as $x \to -\infty$ $f(x) \to 9$

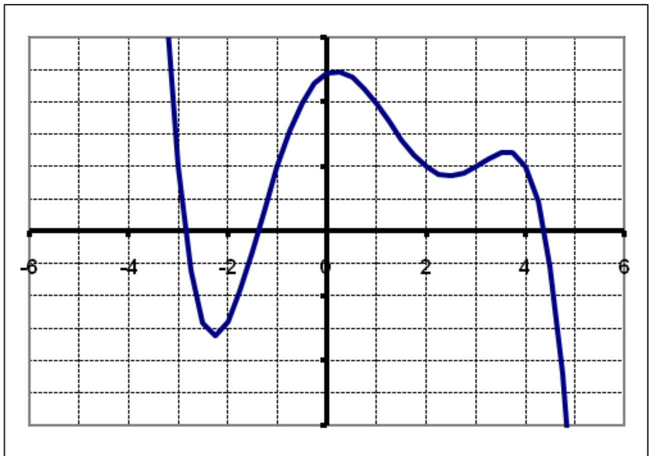
m. The cubic polynomial function with a double root of 2, a zero of -4, and going through (1,24).

n. A fourth degree polynomial function with zeros of $\pm 2\sqrt{3}$ *and* $\pm 2i\sqrt{2}$ and a y-intercept of 24.

o. Any polynomial function where the solution to $f(x) \geq 0$ *is* $-3 \leq x \leq -1$ *or* $x = 2$

Part V: Shape of Polynomial Functions

The graph below is a polynomial function. What degree could it be, and what do we know about the number of real zeros and zeros with imaginary components? Also, what do we know about the leading coefficient?



Answers
Parts I-III: check all of your answers on your calculators. vertices of parabolas:
a. (3,12) b. (-2,3) c. (-2,16) d. (-2,27) e. (1.25, -1.5625) f. (1.5, -12.25)
Part IV: a. $y = -0.5x + 7$ b. horiz: $y = -2$; vert $x = 4$ c. $y = 3(x-2)^2 - 4$
d. $f(x) = -|x+3| + 1$ e. $y = -0.5(x+6)(x-2)$ f. $f(x) = 0.5|x+2| - 4$
g. $f(x) = -0.2(x+5)(x+2)(x-3)$ h. $f(x) = \frac{1}{6}(x+2)^2(x-3)^3$ i. $f(x) = 20 \cdot 0.5^x$
j. $f(x) = 55.3 \cdot 0.737^x$ k. a few possibilities: $f(x) = \log_4(x+1) - 1$ *or* $f(x) = \log_2(x+1) - 2$
l. something like $f(x) = -3 \cdot 1.5^x + 9$ (the 1.5 could be anything >1)
m. $f(x) = 4.8(x-2)^2(x+4)$ n. $f(x) = -0.25(x^2 - 12)(x^2 + 8)$
o. $f(x) = a(x-2)^2(x+3)(x+1)$ where a<0 (could also be a higher degree in many ways…)
Part V: Odd degree that is 5 or higher; three real zeros and at least one pair of zeros with imaginary part; negative leading coefficient.

Final Exam Review Part 5: Word Problems and Application Problems

Be careful to determine when a linear model is appropriate and when an exponential model is appropriate. A **linear model** makes sense when things change by the **same amount** each period. An **exponential model** makes sense when things change by the **same percentage** each period.

1. The path of an Olympic long-jumper can be given by the equation $y = -0.02x^2 + 0.50x$ where y is her height and x is her horizontal distance from the start of her jump (both are in feet). Sketch a graph on your calculator and use it to answer the following questions:
   a. How high was she when she was 5 feet (horizontally) from where she started?
   b. How far did she jump?
   c. What was the maximum height she reached?

2. Katie and Bob love playing Skee-Ball at Chuck-E-Cheese's. They are good and they win many prize tickets with their scores. One day, they went in to their local Chuck-E-Cheese's, carrying the prize tickets they won the previous time. They started to play Skee-Ball. Thirty minutes after they started playing, they had a total of 850 tickets (including those they brought in with them). Seventy minutes after they started playing, they had a total 1330 tickets. Assume that they win tickets at a steady rate, so their number of tickets is a linear function of time.
   a. On a graph relating tickets they have to time since they started playing, plot the two points that are given. Be sure to choose input and output variables carefully.
   b. Write a linear function $C(t)$ that shows the number of tickets they had t minutes after they got there that day. Graph that line on the axes provided.
   c. What is the slope and what meaning does it have in this problem? What units is slope in (? Per ?)
   d. How many tickets did they start with that day?
   e. What is $C(90)$ and what does it mean in this problem?
   f. How many minutes after they get there will they have 1950 tickets, which is enough for the Star Wars Lego set they want?

3. Between 1880 and 1930, US oil production grew at a constant annual rate of 7.9%. In 1930, production was 995 million barrels.
   a. If this growth continued indefinitely, what would production be in 1934?
   b. What was US oil production in 1905?
   c. If growth continued at this 7.9% annual rate after 1930, when would oil production reach 1800 million barrels?
   d. In actuality, US oil production reached 1800 million barrels in 1949. What was its annual rate of growth between 1930 and 1949? Your answer should be the percentage growth rate to TWO decimal places (like 11.25% or 1.47%).

4. A rectangle's length is six inches greater than its width. When you add three inches to its width and subtract four inches from its length, the area falls by eight square inches. What were its original dimensions?

5. A US company that makes clothing has been laying off American workers as they increasingly make their products in Mexico and Asia. Each year, the size of the American workforce falls by 12%. It is currently 12,000 people. How many years will it be until the size of the American workforce halves (in other words, becomes half as large as it currently is).

6. A rectangle's length is twice its width. Its area is eight more than its perimeter. What are its dimensions?

7. A rocket was launched off a tower; it rose for a while and then fell to the ground next to the tower. Its height (in feet) t seconds after it's launch is given by the equation $h(t) = -16t^2 + 80t + 400$. [You may use your calculator however you like for this one—graph the function!]
 a. How tall is the tower?
 b. What is $h(3)$ and what meaning does it have in the context of this problem?
 c. What was the highest it got and when did it reach this height?
 d. When did it land?
 e. What is $h^{-1}(450)$ and what meaning does it have in the context of this problem?

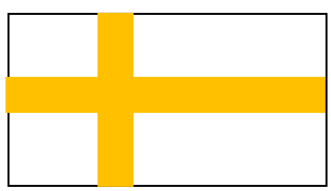
8. Sarah made a $450 investment twelve years ago. The investment is currently worth $1300. Assume it grew at a constant annual percentage rate.
 a. What was the annual rate of growth of her investment?
 b. If growth continues at this rate, how much will it be worth five years from now?
 c. Assuming no change in the investment's growth rate, when will it be worth $3000?

9. Mrs. Cordero keeps a bowl of candy in the math office. Eight school days after she filled it, there were 145 pieces of candy in it. Each school day, twenty-three pieces are eaten.
 a. Write a linear function $C(t)$ showing the amount of candy in the bowl t days after Mrs. Cordero filled it.
 b. What is the slope and what meaning does it have in this problem?
 c. How many pieces were in the bowl immediately after it was filled?
 d. When will the bowl be empty?

10. A rectangle's dimensions are such that if you take away a square (whose side is the shorter side of the rectangle), the rectangle that remains is similar to the original rectangle. (This means the ratio of the longer side to the shorter side is the same for the two rectangles). If the shorter side of the original rectangle is 1, what is the longer side?

11. A square photograph with side 6 inches has a frame around it. The width of the frame is the same everywhere. The area of the frame itself is one-half the area of the photograph. How wide is the frame.

12. The flag of Sweden has a yellow cross on a blue background. The dimensions of the flag are 3 feet by 2 feet. One third of the total area is yellow. How wide are the yellow stripes (in inches to 0.01 inch)?



Answers

1a. when x=5, y=2 feet b. when y=0 x=0 or 25 so 25 feet c. vertex: when x=12.5, y=3.125 feet

2a. t=minutes since they came that day C(t)=number of tickets they have b. $C(t) = 12t + 490$

c. 12 tickets per minute d. C(0)=490 so 490 tickets

e. C(90)=1570; 90 minutes after arriving they have 1570 tickets f. C(121.67)=1950 so 121.67 mins

3a. $995(1.079)^4 = 1348.7$ million barrels b. $995 = x(1.079)^{25}$ *so* $x = 148.7$ million barrels

c. $1800 = 995(1.079)^x$ using logs x=7.8 yrs so 1937-1938 d. $1800 = 995(x)^{19}$ *so* $x = 1.0317$ → 3.17%

4. w=width $w(w+6) = (w+3)(w+2)+8$ so w=14 → 14x20

5. $12000(0.88)^x = 6000$ so x=5.42 years (using logarithms)

6. w=width $w(2w) = 8 + (2w + 2(2w))$ so w=4 (w=-1 makes no sense) → 4-by-8

7a. $h(0) = 400$ *feet* b. $h(3) = 496$; 496 feet after 3 seconds c. vertex! $t = 2.5$ *and* $h(2.5) = 500$

d. $h(t) = 0$ use quad-formula or calc-zero and get 8.09 seconds (negative answer makes no sense)

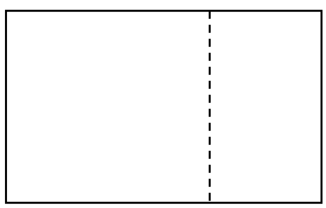
e. time when hgt is 450 feet, so $h(t) = 450$ so t=0.73 seconds or 4.27 seconds

8a. $1300 = 450(x)^{12}$ so 9.24% b. $1300(1.0924)^5 = \$2022.62$ c. 9.46 years from now

9a. $C(t) = -23t + 329$ b. -23 pieces per day c. $C(0) = 329$ pieces d. 14.3 days after filling

10. $\frac{x}{1} = \frac{1}{x-1}$ so $x = \frac{1+\sqrt{5}}{2}$ (can't be negative so only this)

this is the Golden Ratio (phi)

1

1 x-1

11. $(2x+6)^2 = 36 + 18$ so $x = \frac{-6 + 3\sqrt{6}}{2}$

12. If you smooshed the blue parts together into one rectangle, it would be $(3-x)(2-x)$.

So $(3-x)(2-x) = 4$ & x=$\frac{5 \pm \sqrt{17}}{2}$ feet

only lower one makes sense so $\frac{5-\sqrt{17}}{2}$ or 0.438 feet (about 5.26 inches)